PROCEEDINGS

OF THE

ROYAL SOCIETY OF EDINBURGH.

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THE ROYAL SOCIETY OF EDINBURGH.

REGULATIONS REGARDING THE PUBLICATION OF PAPERS IN THE PROCEEDINGS AND TRANSACTIONS OF THE SOCIETY.

The Council beg to direct the attention of authors of communications to the Society to the following Regulations, which have been drawn up in order to accelerate the publication of the Proceedings and Transactions, and to utilise as widely and as fairly as possible the funds which the Society devotes to the publication of Scientific and Literary Researches.

1. Manuscript of Papers.—As soon as any paper has been passed for publication, either in its original or in any altered form, and has been made ready for publication by the author, it is sent to the printer.

2. Method of Publication.—As soon as the final revise of a Transactions paper has been returned, or as soon as the sheet in which the last part of a Proceedings paper appears is ready for press, a certain number of separate copies or reprints, in covers bearing the title of the paper and the name of the author, are printed off and placed on sale. The date of such separate publication will be printed on each paper.

3. Additions to a Paper after it has been finally handed in for publication, if accepted by the Council, will be treated and dated as separate communications, and may, or may not, be printed immediately after the original paper.

4. Brief Abstracts of Transactions Papers will be published in the Proceedings, provided they are sent along with the original paper.

5. Special Discussion of Papers accepted for Publication.—Where a paper has been accepted for publication, the Council may, with the consent of the author, select this paper for Special Discussion. In the case of such papers advanced proofs will be sent to the members of the Society desiring copies, and copies will be supplied to the author for distribution. A paper selected for Special Discussion will be marked with an asterisk (*) and placed first on the Billet for the day of reading. Any following papers for that day may be adjourned or held as read if the discussion prevents their being read.

6. Communications not submitted for Publication, such as Demonstrations of Experiments, Statement of Scientific Problems, etc., may be received by the Council, and may also be selected for Special Discussion. The Council does not undertake to publish any notice of such communications in the Proceedings or Transactions of the Society.

[*Continued on next page.*

PROCEEDINGS

OF THE

ROYAL SOCIETY OF EDINBURGH.

VOL. XXXIX. 1918–19.

I.—The Endowment of Scientific and Industrial Research. By John Horne, LL.D., F.R.S.

(MS. received November 5, 1918. Read November 4, 1918.)

We begin another session in the fifth year of this unparalleled war with the certainty that freedom, justice, and the rights of small nationalities, for which the Allies have so bravely fought, will ultimately triumph. The administration of the affairs of the Society under conditions so abnormal has caused considerable anxiety. The great increase in the cost of paper and printing compelled the Council last session to limit the expenditure on the *Transactions* and *Proceedings*. By adhering rigidly to certain resolutions which they laid down, they have enabled the Treasurer to present a more favourable financial report than was expected. The Council reluctantly declined some papers for publication for the sole reason that they did not wish to augment the Society's liabilities.

I take the liberty of again directing the attention of the Fellows to the necessity of presenting papers in the briefest and clearest form, and of restricting the number of illustrations in accordance with the notice inserted by the Secretary in the billets. It is also essential that authors who are eligible for Carnegie Trust grants should apply for grants from the Carnegie Trust to cover the cost of illustrations.

During last session a committee was appointed by the Council to consider the method of refereeing of papers and awarding of prizes. The report presented by the Committee and adopted by the Council ought, in my opinion, to be communicated to the Fellows. It runs as follows:—

"As regards the refereeing of papers, it was generally agreed that, under ordinary circumstances, when a paper is sent in from a worker in a Research Department, the head of the Department in which the work had been done should be recognised as one referee reporting on the paper. The other referee should be appointed outside the Department. If the reports differ essentially, the Council would then appoint a third referee, and would come to their final decision on the three reports sent in. Papers from investigators outside recognised Research Departments should be reported on by one or more referees as required.

"As regards the awarding of prizes, the Committee considered very carefully the suggestion that the Council 'appoint a man of distinguished eminence, preferably outside Scotland, to serve as standing referee in each main Department of Science, and, at each period of consideration of the award of prizes, forward to him the papers in his own department, and ask him to state whether any one of them is of such outstanding distinction as to merit the allocation of one of the Society's prizes.' It was felt that there were various difficulties in carrying out such a scheme satisfactorily, and one very practical difficulty would be the necessity of paying such a referee a substantial fee. Under the present financial conditions of the Society such an arrangement is obviously out of the question. The Committee accordingly did not see their way to suggest any change in the present method of appointing a committee of experts in the sciences falling within the biological or physical groups."

Various reports have recently been published that demonstrate the valuable results achieved by the endowment of research, and the necessity of increasing this endowment in order to cope with the keen international competition that will ensue when peace is declared.

The Report of the Privy Council for Scientific and Industrial Research for the year 1917–18 demands special consideration. In my brief address at the beginning of last session I gave an outline of the more important developments of this new Government Department, which is charged with administering the sum of one million pounds, covering a period of five years. The great aim of this organisation is that of co-operative industrial research having a direct bearing on the industries of the country. The report just issued shows that great progress has been made in establishing industrial research associations of manufacturers under the Companies Acts, working without the distribution of profits and limited by a normal guarantee. The effort to include some representation of labour in the councils of these research associations has been successful. At a conference held at the Ministry of Labour it was arranged that the Department would

consult the Joint Industrial Council of an industry before establishing a research association for that industry. Upwards of thirty industries are now engaged in the preliminary stage of the work. In the case of widely distributed industries efforts are being made to establish single research associations for the whole kingdom, with local branches dealing with problems relating to particular localities. The advantages of such organised methods are obvious.

Special reference is made to two recent additions to these combinations. One is the British Scientific Instrument Research Association, which was founded through the efforts of the Optical Industry, and has been established on broad enough lines to include all scientific-instrument makers. This group, and especially the optical instrument trade, is regarded as a Key industry, and, on account of its importance, has received exceptional financial support. The Department has guaranteed a sum of £40,000 during the first five years, to be expended in accordance with a scheme of research to be approved each year. The other is the Iron Manufacturers' Research Association, which has resolved to investigate the problems relating to this industry. They have subscribed the necessary funds and ask for no financial assistance from the Government. They have resolved that the results of the researches shall be freely available to each firm, and that "all existing knowledge, trade secrets, and procedures shall be pooled for the common good." It is hoped that other combinations may follow this example, and that most of the research associations will ultimately become independent of financial aid from the State.

The National Physical Laboratory has been transferred to this new Department, which, in future, will be responsible for its maintenance. The experimental research station in connection with the Fuel Research Board is now being erected; when completed, the experiments connected with the problem of the replacement of raw coal by manufactured fuel will be carried out on a comprehensive scale. Brief descriptions are given of the researches connected with food, mine rescue apparatus, the mining of tin and tungsten in Cornwall, timber, and building materials. Special allusion is made to the action of the Home Secretary in requesting the Department to investigate the problems of Industrial Fatigue and questions closely cognate with them. This communication has resulted in the appointment of the Industrial Fatigue Research Board, with Professor Sherrington as chairman. The reference given to it is in the following terms: "To consider and investigate the relations of the hours of labour and of other conditions of employment, including methods of work to the production of industrial fatigue, having regard both to industrial efficiency and to the

preservation of health among the workers." The Department has throughout maintained that an improvement of conditions and efficiency of labour is essential for the effective organisation of British Industry.

A brief review is given of the development of industrial research in our Overseas Dominions, including Canada, Australia, New Zealand, South Africa, and India, and also in the United States of America. The most striking feature in this review is the reference to the recent executive order of the President of the United States which shows his determination to place the organisation of scientific research on a permanent basis. The National Research Council, which was established in 1916, under the auspices of the National Academy of Sciences, before the entry of America into the war, is to be perpetuated. Some of its duties are comprehensive and far-reaching as may be judged from the following selection:—

(1) In general to stimulate research in the mathematical, physical, and biological sciences, and in the application of these sciences to engineering, agriculture, medicine, and other useful arts, with the object of increasing knowledge, of strengthening the national defence, and of contributing in other ways to the public welfare.

(2) To survey the larger possibilities of science, to formulate comprehensive projects of research, and to develop effective means of utilising the scientific and technical resources of the country for dealing with these projects.

(3) To promote co-operation in research, at home and abroad, in order to secure concentration of effort, minimise duplication, and stimulate progress; but in all co-operative undertakings to give encouragement to individual initiative as fundamentally important to the advancement of science.

The section of the report of the Advisory Council dealing with grants to students and research workers demands special notice. The aim of the Department in making these grants is "to select a body of leaders in scientific research, who by choosing and training students in its methods, and by gathering round themselves bodies of competent investigators, will supply the needs of the nation in the future." In making these awards the Department is largely guided by the head of the department who recommends the student on the ground of his having given promise of becoming a competent research worker. During the academic year 1917–18 grants in pure and applied science were made to fifty-eight students, research assistants, and research workers. The expenditure under this head is increasing, and the Department reports that the work done is satisfactory.

At the same time the Advisory Council point out the great danger of an inadequate supply of trained research workers. The language is so strong that I prefer to quote the passage:—

"We make no apology for calling attention again to the grave situation which would arise after the war from the demands for trained research workers if no adequate supply should then be forthcoming. The outlook to-day is at least as serious as it was when we made our first report. We have encouraging evidence on all hands that there is constantly increasing realisation of the need for organised research in connection with industry, but the movement for the formation of research associations will receive a serious set-back if the supply of trained researchers cannot be expanded in proportion to the increasing demands, and there is considerable danger of this happening. We therefore regard the expenditure on grants to students and other persons engaged in research as being an essential part of our organisation of the national resources for the application of science to industry.

"We must, however, again emphasise the partial and insufficient effects of anything which we can do to increase the supply of trained workers. . . . The responsibility for recruiting the army of men and women we need must lie on the education authorities of the country. They are entrusted with the provision and adequate maintenance of those institutions of higher learning which train the students. Unless access for all well-qualified students to our universities and higher technical institutions is made easier than it is at present, and unless they are generously supplied with the large funds which are necessary for their work, our efforts will be foredoomed to failure."

The committee appointed to inquire into the position of Natural Science in the Educational System of Great Britain, with Sir J. J. Thomson as chairman, devoted a separate section of their report to the discussion of this question. They recognise its gravity and urgency. They point out that the schemes of reconstruction which have been prepared require a large number of trained workers, but the supply is wholly inadequate. They quote a statement of Sir George Beilby that from one-fifth to one-half of the 100 millions sterling which represents the national bill for raw coal is being wastefully expended and might be saved; and that the saving can only be effected by the co-operation of a large body of trained fuel experts to carry out the necessary research work, and to introduce and supervise improved methods in all works where fuel is consumed in large quantities.

In order to meet this demand the committee recommend radical changes

in the educational system of the country whereby the number of pupils in secondary schools and the number of university-trained students would be increased. They indicate clearly the steps that ought to be taken to induce able boys to continue at secondary schools from the age of 16 to 18, and thereafter enter the universities or technical colleges. They recommend adequate remuneration in industrial posts for students who have had four or five years' scientific training; maintenance allowances for secondary school pupils who have passed a test examination; entrance scholarships at the universities sufficient to cover the cost of education. Finally, they point out that if the universities are to discharge their responsibilities towards the science students who are coming, and to maintain their position as homes of scientific learning and research, they must receive a measure of financial support much more considerable than any they have received hitherto. This report is a severe but just exposure of the defects of the present educational system of this country, of the lack of appreciation of the value of higher education by the great body of the people, of the apathy of the Government and the moneyed classes in providing adequate endowments for research. *With the aid of the Allies it has taken this country more than four years to defeat the Central Powers in war; a generation may pass before this country recovers lost ground in her educational domain.*

After having read these reports, it occurred to me to ascertain if the scheme of the Carnegie Trust for the encouragement of post-graduate study and research in the Scottish universities furnishes trained research workers with the qualifications required by the Advisory Council. I have had an opportunity of reading the reports published by the Trust extending over a period from 1903 to 1913, and I have looked at the List of Publications, not yet published, covering the period from 1913 to 1918. It appears that the fellows and scholars in chemistry outnumber the total fellows and scholars in all the other branches of the mathematical and physical sciences. Sir James Dobbie suggests that this may be accounted for to some extent by the fact that the comparatively fresh field of physical chemistry offers certain attractions to students who formerly would have devoted themselves to purely physical research. It may also be partly accounted for on the ground that this branch of study is a stepping-stone to industrial posts. The examination of the reports further shows that a larger number of Carnegie Trust fellows and scholars in chemistry belong to St Andrews University than to any other educational centre in Scotland. On inquiry, I find that several factors have been instrumental in achieving this result:—(1) a well-equipped and well-endowed research laboratory;

(2) a limited number of students; (3) the prominence given to research in organic chemistry; (4) the provision of posts for research workers. Professor Irvine, the present occupant of the chair, has supplied me with information on these points.

The laboratories, built and equipped at a cost of £12,000, are so commodious that the part devoted to research work is separate from the teaching laboratories. The private endowment, amounting to about £7500, was founded by the late Professor. It has secured complete freedom of action to the head of the department. It has never been necessary to apply to the University Court for help. When an expensive research is contemplated it can be begun without delay. I understand that this endowment is on the point of being largely increased by means of a private bequest. The limited number of students is also an advantage, as more time can be spent in personal supervision of the advanced workers. By this method, students who seem to be capable of undertaking research work are discovered, and, in the event of their deciding to follow this line, special preparation for it is enforced. Professor Irvine considers that they have been fortunate in selecting the chemistry of sugars in that department, as it furnishes a large and consistent scheme of research, readily divisible into sections, each of which is within the compass of an individual worker. For the pure organic chemist there are plenty of constitutional and synthetical problems, while for the physical chemist researches are available in which exact determinations such as conductivities are required. The man with a biological bent can also find an outlet in studying the natural sources of sugar compounds and the action of ferments on the products. Another feature of that department concerns the future of the research students. Close touch is kept with manufacturing firms requiring research chemists, and a list of the workers, with their special qualifications, is forwarded each year to firms likely to require such men. Even before the war the workers leaving the laboratory obtained suitable research posts.

These details are given to show how a science department can furnish trained research workers, provided it is well equipped and well endowed, with a staff sufficient to cope with the limited number of students, and with a leader who is bent on establishing a school of research, and, at the same time, strives to pass on the workers to industrial posts.

The science departments of the larger Scottish universities do furnish trained research workers, but they labour under considerable disadvantages. In many cases the laboratory accommodation is inadequate, the class-rooms are crowded, the staffs are overworked and in some instances underpaid. In the report of the Sub-Committee on Research

presented to the General Council of Edinburgh University in May 1917, it was stated that new buildings are required for the departments of Chemistry, Zoology, Geology, Physiology, Pathology, Bacteriology, Geography, and Experimental Psychology, and that in several of these departments no provision for research exists at present. This position is far from satisfactory. It is further stated that in far too many cases the whole time of an assistant is occupied by teaching routine, and that little or no time is left for original research. This is a grave injustice to the assistant, for promotion is usually determined in these days not only by the teaching power of the applicant but by the value of his original contributions to the science in which he labours.

Our Scottish universities and technical colleges are progressive institutions, but unless they are more generously supported by the State and by private endowments they cannot possibly keep pace with the demands which are now made upon them. It seems to me that, in order to meet this emergency, the Carnegie Trustees ought seriously to consider whether the sums set apart for the development of research and scientific departments might be largely increased.

Before closing I wish to refer briefly to the final report of the Coal Conservation Committee recently issued. It embodies in the appendices the reports of the various sub-committees appointed by that body, viz. the Power Generation and Transmission Sub-Committee, the Geological Sub-Committee, the Mining Sub-Committee, and the Carbonisation Sub-Committee. The Prime Minister has acted wisely in publishing these documents, because they contain recommendations relating to an industry on which the future prosperity of the country largely depends.

The Geological Sub-Committee had to consider "whether with a view to maintaining our industrial and commercial position it is desirable that any steps should be taken in the near future, and, if so, what steps, to secure the development of new coalfields or extensions of coalfields already worked." Valuable evidence bearing on these questions is embodied in the report. In addition, the sub-committee advise an extension of the powers of the Geological Survey in connection with the mapping of the coalfields. In particular they recommend that it should be made compulsory to give notice of the making of any borehole, shaft, or other sinking which is expected to reach, or does reach, a depth of 100 feet; that free access to boreholes, shafts, or other sinkings while in progress, and to all cores and journals of boreholes, should be permitted to the Geological Survey at all times, and that records of the strata passed through be preserved by the Geological Survey; that these records should

be treated as strictly confidential, if desired by the interested parties, for a period not exceeding ten years after their deposition.

I hope that legislation will ere long give effect to the recommendations of the sub-committee, and thus enable the Geological Survey to make the mapping still more accurate, and to preserve complete records of all future boring operations in the coalfields.

(*Issued separately January* 16, 1919.)

II.—Notices of Fellows, Honorary and Ordinary, recently deceased.

[*Contributed by* Mrs Bonavia-Hunt.]

Henry George Bonavia-Hunt, Mus.D., was born in Malta under somewhat romantic circumstances. His father, who as private secretary to the Bishop of Jerusalem had joined the Bishop in Palestine, was plunged into the deepest grief by the death of his wife and infant son. He was returning to England in a very broken condition, but became so ill that it was thought expedient to land him at Malta. Here he was kindly received and cared for by a certain Dr Bonavia, whose young daughter nursed the stricken Englishman back to convalescence through a long and tedious illness. The inevitable followed. The lonely young widower married his devoted nurse, and later returned to England with their first child—the subject of this memoir.

The Bonavia family, originally Roman, had long been settled in Malta, and had given many priests to the Roman communion. Dr Bonavia, however, had been converted to the Protestant faith, to the grief and annoyance of the rest of the family. The infant son of William and Marietta Hunt was surreptitiously baptised in the Roman Church. At an early age he was placed in the care of his paternal grandparents, who were rigid nonconformists. To them he always said he owed his profound knowledge of the Scriptures. As a young man he made a careful study of the doctrines and history of the various religious bodies, with the result that he finally found his place in the Anglican Church, to which he was most sincerely devoted. He was an English Churchman by absolute conviction.

From his Italian ancestors he had inherited a passionate love of music and poetry; from his grandfather, Dr Bonavia, the strong religious instinct which had enabled that fine old man to face much contumely and loss on behalf of his religious convictions, and from his English ancestors a shrewd business faculty which balanced the romantic and artistic strain in his temperament. The struggle between two conflicting sides of his nature was going on during the greater part of his life, but an extraordinarily strong sense of duty made him abandon the most cherished ambitions when he felt called upon to do so in some more paramount interest. In early life he gave himself up to poetry; but when he realised that he could not

support himself as a poet, he turned his attention to journalism and became a successful editor. Music, for which he had a passionate love, was also a very tempting lure. He gave much time, chiefly at night at the end of days spent in his editorial office, to a close study of the different branches of the art, and eventually took his Bachelor's degree at Oxford and the Doctor's degree at Dublin. Finally he more or less gave up his ambitions both in literature and music, to devote himself to the laborious toil of a parish priest, to which he felt strongly called.

A side interest which developed into one of the strongest interests of his life was the cause of education. He began working for the better education and status of the musical profession, founding Trinity College, London, with the idea of affording facilities for an Arts education in conjunction with the musical studies, which as a rule absorbed all the time and attention of musical students. Among the details of administration which he thought out and inaugurated was the scheme of local examinations, now almost universally adopted by the great teaching bodies. His *Concise History of Music*, for many years the accepted text-book for students, was written entirely at night, at a period when his days were filled with strenuous literary work, in addition to his studies as warden of Trinity College, London—an office which, though honorary, so far as emoluments were concerned, was discharged with rigorous fidelity, to the best interests of the college and of the musical profession generally.

Just as his wide musical interests brought him into touch with many great musicians, so in his editorial capacity he made friends among the most distinguished literary men and women of his day. Very diverse were the views and interests represented; on the one side such intensely religious minds as Tennyson, Thomas Guthrie, Norman Macleod, Sarah Tytler, Jean Ingelow, Katharine Tynan, etc.; while in another group such men as Huxley, Darwin, Spencer, Clifford, and Romanes, who were profoundly influencing the thoughtful younger minds of that period. It was probably the fact of their growing influence that decided him to take the definite step of offering himself for holy orders, that he might have a better opportunity of defending the vital truths that were being so ruthlessly assailed, and turning to the best account his undoubted gift of oratory. There were formidable obstacles in the case of a young man, already the father of a small family, who was struggling along on the meagre income of a sub-editor. With characteristic tenacity of will and purpose these were finally overcome. He graduated at Christ Church, Oxford, and in due course was licensed to the royal parish of Esher, where he was chosen by the rector to preach before the Duke and Duchess of

Albany on their stay at Claremont immediately after their marriage. While curate at St James's, Piccadilly, he preached before many distinguished men—Gladstone among the number. Gladstone was an attentive listener. His habit of fixing a piercing eye upon the preacher was disconcerting. But though a keen critic he was a kindly one, and his approval of the young preacher was expressed in terms which led to extravagant hopes of preferment in the minds of friends. The only step in this direction was the suggestion of a royal chaplaincy, but when it came Dr Hunt had already pledged himself to the forlorn cause of a London church which had fallen to the lowest depths of decay, a building falling to pieces, a congregation of about twenty persons, and funds in a state of bankruptcy. About this time he took a step which had long been in his mind. In Maltese families it is no uncommon practice for the eldest son to add his mother's family name to his own patronymic. Dr Hunt resolved to perpetuate the Bonavia tradition in his own family by linking the names as a surname.

He had now entered upon the most strenuous years of his extraordinarily active life. Trinity College, London, under his wardenship was rapidly expanding into an extensive organisation with ramifications all over the Empire. As chairman of the School Board for Willesden he was taking an active part in the education of one of the biggest London centres, and in this connection he founded the Kilburn Grammar School to fill the crying need for a secondary school in this densely populated neighbourhood. He was also editing the *Quiver*, *Cassell's Magazine*, and *Little Folks*, which last magazine he started and made a great success; and was restoring and filling his church and bringing it into a prominent and honourable position in the religious life of the place. His preaching was vigorous, animated, and original, with a wealth of illustration and felicity of phrase which never failed him.

As a Freemason he passed through all the grades of the craft, and inaugurated a lodge in connection with Trinity College. Musical composition served him as a recreation from sterner duties. He composed much church music, among which were some very beautiful hymn tunes and "services." At different periods of his life various branches of learning and research presented an irresistible appeal. For some years he held the post of Lecturer in Musical History to the University of London, which post he only resigned when he left London to take charge of the large and important country parish of Burgess Hill, near Brighton. He was elected to fellowships of various learned societies, but music and literature held him most firmly, and the honour that he had most coveted, and always

most highly prized, was his Fellowship of the Royal Society of Edinburgh, to which he was elected in 1886 in connection with his research in musical history. He died on September 27, 1917.

William Caldwell Crawford was born in Glasgow on December 2, 1842. He was educated at the Collegiate School, Glasgow, and graduated Master of Arts in Glasgow University, where he distinguished himself as a prizeman under both Professor Edward Caird and Lord Kelvin. He subsequently studied in Berlin with Helmholz and Glan, and also spent some time at Heidelberg and Jena studying physics and chemistry, but finally attended many courses at the Sorbonne. He was a good linguist in German and French, and was particularly interested in zoology, botany, and microscopy, and was an enthusiastic member of the Edinburgh Field Naturalists' Club.

He was elected a Fellow of the Royal Society of Edinburgh in 1887, and died on June 21, 1918.

[*Contributed by* Lord Salvesen.]

Alexander Smith Kinnear, first Baron Kinnear, was born November 3, 1833. After studying at the Universities of Glasgow and Edinburgh he became in 1856 a member of the Faculty of Advocates. He acquired a large practice at the Scottish bar, especially as a senior counsel; and by 1878 he had attained such a recognised position that he was retained as one of the counsel for the liquidators in the series of litigations arising out of the liquidation of the City of Glasgow Bank. In 1881 he was elected Dean of Faculty, and in the following year was appointed a Lord of Council and Session. In that capacity he served until 1913, when he resigned his seat; but during the next three years he took a prominent part in the disposal of appeals in the House of Lords, especially in connection with Scottish cases. As a practising lawyer he was especially distinguished for his mastery of legal principle and his facility of expression in exposition, and was unrivalled in his knowledge of the Scottish system of land rights. As a judge he proved himself one of the most eminent of his day, which was rich in great lawyers; and many of his judgments are masterly expositions of the law on the subject to which they relate. While acting as a judge of the First Division he rendered notable public service by acting as Chairman of the Universities Commission under the Universities Act from 1889 to 1897; and also as a member of the Church Commission of 1905, which was appointed to regulate the distribution of the assets of the Free Church of Scotland between the

majority who adhered to the union with the United Presbyterian Church and the minority who declined to follow them into the union. His public services were recognised by his elevation to the peerage in 1897 under the title of Baron Kinnear. Before his appointment to the bench, Lord Kinnear was a Liberal in politics, but his tastes were more scholarly than political, and he took little part in political controversies. He became a Fellow of the Royal Society of Edinburgh in 1883, and remained on the roll until his death, which took place on December 20, 1917.

Edmund Albert Letts, Ph.D., F.I.C., F.C.S., was born at Sydenham, Kent, on August 27, 1852. He was educated at Bishop Stortford School, King's College, London, and the Universities of Vienna and Berlin. In 1872 he became chief assistant to Professor Crum Brown, University of Edinburgh, and four years later was appointed Professor of Chemistry, University College, Bristol. In 1879 he succeeded Thomas Andrews as Professor of Chemistry in Queen's College, Belfast—a position which failing health compelled him to resign in 1917. From 1878 he communicated to this Society a series of papers on Organic Chemistry, his most important contribution being on Benzyl Phosphines and their Derivatives (vol. xxxv, *Trans. R.S.E.*, 1889), for which he was awarded the Keith Prize. During his thirty-seven years' tenure of the Chair of Chemistry at Belfast he devoted his attention to the question of the pollution of rivers, estuaries, and tidal waters. He was recognised as one of the authorities on this question, and, at the request of the Royal Commission on Sewage Disposal, Professor Letts along with Dr W. E. Adeney made an extensive survey of important British estuaries, and the results of their inquiry were published in 1908 as an Appendix to the Fifth Report of the Commission. The relation of the marine alga *Ulva latissima* to the nitrogen content of the water in which it grows occupied Professor Letts's attention, and he was planning a full discussion of this question up to a few weeks before his death.

He was elected a Fellow of the Royal Society of Edinburgh in 1874, and died on February 19, 1918, as the result of a cycling accident in the Isle of Wight.

Kenneth John Mackenzie, M.A., was born on January 30, 1869, and was educated at the University of Edinburgh, and trained as a teacher in the Moray House Training College. He also studied advanced organic chemistry at Heriot Watt College, and was for five years the private research assistant to the Professor of Organic Chemistry. He was joint

author of two papers published by the London Chemical Society, viz. "A Contribution to our Knowledge of Oxonium Compounds," published in 1910, and "Arylidene-dimethylpyrone and its Salts," published in 1914. After filling various posts as a teacher in Edinburgh and Leith, he became in 1899 Principal Lecturer in English Literature and Philology in Leith Technical College. In 1901 he was appointed First Master and Principal Teacher of Science in Leith Academy Higher Grade School, and in 1907 Master of Method in the Junior Student Centre in Leith.

He was elected a Fellow of the Royal Society of Edinburgh in 1911, and died after an operation in a nursing home on May 27, 1918.

[*Contributed by* Professor M'Intosh.]

James Ramsay Tosh was born in Dundee on November 2, 1872, and was the son of one of the curators of the Public Museum, Dundee. He was educated at Donaldson Street School and the Harris Academy, Dundee. He entered the University of St Andrews in October 1889, and qualified for both the M.A. and B.Sc. degrees. In zoology he especially distinguished himself, gaining high honours in both the systematic and practical classes. His college career was especially adapted for training as a science teacher, a profession for which he seems to have had a natural aptitude. His interest in zoology led to his obtaining the Woodall Prize. For some time he worked in the Old Marine Laboratory, familiarising himself with the fauna of the beach, and becoming expert in dredging and in the use of the tow-nets—surface, mid-water, and bottom.

His first paper, "On the Rate of Growth of certain Marine Fishes," viz. the lump-sucker, sea-scorpion, armed bull-head, and Montagu's snake, was published in the Twelfth Report of the Fishery Board. He also for the same Report identified the pelagic ova, larvæ, and young of fishes collected in the cruises of the *Garland* and the *Dalhousie*, as well as some on the East Coast and on board H.M.S. *Jackal* in May, thus familiarising himself with fishery work, which he also extended by frequent voyages on board the trawlers from Dundee. Moreover, he was a skilful photographer and an accurate and neat draughtsman—see, for example, his drawings of the abnormal edible crabs which he described for the *Annals of Natural History*. When appointed to a post in Berwick-on-Tweed, advantage was taken of his ability and interest in the subject by Mr Archer, then Inspector of Salmon Fisheries in Scotland, who arranged for his carrying out a series of researches on the length, weight, sexual differences, and other points in the salmon of the Tweed, which were subsequently contrasted with similar observations by Dr Hock of Helder. Whilst thus engaged

he collected many parasites of the salmon, and so furnished the materials for a communication to the *Annals of Natural History* on this subject. By and by a naturalist was required for the Queensland Government in connection with the pearling industry, and Dr Tosh was appointed to the post. His field of operations lay in the rich pearling grounds off Thursday Island, where he laboured to extend our knowledge of the pearl-shells, their life-history and economic features. Whilst thus engaged he missed no opportunity of adding to the University Museum of St Andrews, and his fine collection of pearl-oysters, the invertebrate fauna of Thursday Island, and the spirit-preparations of *Ceratodus* from the Burnet and Mary rivers show his success. He returned to this country in 1905, and was appointed Assistant Professor and Lecturer in Zoology in the University of St Andrews, a post which he held for nine years, when he again visited Australia in connection with the pearling industry, returning in 1915 to promote a syndicate for its extensive treatment on a scientific basis. He also made successful experiments, with the aid of Mr Bagot, in polishing the Queensland pearl-oysters, by a special process—which he no doubt would have largely made use of in his proposed syndicate. He was, however, required on active service in Mesopotamia, chiefly in connection with the Ambulance Corps, and there he fell a victim to "heat-stroke" at the comparatively early age of 45.

Dr Tosh was a popular and successful teacher both in school and college, and he spared no pains to give practical as well as systematic instruction. He was an adept in section-making and in all the modern technique, and his students excelled in this respect. The excursions he made with them on board ship, or to marine laboratories, will long be remembered, were it only for the information gained from their genial leader.

He was elected a Fellow of the Royal Society of Edinburgh in 1911, and died in 1917.

Walter G. B. Dickinson, F.R.C.V.S., was born at Boston, Lincs., on April 22, 1858. He was educated at Boston Grammar School, and studied at the Ecole Vétérinaire Alfort, Paris, and at the New Veterinary College and Surgeons' Hall, Edinburgh. He was prize essayist and gold medallist in 1881, and thereafter succeeded to his father's business. He was a man of rare judgment, and a most skilful operator. He held several public appointments, including that of Veterinary Inspector to the Holland County Council under the Contagious (Animals) Diseases Act, and was veterinary adviser to the Royal, Alliance, and Yorkshire Insurance Com-

panies. He was a past President and Vice-President of the Lincs. Veterinary Medical Society, and the author of several professional works. His keenness and enthusiasm as a Territorial officer are well known. He joined the Boston battery as Veterinary Lieutenant in 1902, and was promoted Captain in 1904, and gazetted Major a few years later. As a Veterinary Major in the Territorial forces Mr Dickinson gave valuable help to the Government in the inspection and purchase of horses for military purposes. It was on August 6, 1914, while returning from work of this kind, that he was seized with sudden faintness and expired almost immediately.

He was elected a Fellow of our Society in 1904.

(*Issued separately May* 9, 1919.)

III.—Researches in Optical Activity. Part I: The Temperature-Rotation Curves for the Tartrates at Low Temperatures.

By **T. S. Patterson**, D.Sc., Ph.D., Waltonian Lecturer and Lecturer in Organic Chemistry, University of Glasgow; and **K. L. Moudgill**, B.Sc., late Robert Donaldson Scholar, University of Glasgow. *Communicated by* Professor Andrew Gray, F.R.S.

(MS. received August 15, 1918. Read November 4, 1918.)

But a few years ago the diverse phenomena of optical activity, such as the changes of rotation which occur with alteration of temperature, of colour of light, of solvent, or of concentration in a solvent, appeared, in spite of the great accumulation of relative data, to be practically independent of each other, and gave very little hope of satisfactory generalisation. Quite recently, however, the possibility of bringing into one scheme all these different branches of the subject has become more than a mere aspiration, and the progress which has been made by several investigators reveals clearly the existence of a deep-seated and far-reaching regularity underlying the remarkable sensitiveness to external conditions of the phenomena in question.

Some previous work of one of the present authors may be summed up as follows:—

1. The T–R * curves for a set of homologous active compounds, such as the simple tartrates, exhibit maxima which, for the molecular rotation, occur mostly at temperatures between 140° and 220°. The slight change of constitution, therefore, in passing from one to another, appears to cause a comparatively slight shift in the position of the maximum, the importance of the maximum lying, of course, in the fact that it is a singular point which can be recognised, and which, presumably, represents some corresponding or analogous condition in the substances, the behaviour of which is represented by the different curves (*J.C.S.*, 1913, **103**, 148).

2. When these esters are dissolved in inactive media the position of this maximum is, generally speaking, again shifted, some solvents displacing it to a higher temperature and a lower rotation value, others moving it towards a lower temperature and a higher rotation value both in very varying degree (*J.C.S.*, 1908, **93**, 1844 *et seqq.*).

3. In the region of ordinary temperatures the T–R curve for ethyl

* Temperature-Rotation.

ditrichloracetyltartrate exhibits a minimum, exactly the opposite of what had been found for the simple ester (*J.C.S.*, 1912, **101**, 378; 1913, **103**, 152). But the curves for these two substances must surely have something in common, and, taking what has been said above into account, it appears probable that, just as the change of constitution in passing from ethyl tartrate to *iso*butyl tartrate causes a slight change in appearance of the T–R graph, and the change from the homogeneous condition to solution in nitrobenzene causes one much greater, so the considerable change of constitution in passing from ethyl tartrate to ethyl ditrichloracetyltartrate causes a great shift in the position of the maximum, in fact, removes it from the region of ordinary temperatures altogether, bringing into view a minimum which would occur in the ethyl tartrate curve at temperatures at which the ester has not yet been investigated. This induction is strongly supported by the fact that a number of T–R curves are known in which a point of inflection occurs in addition to a maximum (or minimum), for example in those for certain carbinols (Pickard and Kenyon, *J.C.S.*, 1912, **101**, 623), or else in curves obviously tending towards a maximum (or minimum), as is the case for ethyl tartrate in water (Patterson, *J.C.S.*, 1904, **85**, 1129), and for homogeneous ethyl di-*o*-nitrobenzoyltartrate (Frankland and Harger, *J.C.S.*, 1904, **85**, 1571). In these, therefore, we have direct evidence of the connection between the maximum and the minimum.

4. Thus by piecing together evidence of various kinds—the behaviour of the homogeneous ester up to the highest temperatures possible; the behaviour of homologous esters or derived esters; the behaviour of these substances in different solvents—it seems reasonable to conclude that a fundamental form of T–R curve is common to all the tartrates, and that different tartrates at ordinary temperatures exhibit different regions of this fundamental curve, thereby appearing at first sight to have no connection with each other. This may be stated in the two following propositions: (*a*) the T–R curve of an active substance over a wide range of temperature would probably show several maximum and minimum values; (*b*) the influence of change of constitution or the effect of a solvent is, apparently, to displace not merely the maximum, but the whole T–R curve in one direction or the other, as the case may be (*J.C.S.*, 1913, **103**, 158 *et seqq.*; 1916, **109**, 1140–1142).

5. These conclusions, it may be noted, were arrived at by the study of data obtained by using only one colour of light. They are strengthened and justified when data for other colours are taken into consideration. The extended curves for light of different refrangibilities are similar in

form, but the changes in rotation produced by alteration of temperature become greater as the refrangibility of the light increases (*J.C.S.*, 1913, **103**, 165; 1916, **109**, 1145, 1147).

6. In certain cases the T–R curves for different colours of light intersect, but no instance appears to be known in which the intersections occur all at one point. On the contrary, as for ethyl tartrate (*J.C.S.*, 1916, **109**, 1145), the various pairs of curves intersect over a considerable range of temperature. This gives rise to what is known as *anomalous* rotation-dispersion, although it is quite probable that this behaviour will prove to be the rule rather than the exception.

7. It has been shown that this region of intersection shifts about as the result of change of solvent (*J.C.S.*, 1916, **109**, 1147, 1155), change of concentration (*J.C.S.*, 1913, **103**, 167), or of change of constitution (*ibid.*, 165, 166), in much the same manner as does the maximum rotation. Thus, since both the maximum and the region of intersection are displaced in a similar manner, it seems that the curves are probably displaced as a whole.

8. By piecing together, as before, evidence obtained in various ways, it has been shown that intersection of the T–R curves does not necessarily take place on each side of a maximum or minimum. The general behaviour so far as it has hitherto been examined, is represented by those parts of the graphs in fig. 1 marked *a b c d e f g h i k l m n*. Thus in the case of ethyl tartrate below the ordinary temperature, the rotation values are numerically small and the rotation-dispersion is negative—that is, the absolute values of the rotation diminish as the refrangibility of the light increases, at least in the visible spectrum. With rise of temperature the rotation values increase in such a manner that in the neighbourhood of *e f* intersection takes place, the rotation-dispersion becoming visibly anomalous. On further increase of temperature the curves reach maximum values at slightly different temperatures (region *g h*), after which they fall, and passing through points of inflection but without intersection, reach minimum values likewise at somewhat different temperatures (region *k l m*), subsequently rising again, also without intersection, to some maximum which it has not yet been found possible to investigate (*J.C.S.*, 1916, **109**, 1153).

9. Further it is to be noticed very particularly, that although the temperature at which any two T–R curves intersect—say, for example, those for red and violet light in the case of some substance, A—may be very different from the temperature of intersection of the corresponding curves for a derivative or related compound, B, the rotation values at the intersection do not differ much. Thus, for example, in homogeneous ethyl

tartrate the curve for Hg_b cuts that for Hg_g at [M] = +25° and at a temperature of 55° (*J.C.S.*, 1916, **109**, 1145, 1148), whereas in homogeneous *iso*butyl tartrate the two corresponding curves would intersect at about [M] = +40° and at a temperature of about −10° (*ibid.*, p. 1147). The same thing applies to solutions. For *iso*butyl tartrate dissolved in acetylene tetrachloride (p = 48·15) the intersection occurs at [M] = +28·5 and a

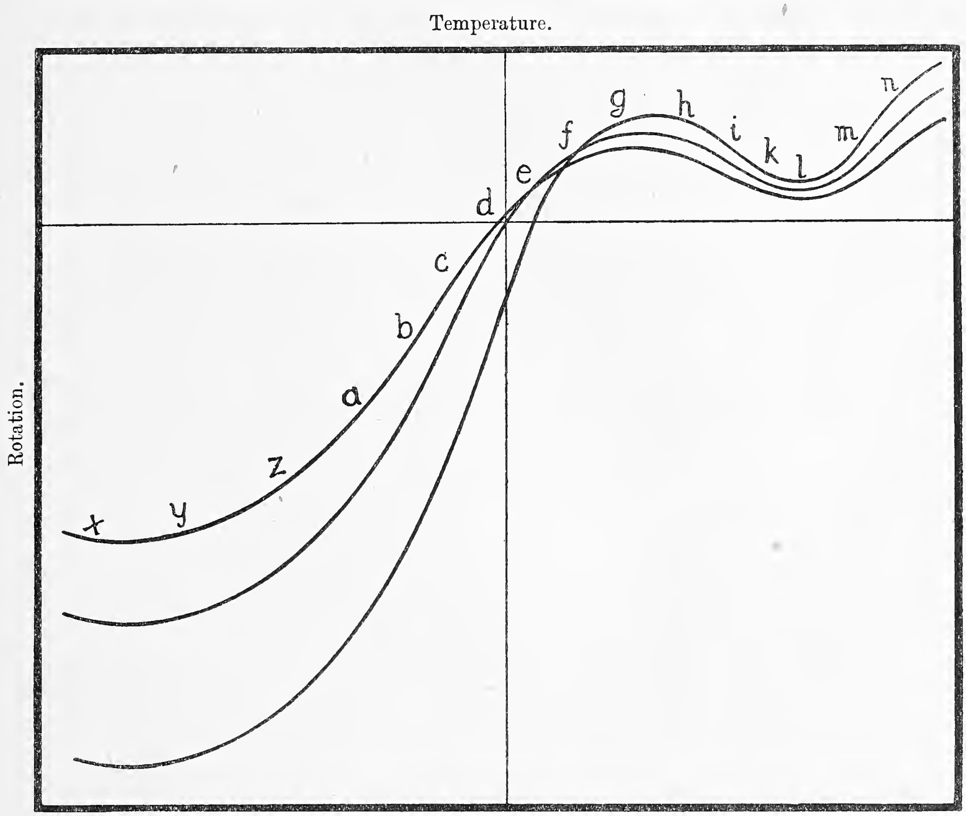

FIG. 1.—General temperature-rotation curves for tartrates.

temperature of 16°, whilst for *iso*butyl diacetyltartrate in *o*-nitrotoluene it takes place at [M] = +33° and a temperature of 104°. The regularity is probably only an approximation, but that it is a fairly close approximation is proved by the fact that so very many data, such as those collected by Pickard and Kenyon, are found to lie along the lines of Armstrong and Walker's characteristic diagram, which is only possible because of the regularity mentioned. The points of intersection of the T–R curves are also the points of intersection of the lines on the characteristic diagram (*ibid.*, p. 1157). A similar remark, however, does not apply to the maxima

or minima; the maximum for violet has apparently neither the same value nor does it occur at the same temperature in homogeneous ethyl tartrate as in ethyl tartrate in some solvent such as nitrobenzene, or in some related ester, either in the homogeneous state or in solution.

The behaviour of the esters of tartaric acid at higher temperatures having thus been examined with some care, it seemed natural next to attempt to follow the trend of these T–R curves towards low temperatures, adopting the same method as before. In the following experiments a start has been made in this direction, but before passing on to our primary purpose we thought it worth while to investigate one rather striking instance of solvent effect connected with the high-temperature end of the diagram. It has been shown by Walden (*Ber.*, 1905, **38**, 371) that cinnamic aldehyde has a very marked effect upon the rotation of ethyl tartrate and of methyl malate, the mere numerical values being greater than those recorded for either of these active substances in any other solvent, on which account we were anxious to ascertain whether the behaviour in the former case fitted in with the views previously developed or not.

A solution of ethyl tartrate in cinnamic aldehyde, p* $=9{\cdot}64$, was therefore prepared, and the rotation examined for six colours of light. Three of these, yellow, green, and violet, were obtained direct from a mercury arc lamp, whilst the other three, dark red (r_1), red (r_2), and blue, which correspond to three fainter lines in the mercury arc, were obtained from a Nernst lamp by a method which has been described elsewhere (*J.C.S.*, 1916, **109**, 1144). The wave-lengths of the light used are given on p. 30, along with the experimental data.

It will be noticed that the rotations at low temperatures are great, and that as the temperature rises the rotation diminishes, which was in accordance with expectation. The T–R curves for this solution are shown in fig. 2, and are there contrasted with those for ethyl tartrate in quinoline, $p=13{\cdot}601$ (*J.C.S.*, 1916, **109**, 1145, 1151). It will be noticed that the curves are, on the whole, somewhat similar, but that those for the cinnamic aldehyde solution lie higher on the diagram than those for the quinoline solution. In quinoline a minimum is apparent, but in cinnamic aldehyde the curves are only tending towards a minimum, which would lie at a distinctly higher temperature than could, meantime, be reached. Now it has been suggested in recent papers that the influence of a solvent should be measured, not so much by the actual value of the rotation in given circumstances, as by the effect which the solvent produces on the whole

* p = grams active substance per 100 grams of solution.

family of T–R curves. If this be so — and there is much evidence in favour of the idea—quinoline must be regarded as a more powerful solvent for this ester than is cinnamic aldehyde, since the shifting of the T–R curve is apparently much greater in the former than in the latter case, and

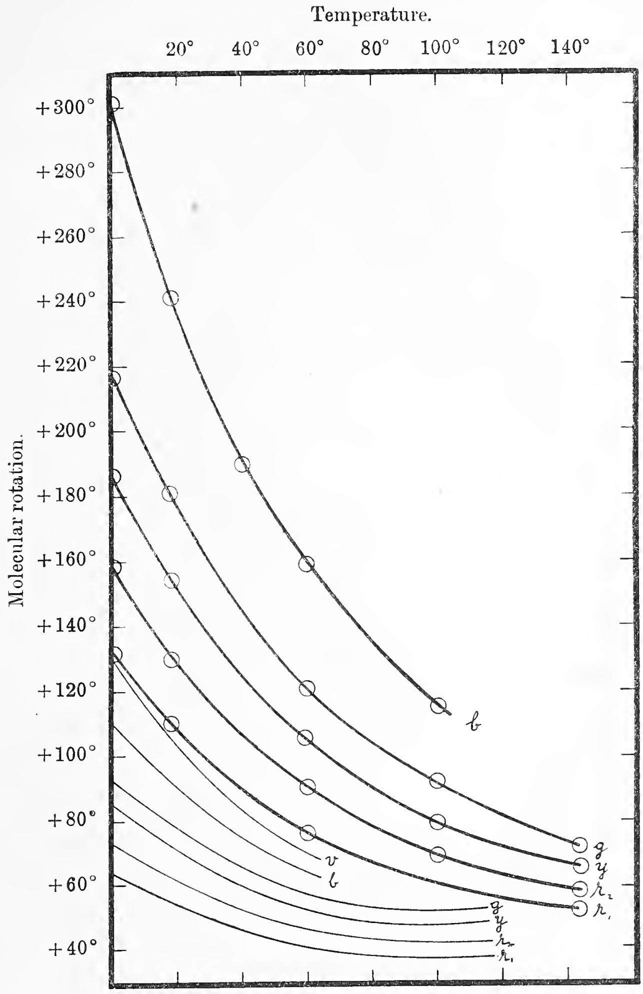

Fig. 2.—Temperature-rotation curves for ethyl tartrate in cinnamic aldehyde, $p=9{\cdot}64$ (heavy lines), and in quinoline, $p=13\ 601$ (thin lines).

it is at least quite possible, indeed it is very probable, that if the quinoline solution were examined at lower temperatures than those for which the diagram applies, the quinoline curves would rise to higher values than those for cinnamic aldehyde. This is also in agreement with the fact that whereas the C–R * curve for ethyl tartrate in quinoline shows a

* Concentration-Rotation.

pronounced maximum (*J.C.S.*, 1909, **95**, 322), Walden's results for ethyl tartrate in cinnamic aldehyde give only a very slight indication of the occurrence of this phenomenon in dilute solution.

We may now turn to the question with which this paper is primarily intended to deal, namely, the course of the T–R curves for tartrates at low temperatures. There are various ways of investigating the problem. Firstly, a substance, homogeneous or in solution, can be examined directly at low temperatures: but this procedure presents a number of difficulties such as the likelihood of solidification of the active substance or of the solvent, or the difficulty of obtaining and maintaining low temperatures. Secondly, the active compound may be dissolved in some solvent which brings into the region of ordinary temperatures the T–R curves, which, in the homogeneous substance, could only be directly observed at very low temperatures. Thus in the case of ethyl tartrate it has already been shown (*J.C.S.*, 1908, **93**, 360; Winther, *Zeitschr. phys. Chem.*, 1907, **60**, 578) that in ethylene bromide the rotation is very considerably depressed, wherefore most probably, in this particular solvent, the whole T–R curve is displaced towards the right of the diagram—that is, in the direction of high temperatures; or, to put it otherwise, ethylene bromide brings into view, at the ordinary temperature, the part of the diagram *a b c* (fig. 1), which, in homogeneous ethyl tartrate, could only be observed at decidedly lower temperatures. The behaviour of ethyl tartrate in ethylene bromide at moderate temperatures ought therefore to reveal the behaviour of the homogeneous ester at temperatures lower still. Thirdly, the behaviour of some related substance can be studied either in the homogeneous condition or in solution. Thus in passing from ethyl tartrate to *iso*butyl tartrate—a slight change of constitution—the general T–R curves are slightly displaced towards the left, towards lower temperatu es; whereas on converting ethyl tartrate into ethyl diacetyl tartrate—a considerable change of constitution,—the displacement of the general T–R curves is very much greater and is in the opposite direction.

Applying, to begin with, the second of these methods, ethyl tartrate was examined in ethylene bromide solution. Unfortunately, in attempting to get readings at as low a temperature as possible, the solution was cooled rather too far, so that the solvent crystallised out and spoiled the experiment at an early stage. From the data on p. 34, however, it will be seen that the rotation for violet is, in the absolute sense, very much lower than for the other colours examined, and that the rotation increases rapidly with rise of temperature, so that the effect of this solvent is very much the opposite of that of cinnamic aldehyde, whence it may be concluded

that the former is represented by the part *b c* of the general curves (fig. 1), whereas the latter is represented by the region *h i k*. But it is clear that the depressing influence of ethylene bromide is not sufficient to take us into a region of minimum rotation, and since ethylene bromide is either the most powerful, or almost the most powerful, depressing solvent known for this ester, we were forced to turn next to the third method of investigation, namely, that of examining the T–R curves of some other tartrate, in which there is reason to suppose that the change of constitution has brought about a considerable shifting of the family of T–R curves.

Now it has been shown by Frankland and Wharton (*J.C.S.*, 1896, **69**, 1587, also 1309) that the rotation, for yellow light, of ethyl dibenzoyltartrate exhibits a distinct minimum rotation at 60·4°; moreover, this rotation is low in value ($[\alpha]_D^{18} = -59{\cdot}36°$), and it seems clearly possible that the T–R curves for this ester at ordinary temperatures represent the behaviour of ethyl tartrate—the parent ester—at very much lower temperatures. The examination of an active substance for a single colour of light can only yield an indication in regard to this question, but an examination of the T–R curves for various colours of light makes a definite decision possible. Thus if, for example, the minimum in ethyl dibenzoyltartrate correspond to the minimum which exists in ethyl tartrate at about 180°, namely, to the region *k l m* in fig. 1, then we should expect the dispersion to be the same in both cases, namely, positive; the rotation for violet should be greater, in an absolute sense than for red. But if the minimum were such as is to be expected from a continuation of the ethyl tartrate curves towards low temperatures, or for the curves for ethyl tartrate in ethylene dibromide, the rotation for red might be expected to have a higher absolute value than the rotation for violet, the dispersion then being negative. In this way it is possible to decide to which region of the ethyl tartrate curves those for ethyl dibenzoyltartrate correspond. Instead, however, of the ethyl derivative, we prepared *iso*butyl dibenzoyltartrate, and examined its rotation over a range of temperature in the homogeneous condition as well as in solution in cinnamic aldehyde and in ethylene bromide. The results are plotted in fig. 3, and it will be observed that the form of the curves is in agreement with the second suggestion made above. The absolute value of the rotation diminishes from red to violet: the dispersion is negative. There is a distinct minimum in each of the curves, and this minimum passes slightly towards lower temperatures as the refrangibility of the light becomes greater, although the displacement is comparatively small. The rotation tends to rise somewhat rapidly as

the temperature increases; and since the appearance of the curves is not inconsistent with the possibility of intersection in the neighbourhood of zero rotation, the observed behaviour is, so far, in agreement with our

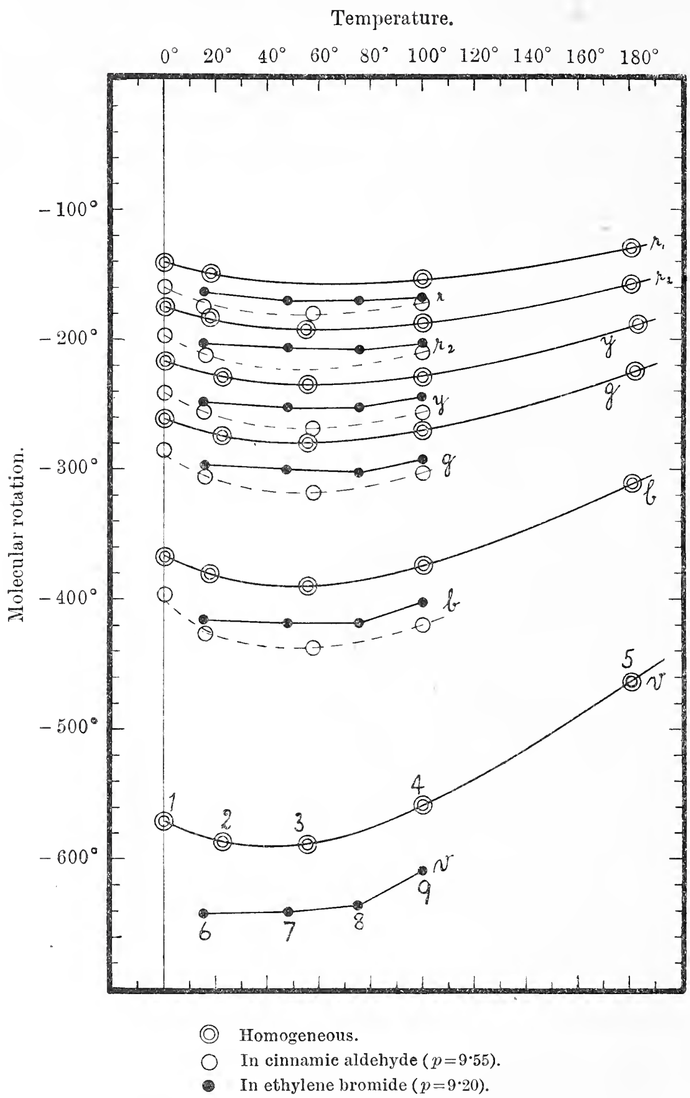

Fig. 3.—Temperature-rotation curves for *iso*butyl dibenzoyltartrate in various solvents.

views. It would obviously, however, be of interest to examine these curves towards the right-hand side of the diagram, and, in the hope of being able to shift them far enough to link them up definitely with those for ethyl tartrate or *iso*butyl tartrate, *iso*butyl dibenzoyltartrate was examined in ethylene bromide and in cinnamic aldehyde solutions,

the curves obtained being also shown in the diagram in fig. 3. But in this we were disappointed, for although the solvents altered the rotation to some extent, the change is not very great. In cinnamic aldehyde the general character of the curves appears to be almost the same as in the homogeneous ester; in ethylene bromide there appears to be some slight difference in form, which we think can hardly be due to experimental error, but to which in the meantime we can do no more than direct attention.

Since the T–R curves for homogeneous *iso*butyl tartrate and *iso*butyl dibenzoyltartrate thus show, directly, no common feature such as, for example, a similar maximum or minimum, or the intersection which is known as anomalous rotation-dispersion, their relationship is not so obvious as might otherwise be the case. That they are connected with one another is rendered clearer by the application of Armstrong and Walker's characteristic diagram (*Proc. Roy. Soc.*, 1913, [A], **88**, 392), the interpretation of which, from the point of view of the T–R curves, has already been discussed by one of us (*J.C.S.*, 1916, **109**, 1180, 1195). From what was there said it is to be expected that if the T–R curves for *iso*butyl dibenzoyltartrate and those for *iso*butyl tartrate or ethyl tartrate are related, as suggested above, then the experimental data corresponding to the region *y z a b c* of fig. 1 should lie along a common line in the characteristic diagram. The points on the curve for ethyl tartrate in ethylene bromide, which clearly belong to this region, should lie along the same line, whereas points belonging to the T–R curves for *iso*butyl dibenzoyltartrate at temperatures below that at which the minimum occurs—that is to say, in the region of *x* (fig. 1)—should not necessarily be expected to lie along the same line on the characteristic diagram. It will be seen from fig. 4 * that this is actually the case. The points marked 1, 2, and 3 (shown also in fig. 3), for *iso*butyl dibenzoyltartrate, obviously do not lie on the line A B in fig. 4. But the points 4 and 5, which are on the part of the curve corresponding to *z y* in fig. 1, do lie on the line A B, and this has almost the same direction as that joining the two points for ethyl tartrate in ethylene bromide C D, both being close to that for ethyl tartrate itself, E F. Considering that we are comparing the behaviour of ethyl tartrate

* The diagram is drawn according to the author's modification of Armstrong and Walker's method. Rotation values for Hg_g are plotted along the horizontal axis (the reference line), the differences between the rotation values for Hg_v and Hg_g, and Hg_y and Hg_g respectively being then plotted, according to sign, vertically above or below the corresponding values for Hg_g. The actual rotation value at any point on the diagram is the horizontal distance from the zero-point, plus (or minus) the vertical distance from the reference line. (See *J.C.S.*, 1916, **109**, 1181.)

with that of the dibenzoyl derivative of a homologous ester, the agreement is on the whole very satisfactory, and it may therefore be concluded that the curves in fig. 1 represent the influence of temperature-change upon the derivatives of tartaric acid generally; that on extending the

The diagram is for green and violet, the reference colour being green.

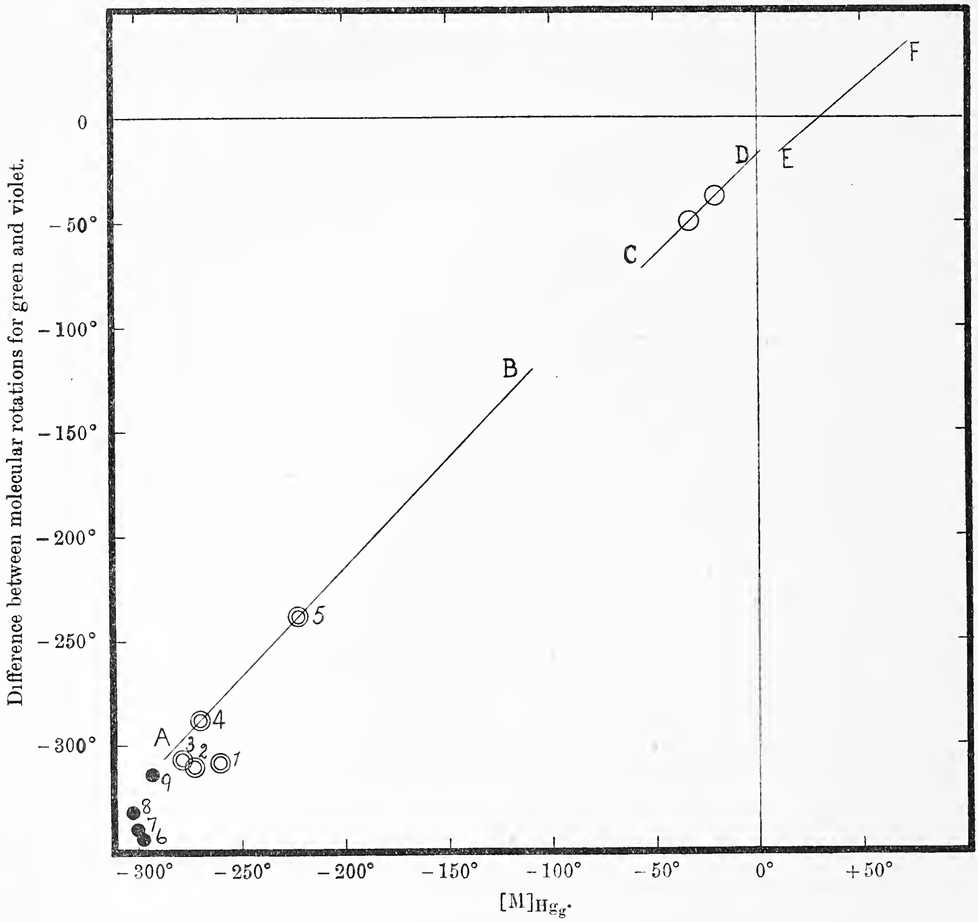

Fig. 4.—Characteristic diagram for ethyl tartrate (homogeneous and in ethylene bromide, $p = 9{\cdot}84$), and for *iso*butyl dibenzoyltartrate (homogeneous and in ethylene bromide, $p = 9{\cdot}20$).

diagram towards the left a very deep minimum is reached in the neighbourhood of low temperatures.

A slightly different way in which the connection between *iso*butyl dibenzoyltartrate and the ethyl tartrate curves might be shown, is as follows:—It is reasonable to suppose that if this relationship exists, the dispersion coefficient should remain constant with varying circumstances, and should be the same for both esters. But it has been shown in a

recent communication (*J.C.S.*, 1916, **109**, 1183) that constancy can be expected only if the dispersion coefficients be calculated from what was called a rational zero. The rational zero (taking molecular rotations) for the Hg_g and Hg_v T–R curves—*i.e.* the rotation value at their point of intersection—for homogeneous ethyl tartrate was found to be 29·4°, and when the dispersion ratio was calculated from this rotation value as zero the number 2·0989 was found (*ibid.*, 1191). The rational zero for the green and violet lines for *iso*butyl dibenzoyltartrate cannot meantime be determined directly, but, assuming the dispersion ratio to remain constant over the requisite interval, the rational zero can obviously be calculated from the rotation data at 100° and 182°, which are as follows:—

Iso*butyl Dibenzoyltartrate.*

Temperature.	100°.	182°.
$[M]_v$ . . .	−559°	−463°
$[M]_g$ . . .	−271	−223

Since the dispersion coefficients at these two temperatures should, by hypothesis, be the same, we must have

$$\frac{559^\circ + x^\circ}{271^\circ + x^\circ} = \frac{463^\circ + x^\circ}{223^\circ + x^\circ},$$

where x° is the value of the rational zero. From this equation $x^\circ = 17^\circ$, and setting this value in either of the above expressions, the dispersion coefficient 2·00 is obtained, very nearly the same as that of ethyl tartrate for these two colours of light. Again, taking the data for ethyl tartrate in ethylene bromide, we have in a similar manner

$$\frac{81^\circ + x^\circ}{32{\cdot}86^\circ + x^\circ} = \frac{58{\cdot}06^\circ + x^\circ}{20{\cdot}53^\circ + x^\circ},$$

from which $x^\circ = 18{\cdot}7^\circ$, almost the same value as found for *iso*butyl dibenzoyltartrate; and, accepting this as the rational zero, the dispersion ratio is 1·96, again much the same as in homogeneous ethyl tartrate for the same two colours of light.

[Experimental Data.

Experimental Data.

Ethyl d-*Tartrate in Cinnamic Aldehyde.*

$p = 9{\cdot}644$.

Densities :—

t . . .	0°	15° *	17·9°	37·5° *	59·7°	62° *	100° *	144°
d . . .	1·0808	1·0685	1·0662	1·0507	1·0327	1·0308	0·9997	0·9637

* Experimental.

Length of tube used, 50 mm.

	t.	α (100 mm.).	[α].	[M].
r_1, λ=6716·3 . .	0°	+6°·704	+64°·31	+132°·5
	17·9	5·488	53·37	109·9
	59·7	3·696	37·11	76·4
	100	2·816	29·21	60·1
	144	2·336	25·13	51·7
r_2, λ=6234·3 . .	0	+8·008	+76·75	+158·1
	17·9	6·474	62·06	129·7
	59·7	4·392	44·10	90·8
	100	3·270	33·91	69·9
	144	2·596	27·93	57·5
y, λ=5790·3 . .	0	+9·494	+91·08	+186·6
	17·9	7·690	74·88	154·1
	59·7	5·178	51·99	107·1
	100	3·726	38·64	79·6
	144	2·964	31·95	65·8
g, λ=5460·7 . .	0	+10·990	+105·43	+217·2
	17·9	9·040	87·91	181·1
	59·7	5·814	58·38	120·3
	100	4·314	44·74	92·2
	144	3·242	34·88	71·7
b, λ=4959·7 . .	0	+15·300	+146·79	+302·4
	17·9	12·040	117·09	241·2
	59·7	7·674	77·05	158·7
	100	5·390	55·90	115·1
	144	4·148	44·63	91·9
v, λ=4358·3 . .	0	+21·688	+208·1	+428·6
	17·9	15·538	151·1	311·3
	Unreadable at higher temperatures.			

Ethyl d-*Tartrate in Ethylene Bromide.*

$p = 9{\cdot}840$.

Densities:—

t	16°*	17·75°	39·7°	43°*	60°*	79·5°*	100°*
d	2·0158	2·0160	1·9745	1·9691	1·9365	1·9006	1·8574

* Experimental.

Length of tube used, 50 mm.

	t.	α (100 mm.).	[α].	[M].
r_1	17·7°	−1·580°	−7·96°	−16·41°
	39·7	0·850	4·37	9·01
r_2	17·7	−2·006	−10·11	−20·83
	39·7	1·112	5·72	11·79
y	17·7	−2·544	−12·82	−26·42
	39·7	1·530	7·88	16·22
g	17·7	−3·164	−15·95	−32·86
	39·7	1·936	9·96	20·53
b	17·7	−4·728	−23·83	−49·09
	39·7	3·156	16·24	33·46
v	17·7	−7·914	−39·90	−82·18
	39·7	5·476	28·19	58·06

Iso*butyl Dibenzoyl*-d-*tartrate.*

Forty-six grams of *iso*butyl tartrate and 100 grams of benzoyl chloride were heated together, under a reflux condenser, in an oil-bath at 140°–150° for about two hours, the heating then being continued at 185° until no further reaction appeared to take place. The excess of benzoyl chloride was removed under reduced pressure, and the residue fractionally distilled. The portion boiling between 200°–280° at 10-mm. pressure was dissolved in benzene and the liquid shaken with sodium carbonate solution, separated, and heated on the water-bath with animal charcoal for about six hours. After drying with fused calcium chloride, the benzene was removed and the ester fractionally distilled. It boiled in the neighbourhood of 240° under 3-mm. pressure.

Iso*butyl Dibenzoyl*-d-*tartrate* (M=470).

Densities:—

t .	0°	17·1°*	17·7°	55·40°	55·5°*	73°*	100°*	181°	182°	183°	193°
d .	1·4800	1·3360	1·1330	1·0990	1·0992	1·0867	1·0644	1·6050	1·0065	0·0075	0·9888

* Experimental.

Length of tube used, 30 mm.

	t.	α (100 mm.).	[α].	[M].
r_1 . . .	0°	−34°·55	−30°·10	−141°·4
	17·7	36·07	31·83	149·6
	100	34·84	32·81	154·2
	181	27·58	27·44	129·0
r_2 . . .	0	−42·83	−37·37	−175·3
	17·7	44·26	39·07	183·6
	55·4	45·29	41·10	193·2
	100	42·89	39·82	187·2
	181	33·55	33·38	156·9
y . . .	0	−53·12	−46·27	−217·5
	22·5	55·04	48·80	229·3
	55·4	55·02	50·06	235·3
	100	51·77	48·75	229·1
	183	40·43	40·12	188·6
g . . .	0	−63·92	−55·68	−261·7
	22·5	65·77	58·31	274·0
	55·4	65·49	59·58	280·0
	100	61·27	57·69	271·2
	182	47·82	47·54	223·4
b . . .	0	−89·92	−78·32	−368·1
	17·7	91·80	81·02	380·8
	55·4	91·07	82·83	389·4
	100	84·36	79·44	373·3
	181	66·48	66·16	310·9
v . . .	0	−139·54	−121·56	−571·4
	22·5	140·70	124·72	586·2
	55·4	137·66	125·24	588·6
	100	126·30	118·93	559·0
	181	99·07	98·58	463·3

*Isobutyl Dibenzoyl-*d*-tartrate in Cinnamic Aldehyde.*

$p = 9·548$.

Densities :—

t . . .	0°	15·7°	17° *	42° *	57·5°	60° *	79° *	100° *
d . . .	1·0738	1·0617	1·0609	1·0417	1·0298	1·0276	1·0128	0·9974

* Experimental.

Length of tube used, 50 mm.

	t.	α (100 mm.).	[α].	[M].
r_1	0°	−3°·482	−33°·96	−159°·6
	15·7	3·772	37·21	174·9
	57·5	3·782	38·46	180·7
	100	3·512	36·88	173·3
r_2	0	−4·284	−41·78	−196·4
	15·7	4·582	45·20	212·4
	100	4·252	44·65	209·8
y	0	−5·262	−51·32	−241·2
	15·7	5·542	54·67	256·9
	57·5	5·628	57·23	269·0
	100	5·196	54·56	256·4
g	0	−6·230	−60·76	−285·6
	15·7	6·610	65·20	306·4
	57·5	6·658	67·70	318·5
	100	6·144	64·51	303·2
b	0	−8·666	−84·52	−397·2
	15·7	9·194	90·69	426·2
	57·5	9·174	93·29	438·4
	100	8·528	89·54	420·8
v	Unreadable.			

Isobutyl Dibenzoyl-d-*tartrate in Ethylene Bromide.*

$p = 9 \cdot 204$.

Densities :—

t	15°	47° *	47·7°	58° *	75·3°	100° *
d	2·0190	1·9584	1·9565	1·9378	1·9034	1·8561

* Experimental.

Length of tube used, 50 mm.

	t.	α (100 mm.).	[α].	[M].
r_1	15°	− 6·466°	− 34·79°	− 163·5°
	47·7	6·558	36·42	171·2
	75·3	6·384	36·45	171·3
	100	6·120	35·83	168·4
r_2	15	− 8·050	− 43·32	− 203·6
	47·7	7·936	44·07	207·1
	75·3	7·784	44·44	208·9
	100	7·406	43·35	203·7
y	15	− 9·838	− 52·94	− 248·8
	47·7	9·696	53·84	253·1
	75·3	9·454	53·97	253·6
	100	8·906	52·13	245·0
g	15	− 11·734	− 63·14	− 296·7
	47·7	11·528	64·03	300·9
	75·3	11·320	64·63	303·7
	100	10·652	62·36	293·0
b	15	− 16·468	− 88·62	− 416·4
	47·7	16·064	89·22	419·3
	75·3	15·621	89·18	419·1
	100	14·648	85·75	403·0
v	15	− 25·340	− 136·27	− 640·9
	47·7	24·582	136·53	641·6
	75·3	23·730	135·46	636·6
	100	22·130	129·58	609·0

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(*Issued separately May* 16, 1919.)

IV.—Note on the Determinant of the Primary Minors of a Special Set of $(n-1)$-by-n Arrays. By Sir Thomas Muir, LL.D.

(MS. received January 21, 1919. Read February 3, 1919.)

(1) The main specialty in the construction of the arrays in question is that the elements of each row are the coefficients of the powers of x in the expansion of a product of the form

$$(x-a)(x-b)(x-c)\ \ldots\,;$$

that is to say, the rows all belong to the type

$$1\,,\ \ \Sigma a\,,\ \Sigma ab\,,\ \ \Sigma abc\,,\ \ldots$$

The variables from which the elements of any row are formed are $n-1$ consecutive members of a series of $n(n-1)$ members, the member considered to be consecutive to the last being the first member; for example, when n is 3 and the $3(3-1)$ independent variables are

$$a\,,\,b\,,\,c\,,\,d\,,\,e\,,f\,,$$

the rows of the arrays are

$$\begin{array}{lll} 1 & a+b & ab \\ 1 & b+c & bc \\ \cdot & \cdot\ \cdot & \cdot \\ 1 & f+a & fa\,. \end{array}$$

Further, these rows in order are taken to form all the first rows of the arrays, then all the second rows, and so on. Thus in the case just referred to the arrays are

$$\left\|\begin{array}{lll} 1 & a+b & ab \\ 1 & d+e & de \end{array}\right\| \quad \left\|\begin{array}{lll} 1 & b+c & bc \\ 1 & e+f & ef \end{array}\right\| \quad \left\|\begin{array}{lll} 1 & c+d & cd \\ 1 & f+a & fa \end{array}\right\|,$$

so that the determinant proposed for consideration is then

$$\left|\begin{array}{ccc} \begin{vmatrix} 1 & a+b \\ 1 & d+e \end{vmatrix} & \begin{vmatrix} 1 & ab \\ 1 & de \end{vmatrix} & \begin{vmatrix} a+b & ab \\ d+e & de \end{vmatrix} \\ \begin{vmatrix} 1 & b+c \\ 1 & e+f \end{vmatrix} & \begin{vmatrix} 1 & bc \\ 1 & ef \end{vmatrix} & \begin{vmatrix} b+c & bc \\ e+f & ef \end{vmatrix} \\ \begin{vmatrix} 1 & c+d \\ 1 & f+a \end{vmatrix} & \begin{vmatrix} 1 & cd \\ 1 & fa \end{vmatrix} & \begin{vmatrix} c+d & cd \\ f+a & fa \end{vmatrix} \end{array}\right|.$$

The particular matter to be investigated is the circumstances under which the determinant vanishes. It is closely connected with an important question in geometry, which has recently been engaging the attention of Professor Hayashi of Sendai.

(2) Unfortunately, from the examination of this first case very little can be learned regarding the higher cases, there being no conditions attached to it at all for evanescence. We only need indicate, therefore, that multiplication of the determinant by

$$\begin{vmatrix} 1 & c & cf \\ -1 & -a & -ad \\ 1 & e & eb \end{vmatrix}$$

in column-by-column fashion gives a product which vanishes by reason of being skew and zero-axial, and that in other less compact ways a similar result can be obtained.*

(3) If we denote the $4(4-1)$ variables in the next case by

$$a\,,\, b\,,\, c\,,\, d\,,\, e\,,\, f\,,\, g\,,\, h\,,\, i\,,\, j\,,\, k\,,\, l\,,$$

the twelve rows thence constructed are

$$\begin{array}{cccc} 1 & a+b+c & ab+ac+bc & abc \\ 1 & b+c+d & bc+bd+cd & bcd \\ . & . & . & . \\ 1 & l+a+b & la+lb+ab & lab\,, \end{array}$$

the arrays are

$$\begin{Vmatrix} 1 & a+b+c & \ldots & abc \\ 1 & e+f+g & \ldots & efg \\ 1 & i+j+k & \ldots & ijk \end{Vmatrix}, \quad \begin{Vmatrix} 1 & b+c+d & \ldots & bcd \\ 1 & f+g+h & \ldots & fgh \\ 1 & j+k+l & \ldots & jkl \end{Vmatrix},$$

$$\begin{Vmatrix} 1 & c+d+e & \ldots & cde \\ 1 & g+h+i & \ldots & ghi \\ 1 & k+l+a & \ldots & kla \end{Vmatrix}, \quad \begin{Vmatrix} 1 & d+e+f & \ldots & def \\ 1 & h+i+j & \ldots & hij \\ 1 & l+a+b & \ldots & lab \end{Vmatrix},$$

and only want of space prevents the immediate visualising of the determinant, Θ say, to be dealt with. All, however, that remains to be mentally pictured is that it has for its $(r, s)^{\text{th}}$ element the minor got from the r^{th} array by deleting the s^{th} column.

(4) At the outset it is of the utmost importance to note, if for no other

* From a purely algebraical point of view the following generalisation is a more interesting result: *The determinant of the primary minors of the arrays*

$$\begin{Vmatrix} 1 & a+b & ab \\ 1 & d+e & de \end{Vmatrix}, \quad \begin{Vmatrix} 1 & b+c & bc \\ 1 & e+f & ef \end{Vmatrix}, \quad \begin{Vmatrix} 1 & c+d & cd \\ 1 & f+g & fg \end{Vmatrix}$$

is equal to

$$(a-g)\,.\,(b-d)(c-e)(d-f)(e-b)(f-c).$$

reason than the consequent overcoming of this difficulty of notation, that the determinant is of a type which is subject to condensational transformation, being changeable into a determinant of the 3rd order in which the elements are minors of the 4th order. Applying this condensation-theorem (*Messenger of Math.*, xxxv, pp. 118–121), and noting that the rows of the new four-line minors all belong to the group of twelve rows which we started with and which we may specify by their ordinal numbers

$$1, 2, 3, 4, 5, 6, 7, 8, 9, t, e, \tau,$$

we obtain for our determinant Θ the form

$$-\begin{vmatrix} 26t1 & 26t5 & 26t9 \\ 37e1 & 37e5 & 37e9 \\ 48\tau 1 & 48\tau 5 & 48\tau 9 \end{vmatrix},$$

or, if we indicate each of the twelve rows by the variables appearing in it, the form

$$\begin{vmatrix} \begin{bmatrix} a & b & c \\ b & c & d \\ f & g & h \\ j & k & l \end{bmatrix} & \begin{bmatrix} b & c & d \\ e & f & g \\ f & g & h \\ j & k & l \end{bmatrix} & \begin{bmatrix} b & c & d \\ f & g & h \\ i & j & k \\ j & k & l \end{bmatrix} \\ \begin{bmatrix} a & b & c \\ c & d & e \\ g & h & i \\ k & l & a \end{bmatrix} & \begin{bmatrix} c & d & e \\ e & f & g \\ g & h & i \\ k & l & a \end{bmatrix} & \begin{bmatrix} c & d & e \\ g & h & i \\ i & j & k \\ k & l & a \end{bmatrix} \\ \begin{bmatrix} a & b & c \\ d & e & f \\ h & i & j \\ l & a & b \end{bmatrix} & \begin{bmatrix} d & e & f \\ e & f & g \\ h & i & j \\ l & a & b \end{bmatrix} & \begin{bmatrix} d & e & f \\ h & i & j \\ i & j & k \\ l & a & b \end{bmatrix} \end{vmatrix},$$

where, for example, the element in the $(1, 1)^{\text{th}}$ place is

$$\begin{vmatrix} 1 & a+b+c & ab+ac+bc & abc \\ 1 & b+c+d & bc+bd+cd & bcd \\ 1 & f+g+h & fg+fh+gh & fgh \\ 1 & j+k+l & jk+jl+kl & jkl \end{vmatrix}.$$

(5) A knowledge of certain properties of this latter type of 4-line minor is thus a necessary preliminary to a knowledge of Θ. Those requisite for our purpose are: *If any two rows have two variables in common, the difference of the remaining two variables is a factor: if in addition one of the two repeated variables occurs in a third row, the determinant is expressible as a product of six differences: and if two*

variables be common to three rows, or one variable common to four rows, the determinant vanishes. These are easily established. The second only of them deserves a word of comment, its intrinsic interest being enhanced a little if the factors be removed in a particular order and the co-factors at each stage of the process have their affinity of form brought to light. Thus, taking the corresponding property in the case of the five-line determinant

$$\begin{bmatrix} a & b & c & d \\ b & c & d & p \\ c & d & q & r \\ d & s & t & u \\ v & w & x & y \end{bmatrix},$$

the factor $p-a$ can be removed and the first two rows left in the form

$$\begin{matrix} 1 & b+c+d & bc+bd+cd & bcd & \cdot \\ \cdot & 1 & b+c+d & bc+bd+cd & bcd \end{matrix};$$

the factors $q-b$, $r-b$ can then be removed and the first three rows left in the form

$$\begin{matrix} 1 & c+d & cd & \cdot & \cdot \\ \cdot & 1 & c+d & cd & \cdot \\ \cdot & \cdot & 1 & c+d & cd \end{matrix};$$

the factors $s-c$, $t-c$, $u-c$ can next be removed and the remaining co-factor left in the form

$$\begin{vmatrix} 1 & d & \cdot & \cdot & \cdot \\ \cdot & 1 & d & \cdot & \cdot \\ \cdot & \cdot & 1 & d & \cdot \\ \cdot & \cdot & \cdot & 1 & d \\ 1 & (vwxy)_1 & (vwxy)_2 & (vwxy)_3 & (vwxy)_4 \end{vmatrix},$$

which is equal to

$$(v-d)(w-d)(x-d)(y-d).$$

(6) We are now in a position to show in a variety of ways that Θ does not vanish identically. Probably the most interesting way is to be found in the evaluation of it for the case where

$$g\,,\,l\,,\,i \;=\; d\,,\,c\,,\,e\,,$$

for then it takes the form of the product of eighteen differences. To see this we have only to note that its $(3,1)^{\text{th}}$ element then vanishes from having two rows alike, that the $(3,2)^{\text{th}}$ element vanishes for the same reason, and that the $(2,2)^{\text{th}}$ element vanishes because it has three rows with two variables in common. Θ thus reduces to one product of three elements, namely, to

$$-\begin{bmatrix} d & e & f \\ e & h & j \\ c & a & b \\ e & j & k \end{bmatrix} \begin{bmatrix} a & b & c \\ c & d & e \\ d & h & e \\ k & c & a \end{bmatrix} \begin{bmatrix} b & c & d \\ e & f & d \\ f & d & h \\ j & k & c \end{bmatrix},$$

which by mere interchange of rows becomes

$$\begin{bmatrix} h & j & e \\ j & e & k \\ e & d & f \\ a & b & c \end{bmatrix} \cdot \begin{bmatrix} b & a & c \\ a & c & k \\ c & d & e \\ d & h & e \end{bmatrix} \cdot \begin{bmatrix} e & f & d \\ f & d & h \\ d & c & b \\ j & k & c \end{bmatrix},$$

and therefore is equal to

$$\begin{aligned} &(k-h)(d-j)(f-j)(a-e)(b-e)(c-e) \\ &.\,(k-b)(d-a)(e-a)(d-c)(h-c)(e-c) \\ &.\,(h-e)(c-f)(b-f)(j-d)(k-d)(c-d). \end{aligned}$$

(7) As regards the actual vanishing of Θ little can be learned from the original 4-line form, save in the unimportant cases where special values are assignable to as many as three of the variables: namely, such values (for example, $a, b, c = e, f, g$) as will make all the elements of a row vanish, such values (for example, $a=e=i=0$) as will make some other sufficiency of elements vanish, and such values (for example, $a, e, i = d, h, l$) as will make two rows identical.

There is one case of this kind, however, for which we must turn for help to the derived 3-line form of Θ, namely, where $j, k, l = a, b, c$. When this substitution is made it will be found that each element of the first column vanishes from having two rows identical. The same happens when a, b, c is put equal to d, e, f, and for the same reason: also a similar result when d, e, f is put equal to g, h, i, or when g, h, i is put equal to j, k, l.

(8) The cases in which special values are given to *two* variables may be summed up in the following pair of propositions:—

(α) *If any one of the variables be fixed on, and the two variables in front of it in the series be put equal in order to the two behind it, the determinant vanishes:* for example, if $c, d = f, g$, or $j, k = a, b$, then $\Theta = 0$.

(β) *If three variables be equated which are so situated in the series that the first and second are separated by two places, and the second and third by either three or four places, the determinant vanishes:* for example, if $a=d=h$ or i, then $\Theta = 0$.

For the establishment of these results little, if anything, more is

wanted than has already been used: and in dealing with each theorem, only one instance need be considered, the others following from it by reason of the fact that the determinant is invariant to the circular substitution of the full set of variables; thus, if it be shown that Θ vanishes when $c=g=l$, it follows that the same happens when $d=h=a$.

Rondebosch, S.A.,
1st *December* 1918.

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V.—Factors of Circulants. By Professor W. H. Metzler.

(MS. received December 18, 1918. Read May 5, 1919.)

1. Among the well-known theorems for the breaking up of a circulant into factors are the following:—

Theorem I.*—The circulant

$$C(a_1, a_2, \ldots, a_n) = \Pi(a_1 + a_2\theta + a_3\theta^2 + \ldots + a_n\theta^{n-1}),$$

where θ is one of the nth roots of unity.

Theorem II.†—

$$C(a_1, a_2, \ldots, a_n) = (a_1 + a_2\theta + a_3\theta^2 + \ldots + a_n\theta^{n-1})(A_1 + A_2\theta^{n-1} + A_3\theta^{n-2} + \ldots + A_n\theta),$$

where A_k is the signed complementary minor of a_k in the first row of C.

In the case where $\theta = 1$, the cofactor of $s = a_1 + a_2 + \ldots + a_n$ in C is $A_1 + A_2 + \ldots + A_n$, and has been expressed in persymmetric form by Catalan,‡ and when the order of C is odd, in symmetric form by Muir.§ A similar theorem is also true for circulants of even order. That is, the cofactor of $s.s'$, $(s' = a_1 - a_2 + a_3 - \ldots)$, in the circulant $C(a_1, a_2, \ldots, a_n)$, of order $2n+2$, can be expressed as a symmetric determinant of order n. A proof quite similar to that used by Muir for odd orders may be used to show this.

Theorem III.‖—Every circulant of order $2m$ can be expressed as the product of a circulant and a skew circulant, each of order m.

Thus,

$$C(a_1, a_2, \ldots, a_{2m}) = C(a_1 + a_{m+1}, a_2 + a_{m+2}, \ldots, a_m + a_{2m}) \times \\ C'(a_1 - a_{m+1}, a_2 - a_{m+2}, \ldots, a_m - a_{2m}).$$

Theorem IV.¶—A circulant of order $n = r.m$ can be expressed as a product of m circulants of order r, each involving an mth root of unity.

* Catalan, "Recherches sur les determinants," *Bull. de l'Acad. R. des Sci., etc., de Belgique*, 1 sér., xiii.

† Stern, "Einige Bemerkungen über eine Determinante," *Crelle's Journal*, lxxiii, pp. 374–380.

‡ Catalan, *loc. cit.*

§ Muir, "On Circulants of Odd Order," *Quarterly Jour. Math.*, xviii, pp. 261–265.

‖ Scott, "Note on a Determinant Theorem of Mr Glaisher's," *Quarterly Jour. Math.*, xvii, pp. 129–132.

¶ Torelli, "Sui determinanti circolanti," *Rendic. Accad. delle Sci. Fis. e Mat.* (Napoli), 1882, pp. 3–11.

THEOREM V.*—Corresponding to every rational factor of x^n-1 there is a rational factor of the circulant of the nth order.

Thus, corresponding to the factors $x+1$ and $x-1$, the factors of the circulant are $a_1-a_2+a_3 \ldots -(-1)^n a_n$, and $a_1+a_2+\ldots+a_n$, respectively. The factors of the circulant corresponding to binomial factors of x^n-1 can be expressed as circulants or skew circulants, while those corresponding to multinomial factors can be expressed as persymmetric determinants.

THEOREM VI.†—The circulant $C(a, a, \ldots, a, b, b, \ldots, b)$ whose elements in the first row are p a's followed by $n-p$ b's is equal to zero when p and $n-p$ are not prime to each other, and is equal to $(a-b)^{n-1}(p \cdot a+\overline{n-p} \cdot b)$ when p is prime to $n-p$.

A simple proof of this theorem depending upon the properties of the roots of unity might be given.

2. The principal object of this paper is to exhibit the rational and real factors of certain forms of circulants. This will be done, for the most part, by considering the factors $(a_1+a_2\theta+a_3\theta^2+\ldots+a_n\theta^{n-1})$ themselves, making use of the properties of the roots of unity.

3. It may be observed in the first place that every circulant can be factored into real linear and quadratic factors. For, since $\theta^k+\theta^{n-k}(k=1, 2, \ldots \overline{n-1})$ is real, the product $(a_1+a_2\theta+a_3\theta^2 \ldots +a_n\theta^{n-1})$ $(a_1+a_2\theta^{n-1}+a_3\theta^{n-2}+\ldots+a_n\theta)$ is a real quadratic expression.

It may be observed next that if, in Theorem III, $a_{m+1}=a_{m+2}=a_{m+3}=\ldots$ $=a_{2m}=0$, then

$$C(a_1, a_2, \ldots, a_m, 0, 0, \ldots 0)_{2m}=C(a_1, a_2, \ldots, a_m)C'(a_1, a_2, \ldots, a_m) \quad (1)$$

4. The circulant C, of order $n=r \cdot s$, where $a_h=a_{kr+h} \begin{Bmatrix} h=2, 3, \ldots r \\ k=1, 2, \ldots \overline{s-1} \end{Bmatrix}$, has for its value

$$C=\frac{C^r(a_1, a_{1+r}, \ \ldots \ a_{1+\overline{s-1}.r}) \cdot C(a_1+a_{1+r}+ \ \ldots \ +a_{1+\overline{s-1}.r}, sa_2, sa_3, \ldots sa_r)}{(a_1+a_{1+r}+ \ \ldots \ +a_{1+\overline{s-1}.r})} \quad (2)$$

The theorem and method of proof may be illustrated by taking the case where $n=12$. We have $n=3{\cdot}4$, or $n=2{\cdot}6$, and starting with the former we have the following relations between the elements:

$$a_2=a_5=a_8=a_{11}$$
$$a_3=a_6=a_9=a_{12}.$$

* Muir, "On the Resolution of Circulants into Rational Factors," *Proc. Roy. Soc. Edin.*, xxi, 1896, pp. 369–382.

† Muir, "A Special Circulant considered by Catalan," *Proc. Roy. Soc. Edin.*, xxiv, Part 6, 1903.

The factor

$$a_1 + a_2\theta + a_3\theta^2 + a_4\theta^3 + a_5\theta^4 + a_6\theta^5 + a_7\theta^6 + a_8\theta^7 + a_9\theta^8 + a_{10}\theta^9 + a_{11}\theta^{10} + a_{12}\theta^{11}$$

becomes

$$(a_1 + a_4\theta^3 + a_7\theta^6 + a_{10}\theta^9) + a_2(\theta + \theta^4 + \theta^7 + \theta^{10}) + a_3(\theta^2 + \theta^5 + \theta^8 + \theta^{11}),$$

which, on account of $1 + \theta^3 + \theta^6 + \theta^9 = 0$, reduces to

1.	$(a_1 + a_4\theta^3 + a_7\theta^6 + a_{10}\theta^9)$.	

The other factors follow:

2.	$(a_1 + a_4\theta^9 + a_7\theta^6 + a_{10}\theta^3)$	Corresponding to $-\theta$
3.	$(a_1 + a_4\theta^6 + a_7 + a_{10}\theta^6\ )$	,, θ^2
4.	$(a_1 + a_4 + a_7 + a_{10}\qquad) + 4a_2\theta^8 + 4a_3\theta^4$	,, $-\theta^2$
5.	$(a_1 + a_4\theta^9 + a_7\theta^6 + a_{10}\theta^3)$	,, θ^3
6.	$(a_1 + a_4\theta^3 + a_7\theta^6 + a_{10}\theta^9)$	,, $-\theta^3$
7.	$(a_1 + a_4 + a_7 + a_{10}\qquad) + 4a_2\theta^4 + 4a_3\theta^8$	,, θ^4
8.	$(a_1 + a_4\theta^6 + a_7 + a_{10}\theta^6\ )$	,, $-\theta^4$
9.	$(a_1 + a_4\theta^3 + a_7\theta^6 + a_{10}\theta^9)$	,, θ^5
10.	$(a_1 + a_4\theta^9 + a_7\theta^6 + a_{10}\theta^3)$	,, $-\theta^5$
11.	$(a_1 + a_4\theta^6 + a_7 + a_{10}\theta^6\ )$	,, θ^6
12.	$(a_1 + a_4 + a_7 + a_{10}\qquad) + 4a_2 + 4a_3$	,, $-\theta^6$

The product of the first, third, fifth factors is

$$\frac{C(a_1, a_4, a_7, a_{10})}{(a_1 + a_4 + a_4 + a_{10})}.$$

The product of the second, sixth, eleventh, as well as of the eighth, ninth, tenth, gives the same result.

The product of the fourth, seventh, twelfth gives

$$C(a_1 + a_4 + a_7 + a_{10}, 4a_2, 4a_3).$$

Therefore

$$C = \frac{C^3(a_1, a_4, a_7, a_{10}) . C(a_1 + a_4 + a_7 + a_{10}, 4a_2, 4a_3)}{(a_1 + a_4 + a_7 + a_{10})^3} \quad . \quad . \quad (2')$$

Taking $r = 4$ and $s = 3$ so that the relations are $a_2 = a_6 = a_{10}$, $a_3 = a_7 = a_{11}$, $a_4 = a_8 = a_{12}$, we have

$$C = \frac{C^4(a_1, a_5, a_9) . C(a_1 + a_5 + a_9, 3a_2, 3a_3, 3a_4)}{(a_1 + a_5 + a_9)^4} \quad . \quad . \quad . \quad (2'')$$

With $r = 2$ and $s = 6$ the relations are $a_2 = a_4 = a_6 = a_8 = a_{10} = a_{12}$, and we have

$$C = \frac{C^2(a_1, a_3, a_5, a_7, a_9, a_{11}) . C(a_1 + a_3 + a_5 + a_7 + a_9 + a_{11}, 6a)}{(a_1 + a_3 + a_5 + a_7 + a_9 + a_{11})^2} \quad . \quad (2''')$$

With $r = 6$ and $s = 2$ the relations are $a_2 = a_8$, $a_3 = a_9$, $a_4 = a_{10}$, $a_5 = a_{11}$, $a_6 = a_{12}$, and we have

$$C = \frac{C^6(a_1, a_7) . C(a_1 + a_7, 2a_2, 2a_3, 2a_4, 2a_5, 2a_6)}{(a_1 + a_7)^6}$$

$$= (a_1 - a_7)^6 . C(a_1 + a_7, 2a_2, 2a_3, 2a_4, 2a_5, 2a_6) \quad . \quad . \quad . \quad (2^{\text{IV}})$$

If in (2) we put $a_1 = a_{h.r+1}$ $(h=1, 2, \ldots, \overline{s-1})$, then

$$C = 0 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (3)$$

That is, the C of our example vanishes if in (2′) $a_1 = a_4 = a_7 = a_{10}$; in (2″) $a_1 = a_5 = a_9$; in (2‴) $a_1 = a_3 = a_5 = a_7 = a_9 = a_{11}$; in ($2^{\text{iv}}$) $a_1 = a_7$.

If in (2) $a_2 = a_3 = \ldots = a_r$, then we have

$$C = \frac{C^r(a_1, a_{1+r}, \ldots a_{1+\overline{s-1}.r})(a_1 + a_{1+r} + \ldots + a_{1+\overline{s-1}.r} - sa_2)^{r-1}(a_1 + a_{1+r} + \ldots + a_{1+\overline{s-1}.r} + \overline{r-1}.s.a_2)}{(a_1 + a_{1+r} + \ldots + a_{1+\overline{s-1}.r})^r} \quad (4)$$

If in (4) $a_2 = 0$, we have

$$C = C^r(a_1, a_{1+r}, \ldots, a_{1+\overline{s-1}.r}) \quad . \quad . \quad . \quad . \quad . \quad (5)$$

In the case where $s=2$, (4) becomes

$$C = \frac{C^r(a_1, a_{1+r})(a_1 + a_{1+r} - 2a_2)^{r-1}(a_1 + a_{1+r} + 2.\overline{r-1}.a_2)}{(a_1 + a_{1+r})^r}$$

$$= (a_1 - a_{1+r})^r(a_1 + a_{1+r} - 2a_2)^{r-1}(a_1 + a_{1+r} + 2.\overline{r-1}.a_2) \quad . \quad . \quad (4')$$

which when $a_2 = 0$ is

$$C = (a_1^2 - a_{1+r}^2)^r \quad . \quad . \quad . \quad . \quad . \quad . \quad (5')$$

The case of (2) when $s=2$ takes the form

$$C(a_1, a_2, \ldots a_r, a_{r+1}, a_2, \ldots, a_r)_{2r} = (a_1 - a_{r+1})^r . C(a_1 + a_{r+1}, 2a_2, 2a_3, \ldots 2a_r)$$

and if in this $a_2 = a_3 = a_4 = \ldots = a_r$, we have

$$C(a_1, a_2, \ldots a_2, a_{1+r}, a_2, a_2, \ldots a_2)_{2r}$$
$$= (a_1 - a_{r+1})^r(a_1 + a_{1+r} - 2a_2)^{r-1}(a_1 + a_{1+r} + 2.\overline{r-1}.a_2) \quad . \quad (6)$$

which when $a_2 = a_{1+r}$ is an example of Theorem VI.

5. The circulant C of order $2n$, (n even), where

$$\left.\begin{cases} a_{2k} = a_{2k+2} \\ a_{2k-1} = a_{2k+1} \end{cases}\right\} (k=2, 3, \ldots, \overline{n-1}),$$

except that $a_{n+1} \neq a_3$ has for value

$$C = \frac{\{(a_1 - a_{n+1})^n + (a_2 - a_4)^n\}\{(a_1 - 2a_3 + a_{n+1})^n - (a_2 - a_4)^n\}\{(a_1 + \overline{n-2}.a_3 + a_{n+1})^2 - (a_2 + \overline{n-1}.a_4)^2\}}{\{(a_1 - 2a_3 + a_{n+1})^2 - (a_2 - a_4)\}} \quad (7a)$$

The circulant C of order $2n$, (n odd), where

$$\left.\begin{cases} a_{2k} = a_{2k+2} \\ a_{2k-1} = a_{2k+1} \end{cases}\right\} (k=2, 3, \ldots, \overline{n-1}),$$

except that $a_{n+1} \neq a_4$ has for value

$$C = \frac{\{(a_1 - a_3 - a_4 + a_{n+1})^n + (a_2 - a_4)^n\}\{(a_1 - a_3 + a_4 - a_{n+1})^n - (a_2 - a_4)^n\}\{(a_1 + \overline{n-1}.a_3)^2 - (a_2 + \overline{n-2}.a_4 + a_{n+1})^2\}}{\{(a_1 - a_3)^2 - (a_2 - 2a_4 + a_{n+1})^2\}} \quad (7b)$$

The method of proof may be illustrated by using for ($7a$) the same case as was used in art. 4. The relations between the elements are $a_3 = a_5 = a_9 = a_{11}$, $a_4 = a_6 = a_8 = a_{10} = a_{12}$, and the first factor is

$$a_1 + a_2\theta + a_7\theta^6 + a_3(\theta^2 + \theta^4 + \theta^8 + \theta^{10}) + a_4(\theta^3 + \theta^5 + \theta^7 + \theta^9 + \theta^{11}),$$

which, since $\theta^6 = -1$, becomes $(a_1 - a_7) + (a_2 - a_4)\theta$, and similarly for the other factors, $\{(a_1 - a_7) - (a_2 - a_4)\theta\}$, etc.; and the product of all the factors is

$$C = \frac{\{(a_1 - a_7)^6 + (a_2 - a_4)^6\}\{(a_1 - 2a_3 + a_7)^6 - (a_2 - a_4)^6\}\{(a_1 + 4a_3 + a_7)^2 - (a_2 + 5a_4)^2\}}{\{(a_1 - 2a_3 + a_7)^2 - (a_2 - a_4)^2\}} \quad (7a')$$

For the case of $(7b)$ take $n = 7$ and the relations are $a_3 = a_5 = a_7 = a_9 = a_{11} = a_{13}, a_4 = a_6 = a_{10} = a_{12} = a_{14}$. The factors are: $\{(a_1 - a_3 + a_4 - a_8) + (a_2 - a_4)\theta\}$, $\{(a_1 - a_3 - a_4 + a_8) - (a_2 - a_4)\theta\}$, etc.

The product of all the factors gives

$$C = \frac{\{(a_1 - a_3 + a_4 - a_8)^7 - (a_2 - a_4)^7\}\{(a_1 - a_3 - a_4 + a_8)^7 + (a_2 - a_4)^7\}\{(a_1 + 6a_3)^2 - (a_2 + 5a_4 + a_8)^2\}}{(a_1 - a_3)^2 - (a_2 + a_8 - 2a_4)^2} \quad (7b')$$

If in $(7a)$ $a_{n+1} = a_3$, and in $(7b)$ $a_{n+1} = a_4$, both reduce to

$$C = \frac{\{(a_1 - a_3)^n - (a_2 - a_4)^n\}\{(a_1 - a_3)^n + (a_2 - a_4)^n\}\{(a_1 + \overline{n-1}\,.\,a_3)^2 - (a_2 + \overline{n-1}\,.\,a_4)^2\}}{(a_1 - a_3)^2 - (a_2 - a_4)^2}$$

$$= \frac{(a_1 - a_3)^{2n} - (a_2 - a_4)^{2n}}{(a_1 - a_3)^2 - (a_2 - a_4)^2}\{(a_1 + \overline{n-1}\,.\,a_3)^2 - (a_2 + \overline{n-1}\,.\,a_4)^2\} \quad . \quad . \quad . \quad (8)$$

If in (8) $a_4 = a_2$, it becomes

$$C = (a_1 - a_3)^{2n-2}\{(a_1 + \overline{n-1}\,.\,a_3)^2 - (na_2)^2\} \quad . \quad . \quad . \quad (8')$$

If in (8′) $a_3 = a_1$, then

$$C(a_1, a_2, a_1, \ldots, a_2)_{2n} = 0 \ (n > 1) \quad . \quad . \quad . \quad (8'')$$

as is obvious from the determinant itself.

If in $(7a)$ $a_4 = a_2$, it becomes

$$C = (a_1 - a_{n+1})^n(a_1 - 2a_3 - a_{n+1})^{n-2}\{(a_1 + \overline{n-2}\,.\,a_3 + a_{n+1})^2 - (na_2)^2\} \quad . \quad (7a'')$$

which, if $a_3 = a_1$, becomes

$$C = (-1)^{n-2}(a_1 - a_{n+1})^{2n-2}\{(\overline{n-1}\,.\,a_1 + a_{n+1})^2 - (na_2)^2\} \quad . \quad . \quad (7a''')$$

If in $(7b)$ $a_4 = a_2$, it becomes

$$C = \{(a_1 - a_3)^2 - (a_2 - a_{n+1})^2\}^{n-1}\{(a_1 + \overline{n-1}\,.\,a_3)^2 - (\overline{n-1}\,.\,a_2 + a_{n+1})^2\}. \quad (7b'')$$

6. The circulant of order $2n - 1$, where

$$\left\{\begin{array}{l} a_{2k} = a_{2k+2} \\ a_{2k-1} = a_{2k+1} \end{array}\right\} (k = 1, 2, \ldots, n),$$

has for value

$$C = (a_1 - a_2)^{2n}(\overline{n-1}\,.\,a_1 + n\,.\,a_2) \quad . \quad . \quad . \quad . \quad . \quad (9)$$

This may be proved in a similar manner to the others by using the properties of the roots of unity, but a very simple proof using determinants is as follows:—

Starting with the circulant $C(a_1, a_2, a_1, \ldots, a_1)$ and performing on it the following operations: $\mathrm{col}_1 - \mathrm{col}_3$, $\mathrm{col}_2 - \mathrm{col}_4$, ... $\mathrm{col}_{2n-1} - \mathrm{col}_{2n+1}$, $\mathrm{col}_{2n} - \mathrm{col}_{2n+1}$;

followed by the operation of adding all the preceding rows to the $(2n-1)$st, we have a result from which the factor $(\overline{n-1}\,.\,a_1+n\,.\,a_2)$ comes out at once, leaving a determinant whose elements along the secondary diagonal are (a_2-a_1), and all the elements to the left of it are zeros.

It will be observed from (9) and Theorem VI that a circulant of odd order $2n-1$, having as elements $(n-1)$ a's followed by n b's, has the same value whether all the b's follow all the a's or alternate with them.

7. In connection with Muir's paper of 1881, in which he shows that the cofactor of s in the circulant C of order $2n-1$ can be expressed in symmetric form, the use of these same properties of the roots of unity brings out the fact that this symmetric determinant may be factored into linear factors. The consideration of the case of a continuant of order $2n-1=9$ and therefore $n=4$ will serve to illustrate.

The factors of the circulant other than s are

$$a_1+a_2\theta^k+a_3\theta^{2k}+a_4\theta^{3k}+a_5\theta^{4k}+a_6\theta^{5k}+a_7\theta^{6k}+a_8\theta^{7k}+a_9\theta^{8k}=\alpha_k \quad (k=1,2,\ldots 8).$$

The product of the first and eighth (α_1 and α_8) gives

$$\overset{0}{\Sigma}a_1^2+\overset{0}{\Sigma}a_1a_2(\theta+\theta^8)+\overset{0}{\Sigma}a_1a_3(\theta^2+\theta^7)+\overset{0}{\Sigma}a_1a_4(\theta^3+\theta^6)+\overset{0}{\Sigma}a_1a_5(\theta^4+\theta^5)$$

or

$$\text{I.}\qquad A_1+A_2(\theta+\theta^8)+A_3(\theta^2+\theta^7)+A_4(\theta^3+\theta^6)+A_5(\theta^4+\theta^6)=\alpha_{1,8},$$

say, where

$$A_k=\overset{0}{\Sigma}a_1a_k.$$

Similarly, the other pairs, $\alpha_2\ \alpha_7$, $\alpha_3\ \alpha_6$, $\alpha_4\ \alpha_5$, give

$$\begin{array}{ll}
\text{II.} & A_1+A_2(\theta^2+\theta^7)+A_3(\theta^4+\theta^5)+A_4(\ ^3+\theta^6)+A_5(\theta\ +\theta^8)=\alpha_{2,7} \\
\text{III.} & A_1+A_2(\theta^3+\theta^6)+A_3(\theta^3+\theta^6)+A_4(2) \qquad +A_5(\theta^3+\theta^6)=\alpha_{3,6} \\
\text{IV.} & A_1+A_2(\theta^4+\theta^5)+A_3(\theta\ +\theta^8)+A_4(\theta^3+\theta^6)+A_5(\theta^2+\theta^7)=\alpha_{4,5}.
\end{array}$$

These four relations may be written as follows:—

$$\begin{aligned}
&(A_1-A_2)+(A_2-A_3)(1+\theta+\theta^8)+(A_3-A_4)(1+\theta+\theta^8+\theta^2+\theta^7) \\
&\qquad\qquad +(A_4-A_5)(1+\theta+\theta^8+\theta^2+\theta^7+\theta^3+\theta^6)=\alpha_{1,8} \\
&(A_2-A_3)(1+\theta+\theta^8+\theta^2+\theta^7)+(A_1-A_4)-(A_2-A_5)(1+\theta+\theta^8) \\
&\qquad\qquad +(A_3-A_5)(1+\theta+\theta^8+\theta^2+\theta^7+\theta^4+\theta^5)=\alpha_{2,7} \\
&-(A_3-A_4)(1+\theta^3+\theta^6+\theta^3+\theta^6+\theta^9+\theta^9)-(A_2-A_5)(1+\theta^3+\theta^6) \\
&\qquad\qquad +(A_1-A_5)+(A_2-A_4)(1+\theta^3+\theta^6+\theta^3+\theta^6)=\alpha_{3,6} \\
&-(A_4-A_5)(1+\theta+\theta^8+\theta^2+\theta^7)+(A_3-A_5)(1+\theta+\theta^8) \\
&\qquad\qquad -(A_2-A_4)(1+\theta+\theta^8+\theta^2+\theta^7+\theta^3+\theta^6)+(A_1-A_3)=\alpha_{4,5}.
\end{aligned}$$

Here we have not only the elements of the symmetric determinant but the multipliers for the columns which give the factors, and the factors themselves, which are real.

The law of formation of these symmetric determinants is, perhaps, best seen by examining the elements along the principal and parallel

diagonals, and by observing that the consecutive numbers as subscripts alternate in position from the first to the last, second, second last, etc.

From I, II, IV, we have

$$\begin{aligned}
3(A_1 - A_4) &= a_{18} + a_{27} + a_{45} \\
3(A_2\theta + A_3\theta^7 + A_5\theta^4) &= a_{18} + a_{27}\theta^3 + a_{45}\theta^6 \\
3(A_2\theta^8 + A_3\theta^2 + A_5\theta^5) &= a_{18} + a_{27}\theta^6 + a_{45}\theta^3,
\end{aligned}$$

and since $1+\theta^3+\theta^6=0$, we have

$$\begin{aligned}
C(a_{18}, a_{27}, a_{45}) &= 27(A_1 - A_4)(A_2\theta + A_3\theta^7 + A_5\theta^4)(A_2\theta^8 + A_3\theta^2 + A_5\theta^5) \\
&= 27(A_1 - A_4)(A_2^2 + A_3^2 + A_5^2 - A_2A_3 - A_2A_5 - A_3A_5).
\end{aligned}$$

If we write I, II, III, IV as follows:—

$$\begin{aligned}
(A_1 - A_4) + (A_2 - A_4)(\theta + \theta^8) + (A_3 - A_4)(\theta^2 + \theta^7) + (A_5 - A_4)(\theta^4 + \theta^5) &= a_{18} \\
(A_1 - A_4) + (A_2 - A_4)(\theta^2 + \theta^7) + (A_3 - A_4)(\theta^4 + \theta^5) + (A_5 - A_4)(\theta + \theta^8) &= a_{27} \\
(A_1 - A_4) + (A_2 - A_4)(\theta^3 + \theta^6) + (A_3 - A_4)(\theta^3 + \theta^6) + (A_5 - A_4)(\theta^3 + \theta^6) &= a_{36} \\
(A_1 - A_4) + (A_2 - A_4)(\theta^4 + \theta^5) + (A_3 - A_4)(\theta + \theta^8) + (A_5 - A_4)(\theta^2 + \theta^7) &= a_{45},
\end{aligned}$$

then multiply the first, second, and fourth by $(\theta+\theta^8)$, $(\theta^2+\theta^7)$, $(\theta^4+\theta^5)$, respectively, and add, we have

$$\begin{aligned}
6(A_2 - A_4) - 3(A_3 - A_4) - 3(A_5 - A_4) &= a_{18}(\theta + \theta^8) + a_{27}(\theta^2 + \theta^7) + a_{45}(\theta^4 + \theta^5) \\
-3(A_2 - A_4) + 6(A_3 - A_4) - 3(A_5 - A_4) &= a_{18}(\theta^2 + \theta^7) + a_{27}(\theta^4 + \theta^5) + a_{45}(\theta + \theta^8) \\
-3(A_2 - A_4) - 3(A_3 - A_4) + 6(A_5 - A_4) &= a_{18}(\theta^4 + \theta^5) + a_{27}(\theta + \theta^8) + a_{45}(\theta^2 + \theta^7).
\end{aligned}$$

The product of these gives

$$\begin{aligned}
&27(2A_2 - A_3 - A_5)(-A_2 + 2A_3 - A_5)(-A_2 - A_3 + 2A_5) \\
&\quad = a_{18}^2(6a_{27} - 3a_{45} - a_{18}) + a_{27}^2(6a_{45} - 3a_{18} - a_{27}) + a_{45}^2(6a_{18} - 3a_{27} - a_{45}) - 6a_{18}a_{27}a_{45}.
\end{aligned}$$

Syracuse University,
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VI.—The Adsorption Isotherm at Low Concentrations. By A. M. Williams, M.A., D.Sc. *Communicated by* Professor James Walker, F.R.S.

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Several formulæ have been proposed to express the adsorption equililibrium between the amount adsorbed a and the equilibrium concentration c. That most extensively employed was apparently first used by Küster,* and is based on the increasing positive adsorption of the first portion of the (c, a) curve in the case of solutions (see fig. 1). The graph of $(\log c, \log a)$ is found to be approximately a straight line with a gradient less than unity, and hence

$$\log a = \log a_0 + \frac{1}{n} \log c$$

or

$$a = a_0 c^{\frac{1}{n}}.$$

In the case of gases the pressure p usually replaces c in the formula, which is then written

$$a = a_0 p^{\frac{1}{n}}.$$

With gases like hydrogen at ordinary temperatures and other gases at high temperatures the value of the exponent appears to be unity for moderate pressures. The mean value of $\frac{1}{n}$ over the ordinary range of pressures—that is, up to atmospheric—rises with the temperature, as is shown, for example, by Travers † for the adsorption of carbon dioxide by charcoal.

Table I.

t	$\frac{1}{n}$
− 78° C.	0·13
0	·30
35°	·46
61°	·48
100	·52

* See Ostwald, *Lehrbuch der allgemeinen Chemie* (1906), II, iii, p. 253.

† *Proc. Roy. Soc.* (1906), A, 78, p. 9.

If, however, the ($\log p$, $\log a$) curve for any individual temperature be examined it will be found to be distinctly concave to the $\log p$ axis, the gradient increasing as a decreases. This is shown in the second table,

TABLE II.

$\frac{1}{n}$.

t	$p=1$ cm.	10	20
$-78°$C.	0·22	0·12	0·11
0°	·39	·37	·31

which is calculated from Travers' observations. This change in the value of $\frac{1}{n}$ is also clearly seen in the values tabulated by Titoff * and by Richardson,† and is found in practically every published case of gaseous adsorption.

Hydrogen is of the gases ordinarily examined the least adsorbed at any given temperature, and hence the gas most likely to furnish evidence as to the nature of the adsorption isotherm in the region of low values of a. As has already been mentioned, with low pressures the value of $\frac{1}{n}$ in this case appears to be unity at ordinary temperatures, or the adsorption isotherm is represented by the simple formula

$$a = a_0 p ,$$

which is the same as Henry's Law. Even with hydrogen, as the pressure increases there is a distinct fall in the value of $\frac{1}{n}$. With other gases a is small in the region of ordinarily measured pressures only when the temperature is high, and here again we find that with low pressures the value of the exponent appears to be unity. Since the value of $\frac{1}{n}$ or $\frac{\partial \log a}{\partial \log p}$ at different temperatures varies much less with a constant than with p, and at lower temperatures all values rise towards unity as a falls, it seems highly probable that at all temperatures the adsorption isotherm assumes the simple form

$$a = a_0 p$$

for small values of a. The fact that the value of the exponent appears in

* *Zeits. f. physik. Chem.* (1910), lxxiv, p. 641.

† *Journ. Amer. Chem. Soc.* (1917), xxxix, p. 1828.

some cases to *exceed* unity may be ascribed to the difficulties of measurement in the region of low pressures.

In the case of adsorption from solutions we are dealing with a double adsorption, namely, adsorption of solute and adsorption of solvent. As with gases, what is ordinarily measured is the excess of the surface concentration over the concentration outside the adsorbent; but in the case of solutions, the difference between the excess and the actual concentration in the adsorption layer may be considerable. Denoting by u and w the amount in grams of solute and solvent adsorbed per gram adsorbent when in equilibrium with a solution containing c grams solute per gram solution we have the excess of the solute per gram adsorbent given by

$$u_0 = u - w \cdot \frac{c}{1-c} \cdot$$

u_0 is directly measurable from the change in concentration of the solution on immersion of the adsorbent, and is in fact $m \cdot \frac{c_0 - c}{1-c}$, where m is the mass of solution per gram adsorbent, and c_0 the initial concentration. u_0, or an approximation to it, is usually called the amount adsorbed, and will be denoted by a. In practice it has been frequently found that in dilute solutions a is positive and increases with c. Careful examination of the ($\log c$, $\log a$) curve once more indicates that the curve is not a straight line, but is decidedly concave to the $\log c$ axis. This may be exemplified by

Table III.

c	u_0	$\frac{1}{n}$
0 0000122	0·0042	
347	73	0·52
704	90	·31
·000128	·0108	·28
305	130	·23
490	142	·18
916	155	·14
·00124	165	·21
159	171	·14

the following observations on the adsorption at 25° C. of hydrochloric acid from aqueous solution by blood charcoal. Hence if we use $a = a_0 c^{\frac{1}{n}}$ to express the adsorption curve, $\frac{1}{n}$ is again only a mean value of the logarithmic curve gradient, which steadily increases as c diminishes.

An examination of the values of the exponent $\frac{1}{n}$ tabulated for different

substances by Freundlich,* shows that of forty-four values, in only one case is the value 0·50 exceeded—namely, 0·52 with mercuric chloride and blood charcoal in water. Numerous other cases in recent work indicate that the value 0·50 is not exceeded, and this suggests $\frac{1}{2}$ as the limiting value of $\frac{1}{n}$ when $c=0$. It is, however, conceivable that at still greater dilutions than those examined the value of $\frac{1}{n}$ approaches unity, as with gases. It is further to be noted that while in the cases mentioned c may be small, the concentration in the adsorption layer is still not small. Thus, using the fact that $w=0·69$ when c is small, we have from Table III above the values of $\frac{u}{w}$ given in Table IV.

We may expect to obtain smaller values of u_0 for a given c when

Table IV.

c	u_0	$\frac{u}{w}$
0·0000122	0·0042	0·006
347	73	·011
...	...	...
·00159	·0171	·026

we increase the temperature (on analogy with gaseous adsorption), and Freundlich† gives (log c, log a) curves for aqueous acetic acid and charcoal where the gradient is 0·45 at 0° C., 0·6 at 50°, and 0·8 at 94°. Again, Georgievics and Dietl ‡ in studying the time rate of adsorption from aqueous solution of acids by wool at 20° C. give figures which indicate a gradient of 0·70 with acetic acid and 0·75 with propionic acid. Ritzel § shows that the adsorption of uranium-X by charcoal appears to obey Henry's Law, and this is confirmed by Freundlich and Kaempfer.|| In the case of adsorption of metallic salts by silica, Schmidt ¶ showed from his own and from van Bemmelen's ** results that Henry's Law held. The figures in Table V show this in the case of sodium chloride. The author's figures †† in Table VI for aqueous solutions at 25° C. and blood charcoal bear on the question. It will be noted that here u_0 and therefore $\frac{u}{w}$ are

* *Kapillarchemie* (1909), p. 150.

† *Loc. cit.*, p. 171.

‡ *Zeits. f. physik. Chem.* (1914), lxxxvii, p. 669.

§ *Ibid.* (1909), lxvii, p. 732.

|| *Ibid.* (1915), xc, p. 681.

¶ *Ibid.* (1895), xv, p. 56.

** *Jour. f. prakt. Chem.* (1881), xxiii, p. 324.

†† *Trans. Farad. Soc.* (1914), x, p. 155.

TABLE V.

c'	a	$\frac{a}{c'}$	$\frac{1}{n}$
0·157	0·286	1·82	
·190	·335	1·77	0·83
·227	·376	1·65	0·65
·278	·465	1·68	1·05
·350	·546	1·56	0·70
·447	·746	1·67	1·28
·601	·983	1·63	0·98

much smaller than in the case of hydrochloric acid, and these figures suggest that if different substances behave in the same manner for low values of u, the limiting value of the exponent $\frac{1}{n}$ is unity, as with gases.

TABLE VI.

	c	u_0	$\frac{1}{n}$	$\frac{u}{w}$
KCl . .	0·00044	0·00032		0·0009
	266	128	0·77	45
	616	230	·70	95
$MgSO_4$.	0·00043	0·00089		0·0017
	123	24	0·95	47

We will now consider the consequences of such a simple relation as $u = kc$ existing in dilute solutions. It follows that when c is small

$$u_0 = u - wc = (k - w)c,$$

which we may write

$$a = a_0 c,$$

where a_0 may be positive or negative. Near $c = 1$ we have a similar relation of the type $w = k'(1 - c)$, which leads to

$$w_0 = (k' - u)(1 - c),$$

and

$$u_0 = u - w\frac{1}{1 - c} = u - k'.$$

If w_0 is positive near $c = 1$ as u_0 is positive (suppose) near $c = 0$, it follows that near $c = 1$, u_0 is negative. On the other hand, $u_0(1 - c)$ will be positive initially and equal to zero finally. Hence the (c, u_0) and $\{c, u_0(1 - c)\}$ curves will be of the type shown in fig. 1. If u_0 and w_0 are negative when $c = 0$ and $c = 1$ respectively, the resultant curve will be the mirror image of

the first in the c axis. Such a curve has apparently been obtained by Trouton,* as also have curves entirely above the c axis, where u_0 is positive initially but w_0 negative near $c=1$, though his interpretation of the curves is not on the lines here indicated. Curves of the type shown in fig. 1 have been obtained by the author and others, as will now be discussed.

Schmidt † examined the adsorption by charcoal of acetic acid in fairly concentrated aqueous solutions. In his calculations he made the assumption that the mass of the solution remained constant even when several per

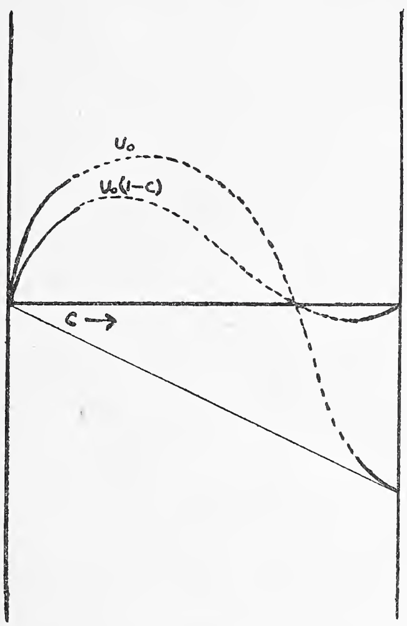

Fig. 1.

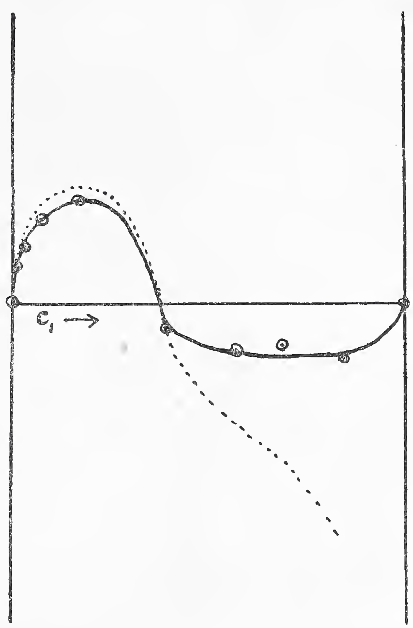

Fig. 2.

cent. as acetic acid was being positively adsorbed—apparently on analogy with the usual constant volume assumption implied in the formula

$$a=v(c_0-c).$$

He thus obtained neither u_0 nor, as he believed, u, but the value $m(c_0-c)$ corresponding to $u_0(1-c)$. The $(u_0(1-c), c)$ curve showed a maximum of $u_0(1-c)$, which Schmidt assumed to indicate a saturation of the adsorbing surface for $c=1$. Schmidt has now abandoned ‡ the formula he proposed to express his supposed results.

The author § showed that in the case in question $u_0(1-c)$ passed

* *B.A. Reports*, 1911, p. 328.
† *Zeits. f. physik. Chem.* (1910), lxxiv, p. 689.
‡ *Ibid.* (1916), xci, p. 103.
§ *Medd. f. K. Vet. Akad. Nobelinstitut* (1913), ii, 27.

through a maximum to decrease near $c=1$, but u_0 steadily increased. He was unable to prove definitely the existence of negative adsorption even in very concentrated solutions ($c=0·96$) with acetic acid and charcoal, but obtained it easily with acetic acid and silica, and subsequently * with blood charcoal and aqueous solutions of potassium chloride and magnesium sulphate.

Dora Schmidt-Walter,† as a result of the author's criticism of Schmidt's papers, tried "to find the influence of the solvent on adsorption," using charcoal and acetic acid in various solvents at great concentrations. Following Schmidt, she calculates first $u_0(1-c)$—c_2 in Table VII—and then by making successive allowances for the amount adsorbed from the

TABLE VII.

c_1	c_2	c_3
1·55	0·439	0·45
3·68	·62	·64
9·09	·90	·99
18·1	1·08	1·2
39·6	−0·225	−0·37
58·0	−0·522	−1·2
68·4	−0·497	−1·6
85·4	−0·633	−4·3

mass of the solution works back to u_0—c_3 in the table. Her observations with water, benzene, and toluene may justifiably be regarded as confirming the author's views. In Table VII are presented her observations with toluene as solvent. c_3 has been calculated by the author, using $c_3=\frac{100c_2}{100-c_1}$, where c_1 is the percentage of acetic acid in the solution. In her curves she makes the correct assumption that $c_2=0$ when $c_1=100$. Fig. 2 reproduces her toluene results. She does *not* graph (u_0, c), and by confusing the author's u_0 with u she attempts to defend Schmidt, and incidentally annexes without acknowledgment certain of the author's views. This is pointed out by Gustafson,‡ who states, "It may now be regarded as beyond doubt that the u_0 curve runs in the manner indicated by Williams." Gustafson obtained negative adsorption with phenol in alcoholic solution and generally confirmed the author's views.

Osaka,§ working with aqueous solutions of different salts and blood

* *Trans. Farad. Soc.* (1914), x, p. 155.
† *Koll. Zeits.* (1914), xiv, p. 242.
‡ *Zeits. f. physik. Chem.* (1916), xci, p. 385.
§ *Mem. Coll. Sci. Kyoto* (1915), i, p. 257.

charcoal, obtained positive adsorption followed by negative adsorption with potassium chloride, and negative adsorption with sodium chloride, potassium sulphate, and sodium sulphate. He corrects for the water adsorbed (w), but as the correction is often much greater than the amount corrected, it is not surprising from its nature that he finds the sequence of adsorption the same as the general sequence of properties of the solutions involved.

Summary.

1. It is pointed out that for very small *adsorptions* the adsorption law appears to be

$$a = a_0 c$$

for both gases and solutions.

2. The general form of the adsorption curve for solutions is deduced from the above conclusion and found to agree with the results of different observers.

The author wishes to thank Professor James Walker, F.R.S., for his assistance in the presentation of this paper.

(*Issued separately May* 26, 1919.)

VII.—The Origin of Anticyclones and Depressions. By Lieut. John Logie, R.A.F., M.A., B.Sc., F.R.A.S. *Communicated by* the late Capt. G. W. Jones, R.A.F.

(MS. received October 17, 1918. Read December 2, 1918.)

I WISH to preface this statement of my views with an acknowledgment of my indebtedness to the works of Major Gold, Mr W. H. Dines, Sir Napier Shaw, Captain Cave, and Mr Lempfert, and more especially to the suggestions of Captain G. W. Jones, R.A.F. Captain Jones was the first person to draw my attention to the fact that changes in the upper wind frequently precede changes in the lower wind. In addition, he has often expressed to me his conviction that the entire theory of the winds requires to be rewritten; that convection and surface friction are of much less importance than is generally supposed, and that we have no reason to regard the variations of the upper winds as less, either in magnitude or abruptness, than those of the lower winds.

The most essential features of my theory, however—namely, the views that the chief cause of depressions and anticyclones is to be sought in the phenomenon of *radiation*; that the first effect of a local decrease of temperature is a diminution of pressure at all higher levels; that cyclones are caused by *cooling*, and anticyclones by the *heating*, of air; that clouds may cause winds, as well as winds causing clouds; and that the motion of the air is most accelerated when depressions are dying out or anticyclones intensifying,—are, I believe, entirely original, as is also the detailed mathematical treatment of the subject.

In this theory two postulates are assumed. As they are the only portion of it not deducible by strict mathematics from well-tested dynamical principles, I state them here for immediate criticism.

(A) When two portions of air, differing slightly in density, are adjacent and in the same level, they tend to mingle and so destroy the difference of density.

(B) When changes of pressure occur at any level in an extensive layer of air, the surrounding air does not "immediately rush in," but only slowly intrudes into the region of diminished pressure.

Regarding the first of these, little comment is necessary. It is the natural consequence of gaseous diffusion aided by the small turbulent motions of the air. Regarding the second, the following points may be noted. (1) It is now many years since Major (then Mr) Gold showed that

the horizontal pressure gradients in the air are effective not so much in increasing its speed as in altering its direction of motion. (2) Mr Shaw and Mr Lempfert, in their *Life History of Surface Air Currents*, gave results of following up the movements of particular masses of air. They showed that these often move many thousands of miles in very complicated paths with but little change in speed, being merely deflected from their courses in passing regions of low or high pressure. (3) Such results are in agreement with the first principles of dynamics. A region of low pressure in the air (while it persists and is stationary) may be regarded as a centre of attractive force, and by the principle of the conservation of angular momentum the moving air cannot pass into it, but must move on past. In the process of its deflection the air will be accelerated in that part of its course in which it approaches the depression, for here there is a component of pressure in the direction of the motion; but as it recedes, if the intensity of the depression is unaltered, all the increase will be lost, and it will finally pass away with a speed slightly diminished on the whole by reason of the effects of friction and of the conduction of heat. (4) We have in the mean pressure of the barometer over a given area a very accurate measure of the mass of air overlying it. The measure is not perfectly accurate, since a small correction (under $\frac{1}{2}$ millibar) may be locally required at the moments of most violent vertical movements in thunderstorms, line-squalls, etc., while the weight of the air itself may be slightly varied by a vertical redistribution of mass. But the ordinary rise and fall of the barometer by several millibars cannot well be attributed to vertical currents, unless we are prepared to consider uniform upward and downward currents of several hundred miles per hour over areas exceeding that of the whole British Isles—a phenomenon not likely to have escaped the notice of aviators, who, in ascending to 20,000 ft., have passed above more than half the mass of the atmosphere. The second cause of inaccuracy, which has been quoted as giving rise to effects "of the same order as those due to differences in latitude," is likewise capable of producing only trifling variations of pressure. It would require a vertical movement equivalent to raising the whole column of atmosphere through two miles before the combined changes of gravity and centrifugality could diminish the pressure by one millibar.

When, therefore, a depression originates, the diminution of pressure must be accompanied by, and may be due to, a general withdrawal of air from the area. Thus our postulate is justified by observation: a diminution of pressure does not, in natural conditions, result in an immediate "inrush of air."

It is a fact of universal observation that the atmosphere, as regards the vertical distribution of temperature, is almost always in a more stable state than the isentropic one. Hence if the temperature of any portion of it is altered, the change in its first stages will not produce convection at all; and even considerable changes may occur without convection effects arising, provided the change of temperature is sufficiently continuous from top to bottom of the region affected to produce only a moderate change in the lapse rate.

The entropy of any mass of air is maintained by a delicate balance between the absorption, radiation, convection, and internal production of heat. Each of these factors is liable to vary independently.

The amount of heat absorbed by the mass depends upon the amount of radiation to which it is exposed and upon its coefficient of absorption. The former may be altered :—(*a*) By expansion or contraction of the mass. If this expansion is adiabatic, not only is the mass exposed to more solar radiation, but also, in consequence of the resulting diminution of temperature, it will itself radiate less actively. Thus its entropy is increased, and if it be again compressed adiabatically to its former volume it will be at higher temperature and pressure than before. (*b*) By the formation of cloud at higher levels, screening off a portion of the solar radiation; or at lower level reflecting back a greater portion of the solar rays than would be returned from the earth through the intervening air. (*c*) By a general translation of the mass bringing it over terrestrial surfaces of a different reflecting or radiating power. (*d*) By a movement in latitude. A poleward movement of a mass of air in the lower reaches of the atmosphere must, in the winter hemisphere, be accompanied by a loss of heat. For not only has the solar radiation to pass through a greater thickness of absorbent air before reaching the mass under consideration, but it has also less time to operate owing to the longer night; while, in addition, the underlying terrestrial surface will generally be cooler as the poleward movement continues. In the summer hemisphere, at any rate in the higher latitudes, it is probable that the increased duration of exposure overwhelms the other two effects and that the entropy increases with poleward movement; but sufficiently exact data for the calculation are lacking.

The coefficient of absorption may be altered by variation of density, temperature, humidity, dustiness, or cloudiness. It should be observed that any increase in the coefficient of absorption tends automatically to diminish the loss of heat by radiation. For the heat radiated from the central portions of the mass is absorbed to a greater extent than before by the outer portions, and so the total effective rate of radiation is diminished.

Thus variations of diathermancy, like adiabatic changes of volume, have a double effect, altering both the rate of absorption and the rate of radiation in such a way that the two results co-operate in increasing or diminishing the entropy of the mass.

The rate of loss of heat by radiation is affected also by the temperature of the mass, and by adjacent clouds or other reflecting screens.

The effects of conduction between the lowest layers of the atmosphere and the underlying terrestrial surfaces have already been discussed *ad nauseam.* The result of steady secular heating from below, such as occurs in the tropics, and of steady cooling from below, as in glacial "anticyclones," will be dealt with later. Such alternate heating and cooling as occurs elsewhere is principally productive of fogs or of convection eddies; the effect on the wind systems may be neglected. As regards the effects of conduction in portions of the atmosphere at higher altitudes, they are slow, but cumulative. Aided by all the small turbulent movements of the air, from the Brownian motion to the eddies of a waterspout, conduction tends, where vertical churning is absent, to the production of isothermal conditions, and where it is present, to the production of isentropic conditions.

Considerable additions or subtractions of heat may occur within a mass of air, in virtue of the condensation or evaporation of moisture or ice. The effect of cloud in a fairly calm layer of air may be roughly compared to the effect of ice particles scattered through water. It tends to diminish the temperature changes which accompany changes of entropy. But whereas the presence of ice in water absolutely fixes the temperature (in the ideal case of perfect stirring), cloud only diminishes changes of temperature. It cannot annihilate them. For if the whole of the absorbed heat could be employed in evaporating a portion of the cloud, then this evaporation would increase the pressure of the aqueous vapour in the clouded mass. But the saturation pressure was already reached, or there would not have been cloud to begin with. Hence only part of the added heat can be used in the evaporation of the cloud, and part must go to raise the temperature of the air and so enable it to retain the additional vapour.

There are a few other sources from which heat can be produced within a given mass of air, such as viscous friction, slow chemical action, electrical discharges, and the friction of meteors. The first of these is effective in diminishing eddy motion, and produces a permanent tendency to stagnation; the others are occasional and random actions, and may be here ignored.

Where so many activities are concerned a perfect balance at all times cannot be expected. Such of the actions as are of a fluctuating, or essentially local, nature may be dismissed from consideration in view of the

slowness of the processes concerned. But in the case of (1) movement in latitude, (2) the formation of extensive cloud-screens, (3) changes of diathermancy due to the formation of thin haze, etc., we perceive causes entirely adequate to produce gradual but appreciable changes of temperature (say, variations of 10–15 degrees absolute) in extensive masses of air.

The next step is to investigate the result of such changes occurring in a layer of air possessing considerable horizontal extension (say, from 10,000 to 500,000 square miles).

The second law of dynamics gives us the well-known equation of vertical equilibrium,

$$\frac{dp}{d\mathrm{H}} = -(g+a)\rho,$$

where H is the height above some fixed level, p and ρ the pressure and density of the air at that height, g the acceleration of a freely falling body in vacuo at that height, and a the vertical acceleration of the air.

We are concerned here with slow, cumulative changes continuing through many hours or even days. In considering the equilibrium of extensive masses of air for such periods of time, the mean value of a must be regarded as negligible in comparison with g. For if we supposed it to remain even for fifteen minutes equal to $\frac{1}{100{,}000}$ of g, the result would be a vertical current exceeding 80 m.p.h. It is doubtful if such vertical speeds are achieved even in the most violent thunderstorms, and they are certainly not long maintained, as the air has considerable vertical stability owing to the low average lapse rate. Thus, a may safely be neglected in comparison with g, giving the simpler equation

$$\frac{dp}{d\mathrm{H}} = -g\rho \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

where g is almost a constant, or, more accurately, a function of H.

For moderate changes which are here dealt with, unsaturated air behaves approximately like a perfect gas. Thus the following equations may be assumed, for the first treatment:—

$$p = \kappa\rho\theta \qquad \text{(Charles's law)},$$
$$p = e^{-\frac{\mathrm{E}}{\kappa}}\theta^{\frac{\gamma}{\gamma-1}} \qquad \text{(Law of adiabatic expansion)},$$

where κ is the volume which unit mass of air would have at unit temperature and pressure if it continued to behave like a perfect gas under these conditions, θ is the temperature on the absolute scale, e is the base of "natural" logarithms, E is the excess of entropy of unit mass of air at pressure p and temperature θ over its entropy at unit

temperature and pressure (subject to the conditions mentioned above), and γ is the "adiabatic constant" for air$=1·41$.

From these two equations it is easy to derive others by means of which any of the four quantities p, ρ, θ, E can be expressed in terms of any two of the others. For convenience of reference the results are here tabulated without proof.

$$p=\kappa\rho\theta=\kappa^{\gamma}e^{\frac{(\gamma-1)\mathrm{E}}{\kappa}}\rho^{\gamma}=e^{-\frac{\mathrm{E}}{\kappa}}\theta^{\frac{\gamma}{\gamma-1}} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

$$\rho=\frac{p}{\kappa\theta}=\frac{1}{\kappa}e^{-\frac{(\gamma-1)\mathrm{E}}{\gamma\kappa}}p^{\frac{1}{\gamma}}=\frac{1}{\kappa}e^{-\frac{\mathrm{E}}{\kappa}}\theta^{\frac{1}{\gamma-1}} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (3)$$

$$\theta=\frac{p}{\kappa\rho}=e^{\frac{(\gamma-1)\mathrm{E}}{\gamma\kappa}}p^{\frac{\gamma-1}{\gamma}}=\kappa^{\gamma-1}\rho^{\gamma-1}e^{\frac{(\gamma-1)\mathrm{E}}{\kappa}} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (4)$$

$$\mathrm{E}=\left(\frac{\kappa}{\gamma-1}\right)\log_e\frac{p}{\kappa^{\gamma}\rho^{\gamma}}=\kappa\log_e\frac{\theta^{\frac{\gamma}{\gamma-1}}}{p}=\kappa\log_e\frac{\theta^{\frac{1}{\gamma-1}}}{\kappa\rho} \quad . \quad . \quad . \quad . \quad (5)$$

Substituting in equation 1 the first value of ρ from equation 3, we find

$$\frac{dp}{d\mathrm{H}}=-\frac{gp}{\kappa\theta}$$

$$\therefore \int_{\mathrm{B}}^{p}-\frac{\kappa dp}{p}=\int_{0}^{\mathrm{H}}\frac{gd\mathrm{H}}{\theta}$$

$$i.e.\ \kappa\log_e\frac{\mathrm{B}}{p}=\int_{0}^{\mathrm{H}}\frac{gd\mathrm{H}}{\theta} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (6)$$

a well-known equation.

Hence if B, the barometric pressure at the level $\mathrm{H}=0$ (ground level) is known, and if θ, like g, is given as a function of H, the pressure (and by inference the density and entropy) at every height is absolutely determined.

Now let us assume that over some area A the air between heights x and y is cooled, and over another area C the air in the same layer is heated, the temperature being unaltered at all other heights and in the surrounding undisturbed area U (see fig. 1). The temperature changes are assumed to be such that the lapse rate is not made to exceed the adiabatic rate and that no condensation occurs.

In the absence of an "immediate inrush of air" over C, and an "immediate expulsion of air" over A, the barometric pressure at level $\mathrm{H}=0$ will remain B.

For values of $\mathrm{H}<x$, $\int_{0}^{\mathrm{H}}\frac{gd\mathrm{H}}{\theta}$ is unchanged.

$\therefore$ p, ρ, and E are unchanged, both over A and over C.

For values $x<\mathrm{H}<y$, θ is increased over A and diminished over C.

U

Temperature.

Steady. Increased. Unchanged.

Pressure Increased.
Upward Movement.

A Conditions Unchanged.

Density Reduced. Density Increased.

Entropy Increased. Entropy Reduced.

U $H = x$ $H = y$

Pressure Reduced.
Downward Movement.

C Conditions Unchanged.

Density Increased. Density Reduced.

Entropy Reduced. Entropy Increased.

Temperature.

U Steady. Decreased. Unchanged.

FIG. 1.—First effects of Temperature Changes which do not make the Lapse Rate exceed the Adiabatic Lapse Rate.

$$\therefore \int_0^{\mathrm{H}} \frac{g d\mathrm{H}}{\theta} = \int_0^{x} \frac{g d\mathrm{H}}{\theta} + \int_x^{\mathrm{H}} \frac{g d\mathrm{H}}{\theta}$$

is diminished over A and increased over C.

Therefore by equation 6, over A p is increased, over C p is diminished. For values $\mathrm{H} > y$,

$$\int_0^{\mathrm{H}} \frac{g d\mathrm{H}}{\theta} = \int_0^{x} \frac{g d\mathrm{H}}{\theta} + \int_x^{y} \frac{g d\mathrm{H}}{\theta} + \int_y^{\mathrm{H}} \frac{g d\mathrm{H}}{\theta},$$

and the middle term is diminished over A, but increased over C by the change of θ, the other terms being unaffected. Hence $\int_0^{\mathrm{H}} \frac{g d\mathrm{H}}{\theta}$ is altered as

for values of H between x and y, and p is increased over A and diminished over C.

Thus anticyclonic conditions are produced in the district A at all levels above $H=x$ and cyclonic conditions over C, as indicated in fig. 1.

These pressure changes at any given level are caused by a general elevation of the air over A due to expansion in the heated portion, and by a general lowering of the air over C due to contraction in the cooled portion. That such expansion and contraction actually occurs follows at once from equation (1). For the value of p at $H=x$ is unaltered, while at all greater heights p is increased over A; hence the mean value of $-\frac{dp}{dH}$ is diminished for the range $x-y$, and this demands that the actual value must be diminished in some part at least of the range, and certainly, in the region just above $H=x$; and g is a function of H only; therefore ρ must be diminished, *i.e.* expansion has occurred.

This upward movement of the air over A (and downward movement over C) distorts the isobaric surfaces, but as the total movement is not great the tendency for the air to flow out of A and into C is not sufficient to produce rapid movement, and I shall show later that an opposite effect arises which may (and generally does) more than counterbalance this tendency.

At levels above $H=y$ in the region over A the pressure is increased, and the temperature unaltered; hence from equations 3 and 5 the density is greater and the entropy less than before.

In the region of expansion lying just above $H=x$ the pressure is increased and the density diminished, hence the entropy is increased (equation 5).

By similar reasoning it may be shown that over C there is an increase of density and a diminution of entropy in the region just above $H=x$, and a decrease of density and increase of entropy at all levels above $H=y$. All these effects are indicated in fig. 1.

So far, the changes considered have been those occurring at definite levels; it is interesting to compare with these the changes in definite portions of air. Our postulate that there has been as yet no intrusion or expulsion of air from the areas considered (*i.e.* no horizontal compression or expansion), combined with the condition that the lapse rate does not exceed the adiabatic lapse rate, enables us to recognise a definite portion of air by the condition that its pressure—due to the weight of superincumbent air—is constant (this statement neglects the effect of the small variation of g with height, see above).

Below $H=x$ the air does not move.

At all higher levels in area A the pressure is increased (see above), hence the isobaric surfaces have risen. Therefore the air has risen.

Let ρ denote the density of a portion Q of the air at height H before the change of temperature began, ρ' the density of air at that height after the change, ρ'' the density of the air Q which has now been raised to a higher level. In the region just above $H=x$, $\rho'<\rho$ (proved above), also $\rho''<\rho'$ (since the lapse rate does not exceed the adiabatic, and ρ'' refers to air at a higher level than ρ').

$$\therefore \rho''<\rho.$$

Therefore the air in this region has actually expanded. Also, from equations 4 and 5, since p is constant and ρ is diminished, both θ and E are increased.

Consider air which is above the level $H=y$. It has risen, yet the temperature has been adjusted to the new level. If the lapse rate was normal, and $H=y$ is within the troposphere, this means that the temperature θ is reduced. But the pressure is unaltered. Therefore the density is increased (equation (3)) and the entropy is reduced (equation (4)). For portions of air in the stratosphere the temperature remains unchanged or slightly rises (as also in regions of temperature inversion), hence the density is unchanged or diminished, and the entropy unchanged or increased.

Hence it appears that except in the stratosphere or in regions of temperature inversion the changes in definite portions of air are similar in kind to, though of a different magnitude from, the changes at different levels, as shown in fig. 1. It should also be observed that in general the level $H=y$ bounding the temperature change will not lie in the troposphere, since this would imply a simultaneous addition of heat below and withdrawal of heat from above. *Mutatis mutandis* the same conclusions will apply to area C, except that, as will appear later, it is more likely for a withdrawal of heat from the lower air to be associated with an addition of heat to the upper.

Instead of fixing our attention on temperature, it is interesting to consider the entropy as the varying quantity. If we suppose one layer of air to have its entropy increased, and the remaining air above and below to undergo no change, the principle of constant pressure for each portion of the air gives at once these results.

Below region of change, no alteration.

In region of increased entropy, air increases in temperature, decreases in density (equations (3) and (4)).

Above region of increased entropy, no change in definite portions of air, but pressure, temperature (except in the stratosphere), and entropy at any given level are altered, as shown in fig. 2, in consequence of vertical displacement of the air.

This probably represents an occurrence more frequent in natural conditions than that shown in fig. 1.

In virtue of these pressure changes, if the surrounding air was originally

		Air which has gained Heat.	Air Unchanged but Elevated.	Stratosphere.
A	Conditions Unchanged.	Pressure Increased.		
		Temperature Increased.		Unchanged.
		Density Decreased.	Density Increased.	
		Entropy Increased.	Entropy Decreased.	

U H = a H = b

		Air which has lost Heat.	Air Unchanged but Lowered.	Stratosphere.
C	Conditions Unchanged.	Pressure Decreased.		
		Temperature Decreased.		Unchanged.
		Density Increased.	Density Decreased.	
		Entropy Decreased.	Entropy Increased.	

Fig. 2.

at rest or in uniform motion, an anticyclonic circulation will arise over A, and a cyclonic circulation over C, at heights greater than H $=x$. For the pressure differences in horizontal planes will tend to set the air in motion outwards from A and inwards towards C. But the air over A shares in the earth's rotation. It has, therefore, an angular momentum about an axis parallel to the earth's polar axis and passing through its own centre of mass. In the process of expansion this angular momentum is unaltered, and therefore the angular velocity is diminished (since the moment of inertia increases) and will become less than that of the underlying terrestrial surface. To an

observer on the earth it will, therefore, appear as a whole to revolve relatively to the earth in an opposite direction to that of the earth's rotation, *i.e.* to circulate anticyclonically. Conversely, the contracting air over C, as it yields to the centripetal pressure, will commence to circulate cyclonically. It should be observed that the energy of the winds is, therefore, not derived exclusively from the solar heat, but in part from the earth's rotation. In every wind system part of the terrestrial kinetic energy of rotation is being converted into heat.

In general, however, the motion of the air will not be uniform. If the existing currents are readily deviated into the required type of circulation, their inertia will intensify the anticyclonic or cyclonic effect. If, however, they are opposed to the motion induced by the thermal changes (*i.e.* if the area A was originally a region of barometric depression or B a region of high pressure), the pressure which every horizontal wind exerts to the right in the northern hemisphere (or to the left in the southern hemisphere) will cause the rapid extinction (or even prevent the formation) of the barometric gradients due to the temperature changes, and the result will be merely a diminution in intensity of the depression originally existing over A, or the anticyclone over C.

The circulatory air movements tend (in virtue of the principle of side pressure quoted above) to maintain the existing inequality of pressure and delay the entrance of air into the region over C, or its exit from the region over A. They would not of themselves suffice to prevent or reverse the effect were it not for the inequalities of density, which are not instantaneously destroyed by the process of mingling. These inequalities of density give rise to a further set of effects arising from the principle that in any fluid in motion denser portions tend to move towards regions of higher pressure, and rarer portions towards regions of lower pressure. As this generalisation of the principle of the centrifuge is apparently new, I attach a formal proof.

Let A, L, M (fig. 3) represent three consecutive portions of a fluid in motion, which pass the point P with the same velocity. A, L, M are taken as spherical portions, all of the same diameter but of different densities—A being of average density, L of less than average density, and M of more than average density. They are supposed to be so near that the pressure conditions do not alter materially between the times of their successively reaching P. As A passes P it is deflected from its undisturbed course by the pressure of the surrounding fluid and follows path PQ, which we may call the "average path." L, reaching P with the same velocity, and being subject to the same pressure-forces (since it is a sphere equal in volume to A),

is deflected further from an "undisturbed path" owing to its mass being less and its acceleration therefore greater. Thus it follows a path PR, which brings it nearer to the low pressure. Similarly, M is subject to a less acceleration on account of its greater mass, and its path PS must bring it nearer to the high region. (In applying this principle to the winds it should be constantly borne in mind that the "undisturbed path" for a body held

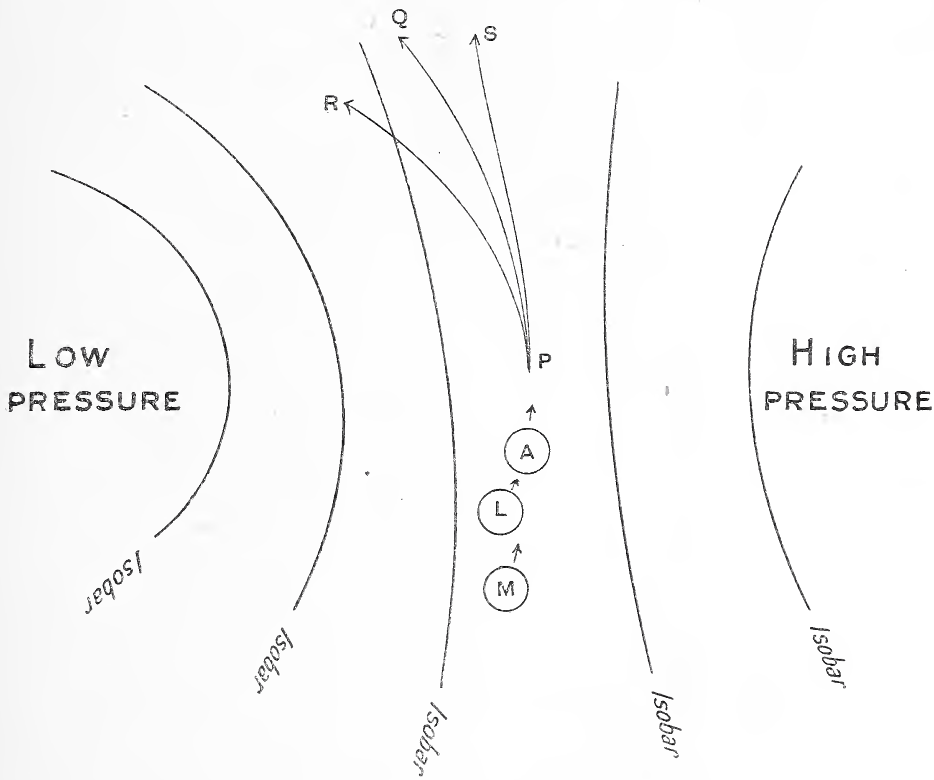

Fig. 3.

by gravitation to a horizontal surface on a rotating spheroid is not a great circle but a curve that deviates continually to the right in one hemisphere and to the left in the other.) Thus the effect of inertia in a moving fluid of varying density is to cause the denser portions to accumulate in the regions of higher pressure, and the more rarefied portions to accumulate in the regions of lower pressure. This is the principle of action of the centrifugal separators used in modern dairies. It also accounts for the relatively abrupt wind changes behind the southern portion of the trough line of a typical depression. In this region (see fig. 4) the circulatory motion is tending to bring the cold dense air of the N.E. current towards the low-

pressure side of the warmer less dense air of the S.W. current. An instability arises, the denser air tending to burst its way through the warmer air, with the result of sudden changes of direction and a squally character of the wind.

On referring to fig. 1 (or fig. 2), it at once appears that the result of the principle just stated is to accelerate the mingling of dense air with the rarefied portion over A, and retard the outward movement of the relatively

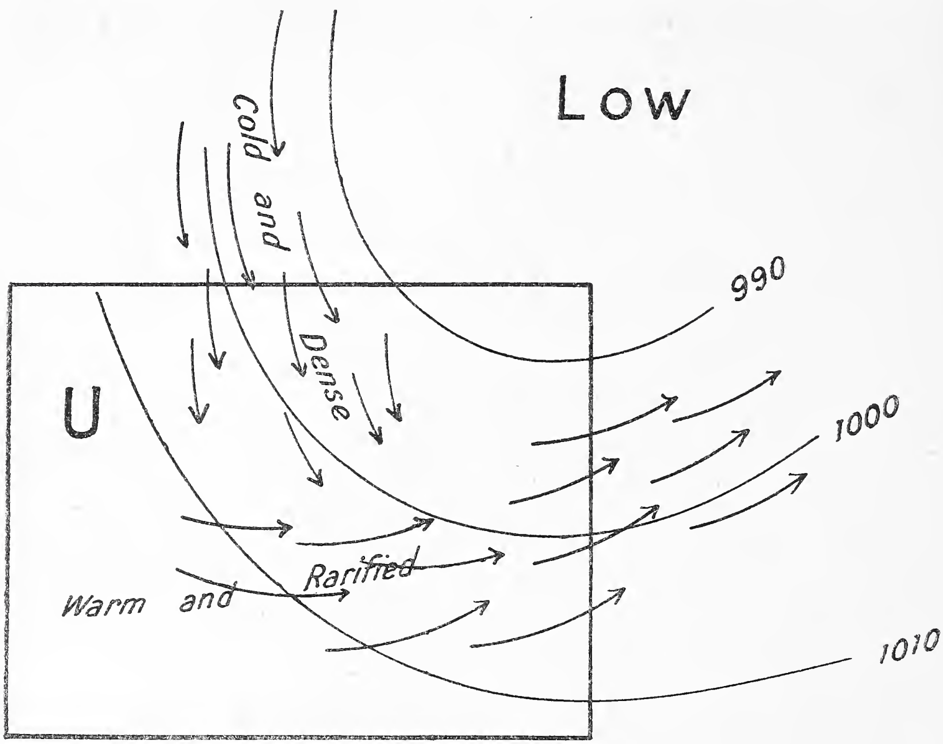

Fig. 4.

denser air above and below. Thus air accumulates over A, the surface pressure is increased, and adiabatic compression raises the temperature of the lower portion, giving rise to a typical anticyclone, as shown in figs. 5 and 6. Similarly the air is expelled from the region of reduced entropy over C more rapidly than it enters the region of increased entropy, the result being the formation of a typical depression. The changes of temperature, pressure, density, and entropy are easily inferred from equations (1)–(5) by reasoning similar to that previously employed, and may be briefly described as a diminution in intensity of the changes above the level $H=x$, and an increase in pressure temperature and density, with but slight change of entropy, at lower levels. The deviations at different levels from

the conditions at corresponding levels in the undisturbed area are as follows:—The pressure over A is increased at all levels. The temperature is also increased at all levels, the amount of this increase being a maximum

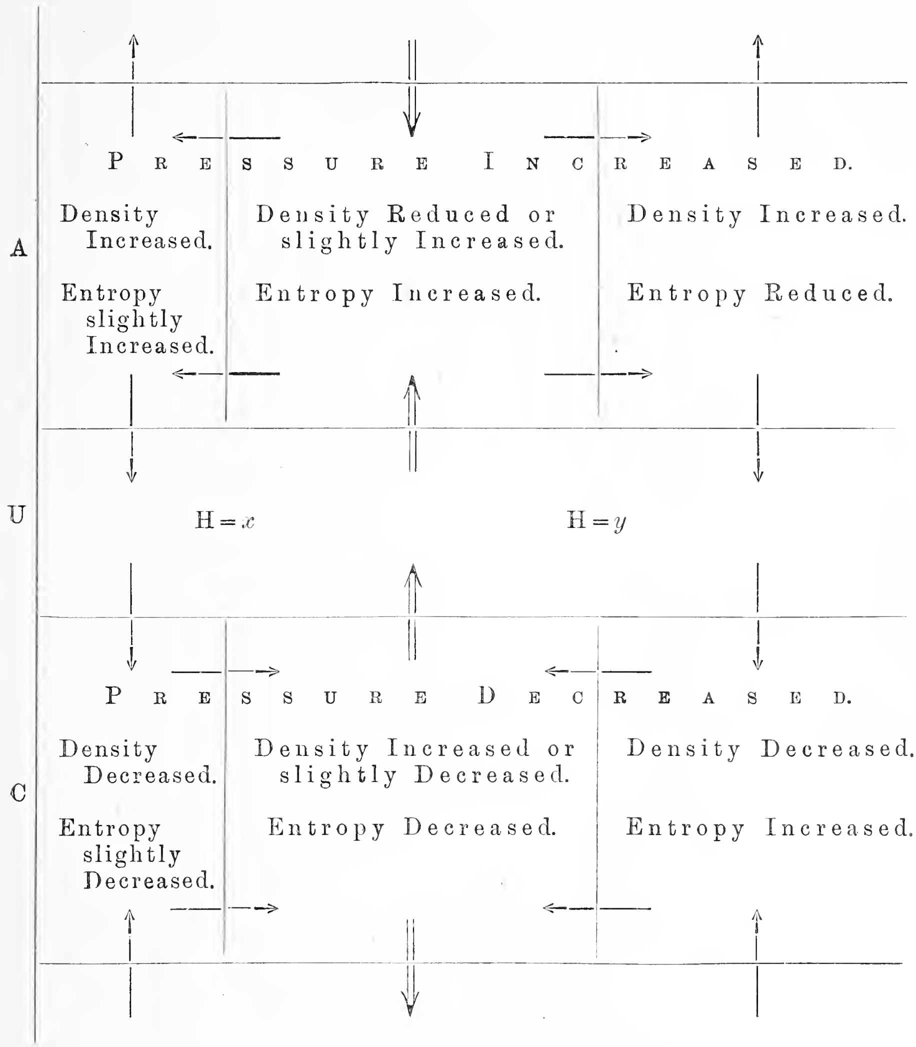

Fig. 5.—Well-established Anticyclone and Cyclone of fig. 1 type.
The arrows show, not the winds, but small components of the winds. Compare fig. 8.
Double-line arrows for dense air.

in the region of heat absorption. The density is increased at all levels, the increase being least (perhaps occasionally negative) in the region of origin. The entropy is increased considerably in the region of origin, and slightly in lower regions (owing to downward movement); but at higher altitudes is reduced by reason of the uplift. In the cyclonic region

C there is a diminution of pressure at all levels. The temperature is also decreased at all levels, the decrease being a maximum at the region of origin. The density is reduced at low levels, less reduced (perhaps occa-

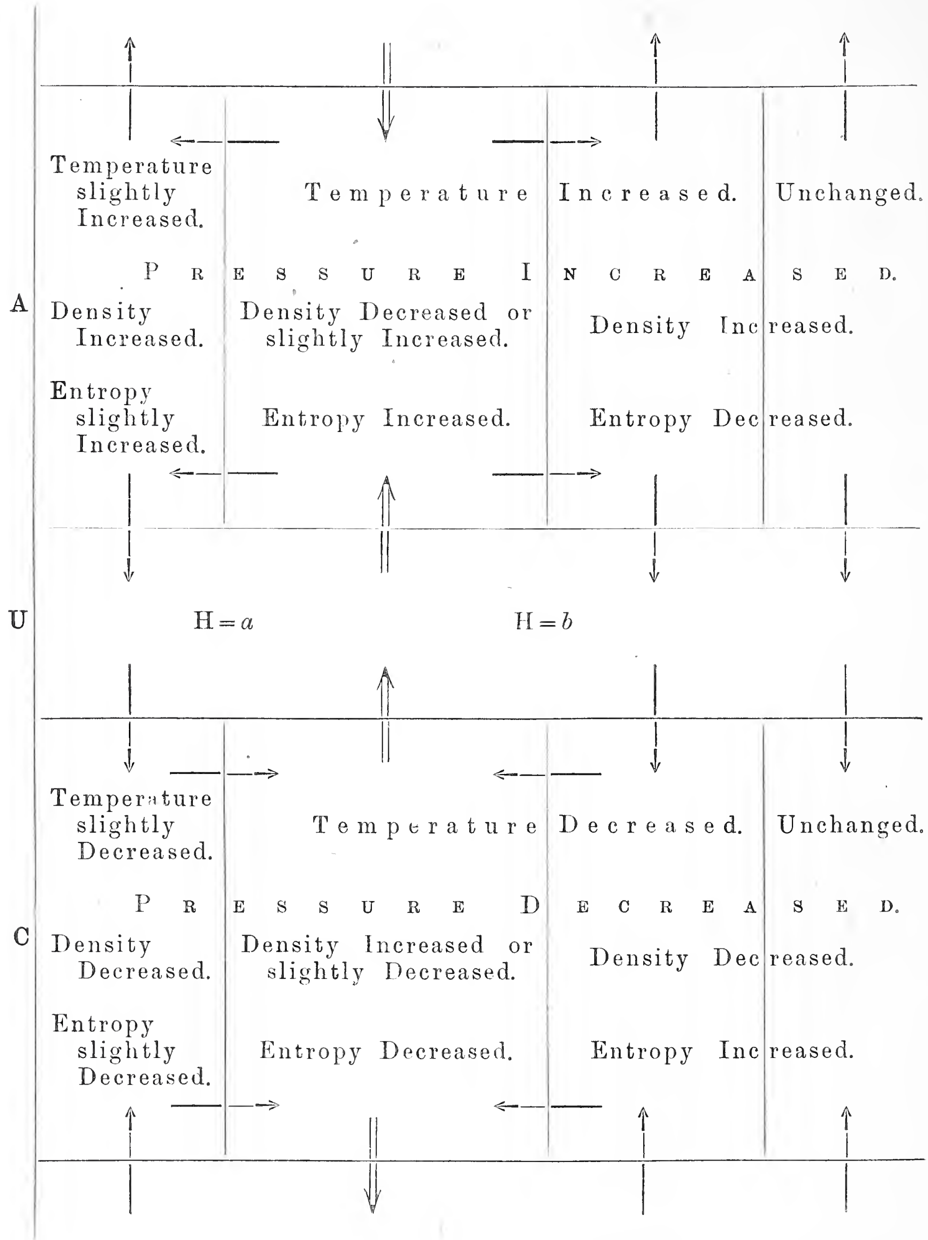

FIG. 6.—Well-established Anticyclone and Cyclone of fig. 2 type.
The arrows show, not the winds, but small components of the winds. Compare fig. 8.
Double-line arrows for dense air.

sionally increased) in the region of origin, and more reduced in higher levels. The entropy is reduced slightly in lower levels, much reduced in the region of origin, but increased (owing to the descent of air from regions of higher entropy) in the upper regions. It will be seen that these results agree well with the figures given by Mr W. H. Dines for the average values of temperature, pressure, and density at various heights in areas of high and low pressure. I have calculated the entropy by equation (5) from the temperature and pressure, the quantities directly observed. The entropy is for one grm. of air, the zero being taken as the entropy at a pressure of one dyne per square centimetre and a temperature of one degree absolute.

TABLE OF AVERAGE VALUES OF TEMPERATURE, PRESSURE, AND DENSITY OF AIR IN REGIONS OF HIGH PRESSURE AND LOW PRESSURE.

(Figures for temperature, pressure, and density from a paper by Mr W. H. Dines, F.R.S., in *Trans. Roy. Soc.*, vol. ccxi, p. 262, and article "Density" in *Met. Gloss. M.O.*, 225, ii.)

H	High Pressure.				Low Pressure.			
	Press.	Tem.	Dens.	E.	Press.	Tem.	Dens.	E.
Km.	Dy/Cm²	°A	Gm/Cm³	Jo/°A	Dy/Cm²	°A.	Gm/Cm²	Jo/°A
10	273000	226	·000421	1·78	247000	225	·000382	1·80
8	366	240	531	1·75	335	227	514	1·72
6	483	254	662	1·73	449	240	652	1·69
4	628	267	818	1·73	591	255	807	1·67
2	807	277	1012	1·66	767	269	992	1·65
0	1031	282	1270	1·61	984	279	1226	1·61

Difference between High and Low.

Ht.	Press.	Temp.	Density.	Entropy.
10	26	1	39	−·02
8	31	13	17	+·03
6	34	14	10	+·04
4	37	12	11	+·06
2	40	8	20	+·01
0	47	3	44	·00

It would appear that the average height of origin is about 4 to 6 kilometres, and it is interesting to note that in the anticyclone at this height the isentropic condition is practically attained. According to the orthodox convection theory it is in the cyclone that such conditions should appear.

Mr Dines has attempted to verify the presence of ascending currents in an area of low barometric pressure and has found them to be of the

very smallest magnitude. This is in opposition to the view that depressions are due to the disturbance of vertical equilibrium, and in agreement with the view suggested above that they arise from thermal changes too slight and continuous to disturb greatly the lapse rate. As a matter of fact, as has been already mentioned, the vertical stability of the air is so great that it is rarely, and never long, disturbed. It is for this reason that, except where disturbed by ground contours or the collision of differently directed currents, the winds flow in almost perfectly horizontal planes.

It has been mentioned above that a portion of the energy of the winds is derived from the rotation of the globe, and it is easy to find an approximate expression for the proportion of energy so derived. If Q (fig. 7) be a portion of the air in a cyclonic whirl closing gradually in on the axis ZY, we may denote its original moment of inertia about ZY, when it was at rest relatively to the earth's surface, by the symbol I. Then its original energy of rotation was $\frac{1}{2}I\omega^2$. If it approaches to a fraction λ of its original distance from ZY, its moment of inertia is diminished to $\lambda^2 I$. If ω' be now its angular velocity, we have

$$I\omega = \lambda^2 I\omega'.$$

$$\therefore \omega' = \frac{\omega}{\lambda^2}.$$

Its energy of rotation about ZY is therefore

$$\tfrac{1}{2}\lambda^2 I\left(\frac{\omega}{\lambda^2}\right)^2 = \frac{\frac{1}{2}I\omega^2}{\lambda^2}.$$

If it were now brought to rest relatively to the underlying surface, its energy would be $\frac{1}{2}\lambda^2 I\omega^2$. Hence the apparent energy of the wind due to its movement relative to the earth's surface is $\frac{1}{2}I\omega^2\left(\frac{1}{\lambda^2} - \lambda^2\right)$, and the portion of this derivable from its original energy of rotation is $\frac{1}{2}I\omega^2(1-\lambda^2)$, the rest being due to the pressure forces impelling it towards ZY. Hence the fraction $\dfrac{1-\lambda^2}{\frac{1}{\lambda^2}-\lambda^2} = \dfrac{\lambda^2}{1+\lambda^2}$ of the energy of the wind is due to the earth's rotation. It appears, therefore, that in the first stages of its movement the light wind in the outer region of the area owes one-half of its energy to the rotation of the earth. As it approaches the centre and increases in strength, an increasingly greater portion of its energy is due to the pressure forces that accelerate the movement. It should be noticed that the greater the angle at which the path cuts the isobars, the greater is the component of force accelerating its speed (or retarding it, if it is moving

towards the high pressure). Yet when the wind has moved half-way to the centre, one-fifth of its energy is still derivable from the earth's rotation.

A portion of the energy is consumed in overcoming frictional resistance,

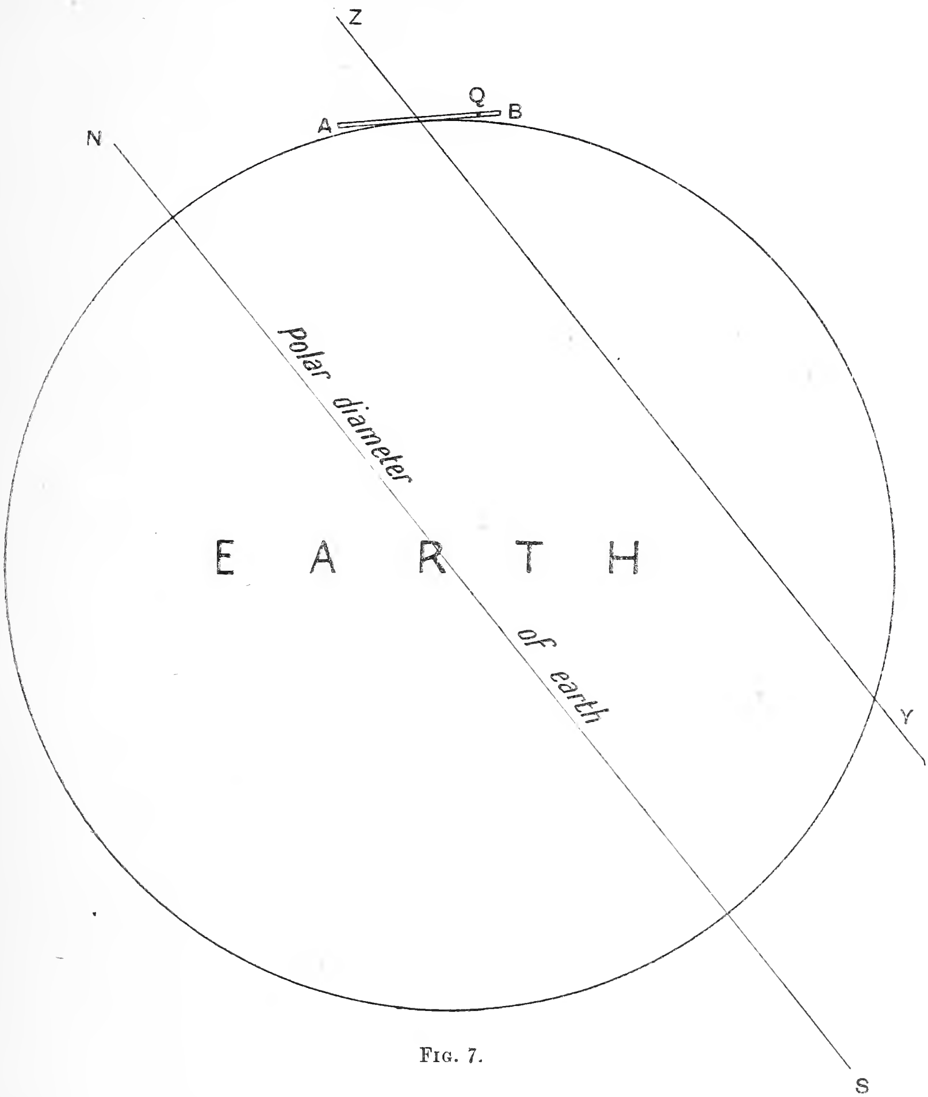

Fig. 7.

and hence when the air finally reaches the region of cooling and is itself expelled towards the high-pressure regions by reason of its increased volume-inertia (*i.e.* density), it will not long maintain its cyclonic circulation but will soon commence to circulate anticyclonically by reason of its outward motion, in spite of the fact that it is moving around a low-pressure area (see fig. 8, which shows the circulation at different levels in a well-

developed cyclone and an anticyclone). A similar effect must occur in the south-east region of a depression (see fig. 4), where detached colder

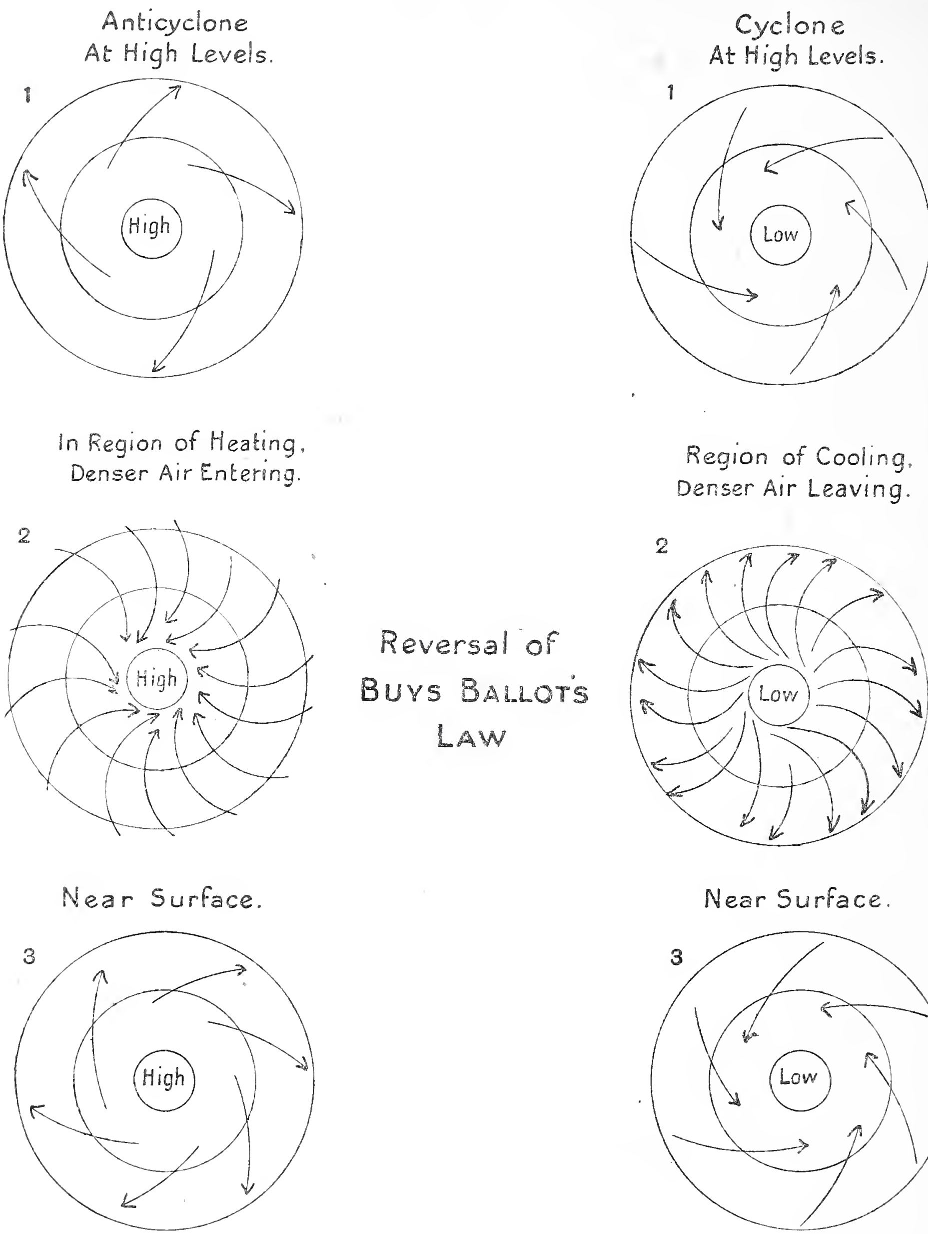

Fig. 8.

portions of the northerly current are bursting their way through the warm southerly current towards the region of higher pressure. The

tendency of these denser masses to press to the right must oppose to a slight degree the general movement of the stream and so promote gustiness.

Glacial breezes, boras, and other katabatic winds are probably to be classed with the winds in the region of cooling of a cyclonic system, and, except in so far as disturbed by ground contours, should form notable exceptions to Buys Ballot's law. Of course, in the extreme case of the disturbance of convectional equilibrium over a steep slope, gravitation plays an important part. But even on fairly level surfaces, or where the cooling effect does not create a "hyper-adiabatic" lapse rate, similar but lighter breezes will also openly defy the law of movement given by Buys Ballot. Lake-breezes and sea-breezes are probably of this type.

It has been shown that the variations of entropy at any level in the air create pressure differences at other levels, which will affect the winds at those levels notwithstanding the comparatively slight amount of actual vertical movement. If we restrict our attention to one horizontal plane, and consider the effect on existing air currents at that level of the creation of a new area of depression or high pressure, it is at once obvious that the portion of an existing current which is approaching the new depression or receding from the new high pressure will be accelerated, while the portion which is receding from the depression or approaching the area of high pressure is retarded. If the new pressure distribution is maintained it has no resultant effect on the speed of the previously existing currents. For the work done by the pressure forces on a small mass M of the air as it moves from an isobar where the pressure is p_1 to one where it is p_2 is

$$-\int_{p_1}^{p_2}(vdp+pdv),$$

$$\therefore -\int_{p_1}^{p_2}(vdp+pdv) = \text{gain of internal energy} + \text{gain of kinetic energy}.$$

If we neglect friction, conduction of heat, and radiation, the conditions are adiabatic, therefore V (and also the internal energy) is a function of p only. Hence in crossing the area enclosed by a given isobar the gain in kinetic energy is

$$-\int_{p_1}^{p_1} f(p)dp - \text{gain of internal energy} = 0 - 0.$$

Hence the air always crosses the same isobar with the same speed, and no permanent acceleration is produced by the pressure forces. Friction and the conduction of heat tend always to reduce the kinetic energy, and so in the absence of radiation effects the air will cross any given isobar with diminished speed on each successive occasion of reaching it.

If, however, a depression comes into existence and then dies out, the air which is passing while it dies out is more accelerated in its approach than it is retarded as it recedes, and is therefore permanently accelerated. Hence the process of dying out of depressions or increase of intensity of anticyclones is associated with a permanent acceleration of those portions of passing currents which come temporarily under their influence. Such accelerated portions of a main atmospheric current, overtaking the portion ahead, are pressed back again, thus setting up an elastic oscillation lengthwise in the stream and producing the phenomenon of squalliness. This quality is therefore not confined to the surface layers of the wind (although admittedly greatest there owing to surface irregularities) but may occur at all levels in consequence of variations of barometric pressure.

As the permanent low-pressure zone in the region of the heat equator will probably be quoted as contradicting this theory of low pressure being due to cooling, I will discuss it now in more detail. In the tropical regions the air is being permanently heated from below and the rate of heating is so great that the vertical equilibrium must often be disturbed, giving rise to convection currents. The air so rising must pass entirely through the isentropic zone, and its inertia will carry it some distance into the overlying regions of greater entropy before it comes to rest. As it falls back to its proper entropic level it abstracts heat from the surrounding air (whose entropy is higher than its own), the conduction of heat being aided by eddies and mingling due to the vertical motion. Thus an effect is produced similar to that which would ensue from mechanical churning. Entropy is withdrawn from the higher layers to the lower, and the lapse rate is much increased. In consequence of this high lapse rate (notwithstanding the increased entropy in the lowest layers), the upper portion of the air is cooled much below the average temperature of air at the same level in surrounding districts. Thus the value of $\int_0^{\mathrm{H}} \frac{g d\mathrm{H}}{\theta}$ is increased and the barometric pressure in the upper regions is reduced. A cyclonic system results, causing an expulsion of the upper cold dense air much in excess of the inflow of air below. Hence low pressure is established in the surface layer also and a permanent low-pressure belt results, in consequence of the mechanical churning set up by the disturbance of the entropic equilibrium. The July world-minimum is over the Himalaya, where the contours aid the churning. The Antarctic plateau (about 10,000 to 13,000 feet) has a similar effect.

It has been stated that the principal causes of secular radiative cooling or heating are probably movement in latitude, formation of extensive

cloud-sheets, and variations of diathermancy. As regards the former, air which is moving poleward, especially in winter time, is almost certainly cooling. Hence arises the continual succession of depressions in the winter time in the Northern Antitrades, which practically control our winter weather. In summer, anticyclones are likelier owing to the increased duration of sunshine. In the trade-wind region, where the winds move towards the equator, anticyclonic conditions prevail. Where the air and underlying earth are cooling, the formation of an extensive cloud-sheet, by reflecting back the terrestrial radiation, will retard the cooling below and accelerate it above. Thus anticyclonic conditions arise beneath the cloud, and cyclonic above. If the cloud is low, the greater mass of air is under cyclonic conditions, the expulsion of air from the cyclone will exceed the entrance of air to the anticyclone, and hence cyclonic conditions will ultimately prevail. If, however, the cloud-sheet forms at a high altitude, anticyclonic conditions will result. [*E.g.* Desert of Gobi in winter.]

The converse is true if the air and ground are in process of heating, and hence (as has been frequently observed) low sheets of stratocumulus are often, in early summer, associated with anticyclonic conditions.

Diminished diathermancy in a layer of air has probably the same effect on lower and higher layers as the formation of cloud, but in the layer of diminished diathermancy itself there will be a rise of temperature, whereas where cloud forms there must be a reduction.

Disturbance of vertical equilibrium may arise in the region of most rapid heating in an anticyclone (*cf.* table of average values given above) and gives rise to summer thunderstorms. The resultant churning must diminish the intensity of the anticyclone. Disturbance by too rapid cooling in a cyclone is a rare occurrence owing to the higher speed of wind movement in such systems, which more rapidly supplies new air to the region of refrigeration. But a few winter thunderstorms may be due to this cause.

After the original causes of the anticyclone or depression have vanished, the established inequalities of density will cause the circulation to persist for a little. Friction and the conduction of heat are, however, constantly withdrawing energy; while the process of mingling (postulate A) tends to destroy the inequalities of density. Average conditions would in time be restored, did not fresh thermal disturbances in adjacent regions set up fresh wind systems.

(*Issued separately June* 19, 1919.)

VIII.—On the Life-History and Bionomics of Myzus ribis, Linn. (Red-Currant Aphis). By Maud D. Haviland, Bathurst Student of Newnham College. *Communicated by* Professor F. O. Bower, F.R.S.

(MS. received November 14, 1918. Read January 20, 1919.)

Synonymy.

Aphis ribis, Linn., Fabr., Schrank, Kalt., Flogel.
Myzus ribis, Pass.? Buckton.
Rhopalosiphum ribis, Koch.
Aphis galeopsidis, Kalt., Walker.
Phorodon galeopsidis, Pass., Buckton.
Myzus whitei, Theobald?
Myzus dispar, Patch?

Introduction.

Myzus ribis, Linn. (red-currant aphis), has attracted the attention of entomologists for more than a hundred and fifty years. But in spite of its abundance, wide geographical distribution, and economic importance as a pest of bush fruits, the complete life-cycle has never been determined. It was with the view of settling the disputed question of the fate of the aphis in the summer, and in the hope that elucidation of the bionomics of this species would throw further light on some uncertain points in the life-history of other migratory *Aphidinæ*—notably on the production of the sexuales—that these researches were undertaken.

I must express my sincere thanks to Professor J. Stanley Gardiner, who gave me facilities to carry out the work in the Zoological Laboratory, Cambridge, and to Miss Stephen, Principal of Newnham College, who permitted the use of her garden for the cultivation of the necessary material.

Mr F. Balfour-Browne has helped me with much useful suggestion and criticism, and I would particularly express my thanks to Mr H. H. Brindley for his advice and assistance. I must also thank the Trustees of the Balfour Fund for a grant awarded to me for the prosecution of the work.

The Nature of the Attack and its Remedy.

Myzus ribis is commonly, though not invariably, associated with the presence of large red blisters on the leaves, but it is still an open question whether these are always caused by the aphides. The deformity and discoloration are generally apparent as soon as the buds open and before the leaves are fully unfolded. The red colour of these blisters is due to anthocyanin, a soluble pigment common in plants. It is possible that

mechanical injury to the tissues, which involves the phloem but not the xylem, may result in an accumulation of sugars and other products of photosynthesis within the affected area, and under certain circumstances these may give rise to anthocyanin. If this is so, the reaction of the plant to the punctures of the aphid's rostrum would account for the malformation without supposing that the injected saliva of the insect acts as an irritant to the leaf cells.

I am inclined to think that most of the damage is done by the stem mother while the buds are opening. My observations go to show that she hatches in April, and has a larval period lasting for three weeks or a month. When she begins to produce young at the beginning of May, the leaves already show the red swellings which characterise the attack of this pest. When once the leaves are fully unfolded, the sucking of the aphides has little effect upon them. The actual damage to the bushes is sometimes very considerable. The functions of the injured leaves are interrupted and the fruit ripens prematurely. Davidson (4) says that the stem mothers of *Myzus ribifoliæ* cause similar blisters on *Ribes glutinosum* in America.

Koch, Kaltenbach, and other older writers attribute the red blisters to *M. ribis*, but Theobald (27, p. 96) is doubtful whether it actually causes them. But he adds (p. 110): "I have never found this species except under the red blisters." This is a mistake, for, as I hope to show, *M. ribis* is equally common under green undeformed leaves. Flogel (11), though he holds that the blisters are caused by the aphides, recognises that they feed also upon normal leaves, but he considers that only the later generations do so. He propounds the curious theory that those feeding under the red blisters have a gregarious disposition (Socialtendenz) which turns to vagrant habit (Dissipationstendenz) in their descendants. The fact really is that if the aphides be taken from the diseased leaves and fed on healthy ones, they distribute themselves all over the surface and *vice versa*. If surplus foodstuffs are massed in the blisters, possibly the aphides are induced to congregate there, while outside they remain scattered because every spot is alike. Moreover, from the first generation onwards, *M. ribis* can be found on healthy leaves where its sucking seems to do little harm, but it is more readily overlooked. I have also found the stem mothers on unblistered leaves, and this inclines me to the view that the blisters when present are the work of the fundatrix on the opening bud. When the *Myzus* is associated with undistorted leaves, it is either that the stem mother has hatched after the buds are open, or else that the lice have migrated thither at a stage when the leaves are impervious to attack.

M. ribis is known to feed upon black currant and gooseberry, but it is less damaging to these than to *Ribes rubrum*. In 1918, black currant and gooseberry growing beside observation bushes of red currant in my garden were unaffected. A colony artificially founded upon gooseberry died out in the first generation after transference; and the third generation on black currant resulted in a swarm of winged forms which left the bushes at the end of May after producing a slight curling of the leaves.

The only effective remedial measure against *M. ribis* is to spray the bushes with soft soap, or nicotine solution, or paraffin emulsion in April as soon as the buds open. Where possible, it is advisable to pick off the blistered leaves by hand, as these afford shelter to the pest; and in case any aphides escape the first spraying, a second should be given early in May. Theobald recommends spraying the bushes with paraffin jelly early in October, to destroy the sexuales; but as my observations go to show that currant-reared sexuales may appear in September, and be succeeded in October by an immigration from the summer host plant, this method is uncertain, for oviposition may have taken place before the remedy is applied, and the egg is impervious to the wash. Ridding the neighbouring ground of such weeds as *Lamium*, *Polygonum*, and *Veronica*, which harbour the aphides in the late summer, might also be beneficial, but little reliance can be placed on this method of control.

The Dimorphism of the Red Blister and Green Leaf Forms.

Myzus ribis has already been so exhaustively described by Kaltenbach (16), Flogel (10, 11), Patch (23), and others, that an account of the general characteristics of the parthenogenetic forms is unnecessary here. But in 1918 I noticed a dimorphism in this species, which, though it has not been treated as such by other observers, seems sufficiently remarkable to be described in detail.

When dealing with four fundatrices in April, a difference had already appeared between two which were found upon red-blistered leaves and two taken from green undistorted leaves. Briefly, the stem mothers on the former were pale yellow, with round abdomens, while those on the latter were green and more oval in shape. One of each form was chosen for breeding the generation series, and the other two were mounted alive in balsam and measured.

The breadth of the head is taken as the standard in calculating the proportions of the fundatrices, as subsequent results with the succeeding apterous generations, have shown that it is the least variable of the dimensions chosen for measurement. It can be seen from Table A that

TABLE A.

The dimensions of two fundatrices from red blister and green leaf respectively.

	Red Blister Form.		Green Leaf Form.	
	Absolute Dimensions.	Dimensions expressed as per cent. of Breadth of Head between Eyes.	Absolute Dimensions.	Dimensions expressed as per cent. of Breadth of Head between Eyes.
	mm.		mm.	
Breadth of head between eyes	·29	100	·28	100
Length of body	2·30	790	2·77	989
Greatest breadth of abdomen	1·38	475	·98	350
Breadth between cornicles	·70	241	·57	203
Length of cornicles	·40	130	·33	117
Total length of antenna	1·85	637	1·40	500
Femur (metathoracic leg)	·65	224	·50	178
Tibia (metathoracic leg)	·90	310	·90	321

the form from the green leaf is actually, as well as proportionately, longer and narrower than that from the blistered leaves—a difference appreciable even to the naked eye.

TABLE B.

The average dimensions of twelve apterous females from red blister compared with those of twelve similar forms from green leaf.

	Red Blister Form.		Green Leaf Form.	
	Average Absolute Dimensions.	Dimensions expressed as per cent. of Width of Head between Eyes.	Average Absolute Dimensions.	Dimensions expressed as per cent. of Width of Head between Eyes.
	mm.		mm.	
Width of head between eyes	·31	100	·31	100
Length of body	2·01	648	2·18	700
Greatest breadth of abdomen	·95	300	·83	267
Breadth between cornicles	·67	200	·50	161
Total length of antenna	2·36	671	2·37	764
Length of cornicles	·30	96	·31	100
Length of femur	·56	180	·57	183
Length of tibia	1·31	422	1·41	454

This disparity is maintained in the succeeding apterous generations (Table B). Henceforth, for the sake of brevity, the two forms will be distinguished as "red blister" and "green leaf" respectively. It will be seen that the most striking difference again occurs in the length and breadth of the abdomen. The dimensions of this part in insects are

variable and depend much on the amount of "telescoping" of the posterior segments in the individual. This holds good also for the Aphidæ, but it does not explain the divergence between the two forms. It merely shifts

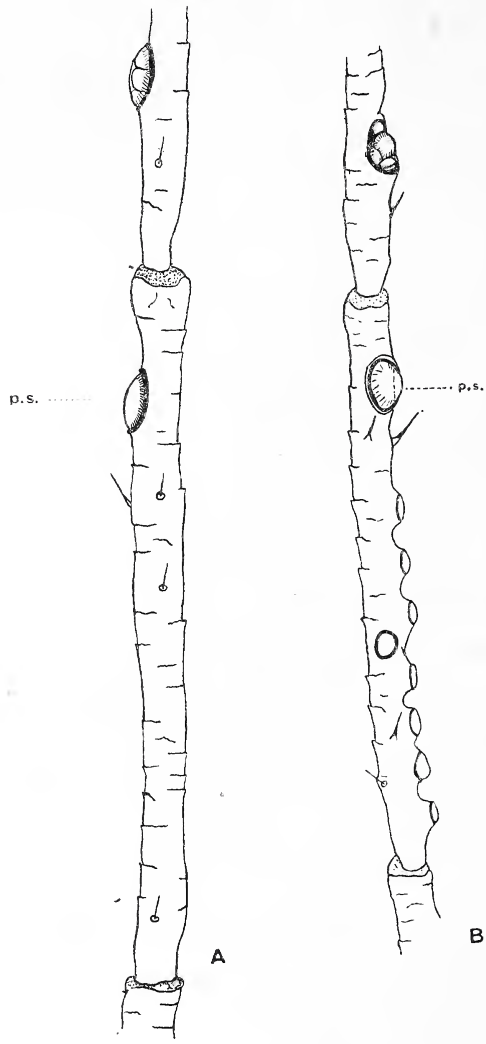

FIG. 1.—Fifth joint of the antenna of two winged females. A, from green leaf, and B, from red blister.

p.s. = permanent sensorium.

the difficulty of explanation to the problem of why an aphid which feeds on red blister should retract the cauda further than one which does not. Actually the difference might be expected to tend in the opposite direction, for if the blisters contain specially abundant food (and apart from the question of shelter, I think there can be no doubt that they are

the favourite feeding-ground), greater extension of the abdomen might be expected there than is found in the forms from normal leaves. That any aphides remain on the green leaves may perhaps be explained by the overcrowding of the blisters, which are filled as early as the first generation.

The colour difference between the forms is quite striking, but I am inclined to think that it is merely transitory, for if both forms be fed for a time on the same food, the distinction disappears. How far the dimensions of the abdomen remain constant under such conditions I have not yet determined for the apterous forms.

The same disparity in abdominal dimensions is not maintained among the winged females; but, on the average, forms from green leaves are rather the larger as regards the body and wing length, while those from red blisters have longer antennæ both actually and proportionately.

The difference in body dimensions is correlated with a difference in the number and arrangement of the sensoria on the fifth and sixth joints of the antenna. These sensoria have been described by several writers, but, so far as I know, Flogel (11) is the only one who has investigated their histology. They consist of a circular opening closed by a membrane, like a tympanum, and are surrounded by a fringe of minute hairs. Flogel says that the peculiar cells lying beneath this membrane are supplied by a branch of the antennary nerve, and the whole structure is evidently specialised for a particular function. He calls it an organ of smell (Geruchsorgan), but the structure is unlike that of any known olfactory organ, and the researches of Haüser (14) on the Heteroptera, and some as yet incomplete experiments of my own on *Macrosiphum pelargonii* (Aphidæ) tend to show that the sense of smell, if it exists at all in the Hemiptera, is not located in the antennæ.

In *Myzus ribis* there are two kinds of sensoria on Joint V. At the distal end is a large sensorium which may be called the permanent sensorium. A similar sensorium is found at the base of the spur on Joint VI, but under the high power of the microscope this is found to be a compound structure, composed of several small ones crowded together. In addition, in the red blister form, six to nine smaller sensoria occur along the shaft of Joint V. These do not appear until after the final moult, and may be called supplementary sensoria. On the other hand, in females from green leaves, besides the permanent sensorium, Joint V bears typically only imbricating scales, and a few stiff hairs which are possibly sensory in function. It is, however, not unusual to find a green leaf female which has three or four supplementary sensoria on Joint V,

Table C.

Dimensions of winged females of *Myzus ribis* from red blister and green leaf, of *Ribes rubrum* from *Galeopsis tetrahit*, and of *Aphis galeopsidis* of Kaltenbach.

Average of twelve individuals in each case.

	Red Blister Form.		Green Leaf Form.		Fourth Generation after Transference to *Galeopsis tetrahit.*		*Aphis galeopsidis* Kalt.	
	Average Absolute Dimensions.	Dimensions expressed as per cent. of Length of Fore-wing.	Average Absolute Dimensions.	Dimensions expressed as per cent. of Length of Fore-wing.	Average Absolute Dimensions.	Dimensions expressed as per cent. of Length of Fore-wing.	Average Absolute Dimensions.	Dimensions expressed as per cent. of Length of Fore-wing.
	mm.		mm.		mm.		mm.	
Length of forewing	2·31	100	2·61	100	3·06	100	3·27	100
Length of body .	1·59	68	1·84	70	2·10	68	2·35	71
Greatest breadth of abdomen.	·52	22	·59	22	·68	22	·71	21
Breadth between cornicles.	·32	14	·30	11	·36	11	·37	11
Total length of antenna.	1·99	86	1·87	71	2·87	93	3·03	92
Length of cornicles	·23	10	·19	7	·20	6	·22	6

but these are always small and never tuberculate, as in the red blister forms. Fig. 2 shows the range of variation in this feature in seventeen sister females—granddaughters of a female with the typical form of open-leaf joint. Five of these had no sensoria on Joint V of either antenna. Of the remainder—

4 had 4 sensoria
4 „ 3 „
3 „ 2 „
1 „ 1 sensorium.

But of these twelve females, only three had additional sensoria on each antenna. In the rest, the other antenna was of the typical green leaf type.

Besides the absence of sensoria on Joint V, the green-leaf form differs from that from the red blister in the smaller size of the sensoria on Joint III and Joint IV. This variation in the antenna is not alluded to by the older writers on *Myzus ribis*. Buckton does not mention it, and

Flogel (10, 11), who described the structure of the species minutely, asserted positively that "on the fifth joint are about eight sensoria."

(For the views of later observers see section *Myzus whitei* and *Myzus dispar.*)

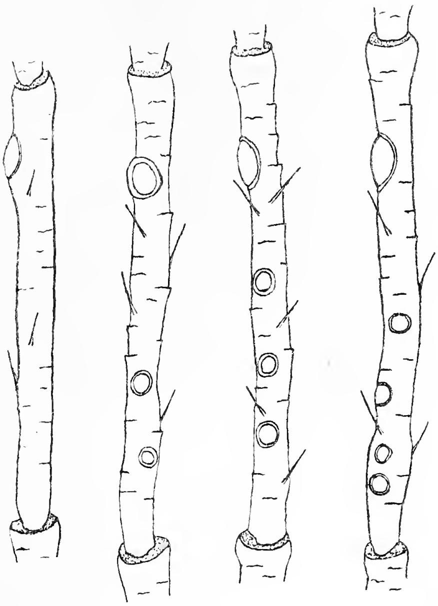

Fig. 2.—Range of variation in Joint V of the antennæ, in respect of the sensoria, of seventeen sister females of the green-leaf type.

Between June 8th and June 23rd I made a collection of winged forms from my garden; the results are set out in the table below.

Table D.

Collections of winged females from red blister and green leaf to show the type of antenna.

	Total Number collected.	Red Blister Type.		Green Leaf Type.		Undetermined.	
		Number.	Per cent. of Total.	Number.	Per cent. of Total.	Number.	Per cent. of Total.
Red blister collection .	174	173	99·4	1	·05	...	...
Green leaf collection .	73	6	7·19	65	78·2	12	14·3

The occurrence of the red blister type in the collection from green leaves may possibly be explained by the fact that the winged females are

more vagrant than the apterous, and crawl away soon after emerging; but as the blisters are already crowded with nymphs and larvæ, they cannot establish themselves elsewhere. Hence it is not surprising that some were taken resting on the surrounding leaves. The green leaf collection is smaller than that from the red blister because the former migrants emerged a few days earlier than the latter, and their maximum swarm was already past.

Correlated with the absence of supplementary sensoria on Joint V is the greater distance of the permanent sensoria of V and VI from the articulation of these two joints. This will be best understood by reference to fig. 3, where the curve of error of sixty females from red blisters is compared with that of the same number from green leaves; or to Table E, which shows the average distance between the articulation and the sensoria of both joints.*

Table E.

Variation in respect of the distance between the permanent sensoria of, and the articulation between, Joint V and Joint VI.

	Total Number taken.	Average Distance between Sensorium of V and Articulation.	Average Distance between Sensorium of VI and Articulation.	Average Total Distance between the two Permanent Sensoria.
		mm.	mm.	mm.
Red blister collection .	60	3·20	7·09	10·39
Red blister stock transferred in first instar to green leaves.	38	3·23	7·24	10·26
Red blister stock, first generation reared on green leaves.	60	3·40	7·52	11·34
Green leaf collection . .	60	5·42	8·13	14·14

The foregoing data show that there is foundation for the distinction between red blister and green leaf types; and the generation series of observations show conclusively that they are dimorphic forms of the same

* Some writers consider that the form of the cornicles is of taxonomic value in determining *M. ribis* and allied species; but after examination of all the material at my disposal, it is evident that forms with clavate and cylindrical cornicles may occur in the same brood, and that food does not seem to be entirely the determining factor. There is certainly a decided preponderance of cylindrical cornicles among forms with the red blister type of antenna, and of clavate cornicles among those from green leaves; but the two features are not altogether correlated, and at present I am inclined to regard the difference in cornicles merely as an individual variation.

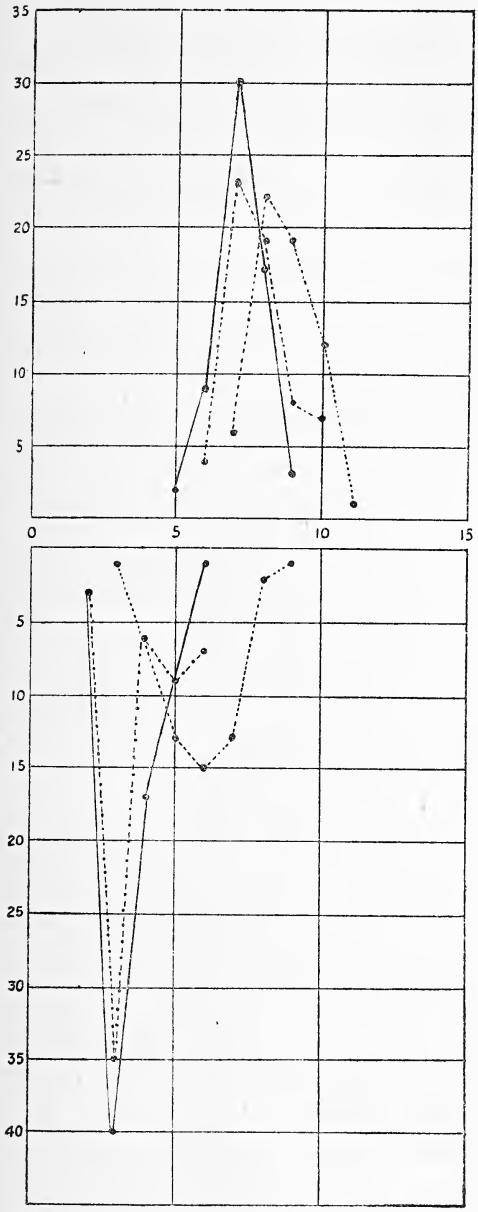

FIG. 3.—Curves showing the relations of the permanent sensoria to the articulation of Joints V and VI. Sixty individuals in each curve.

———— = Red blister form.
- - - - - = ,, ,, First generation on green leaf.
………… = Green leaf form.

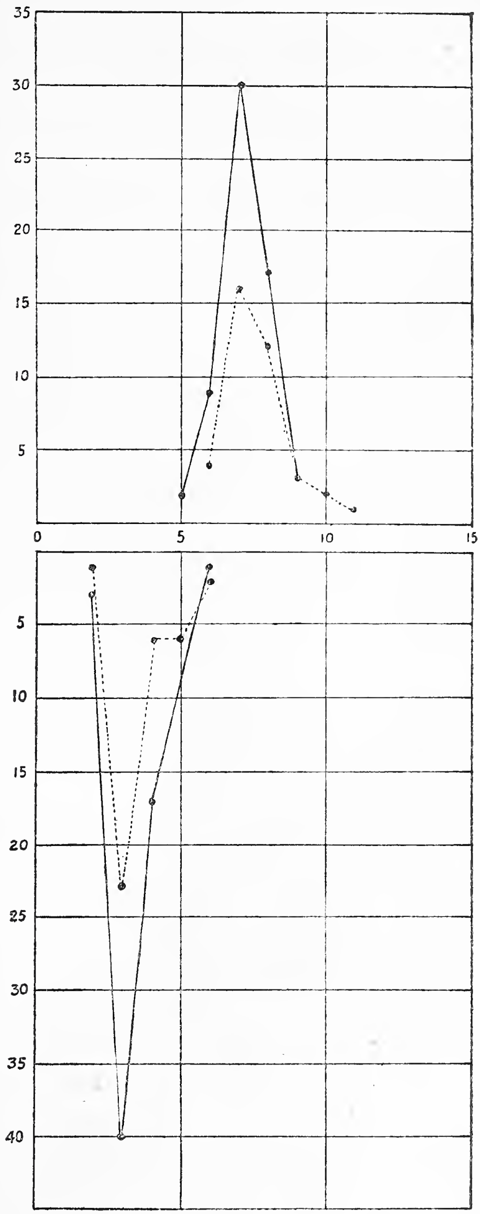

FIG. 4.—Curves showing the relations of the permanent sensoria to the articulation of Joints V and VI.

———— = Red blister form (60 individuals).
………… = Their progeny transferred in first instar to green leaves (38 individuals).

The abscissæ represent the difference in $\frac{1}{100}$ mm. between the sensorium and the articulation. The lower curves refer to the fifth, and the upper to the sixth, joint.

species. Here the descendants of two stem mothers were followed down in each case through four lines of descent. Six of these lines of both stocks were fed entirely on green leaves, but the seventh and eighth (by a coincidence in both instances it was the youngest of both series) were fed on blistered leaves. The result was that all the latter had the red blister type of antenna, while all the rest, irrespective of generation, had the green leaf type. That position in the brood had nothing to do with the modification is shown by the A II, B II, A III, and B III lines of descent. Here

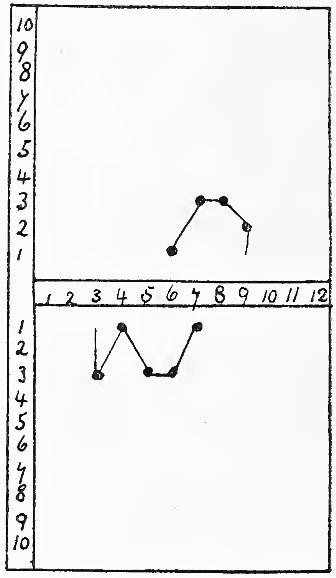

Fig. 5.—Curve showing the relation of the permanent sensoria to the articulation of Joints V and VI in nine females of red blister stock reared for three generations on green leaves.

The abscissæ represent the difference in $\frac{1}{100}$ mm. between the sensorium and the articulation. The lower curves refer to the fifth, and the upper to the sixth, joint.

(see generation series) an attempt was made to check the eldest of eldest and youngest of youngest lines by choosing alternately the eldest and youngest of the brood as parent to the next generation.

All the winged forms of these four "control" lines of descent have the green leaf type of antenna.

The question arises what is the cause of the modification of the antenna in this species? Taking as a working hypothesis that it is due to something in the food, I tried the effect of rearing thirty-eight young of a female of red blister stock on green leaves from their first instar; but the result was on the whole negative, although the range of variation in the

sixth joint was rather greater (fig. 4). In dealing with sixty females of red blister stock, whose apterous parents had been transferred to green leaves (=first generation on green leaf) the result was a little more conclusive (fig. 3). The mode remains the same, but the curve has a tendency to form a "shoulder" falling near the mode of the green leaf curve. The second generation on green leaves were all apterous, and in the third generation only nine winged forms appeared. This number is too small for any conclusion to be drawn, though a tendency to shift the mode of the curve towards the mode of the green leaf type is apparent (fig. 5).

A defect in these feeding experiments in 1918 was that I did not recognise the significance of the character of the food until the production of the winged forms was at its height. This occurred in the third and fourth generations, after which, for the rest of the summer, reproduction was limited to apterous forms which do not show the required characters. Next year it is hoped to carry out the feeding experiments from the first, and thus obtain more data on the subject.

Table F.

Table for comparison with Table D, to show the effect of rearing the red blister form on green leaves. Only the presence or absence of supplementary sensoria is considered here. For the position of the permanent sensoria, see Table E.

	Total Number examined.	Red Blister Type (many Sensoria).		Green Leaf Type (few Sensoria).		Undetermined.	
		Number.	Per cent. of Total.	Number.	Per cent. of Total.	Number.	Per cent. of Total.
Red blister form: transferred in first instar to green leaf.	47	36	76	10	21	1	·01
Red blister form: first generation on green leaf.	73	56	76	17	23	...	...
Red blister form: third generation on green leaf.	9	...	...	9	100	...	...

Kelly (18) recorded a difference in the ratio of Joint III to Joint IV of the antenna between forms of *Aphis rumicis* reared on opium poppy and nasturtium respectively; but he did not ascertain whether this was due to the nature of the food, or to dealing with two different strains of aphides. The conclusion from his further researches among the progeny of poppy-fed females only, was that the progeny of somatically different mothers tend on the average to be alike, and that somatic variations in the par-

thenogenetic line are not inherited; but the number was small, and it was not shown whether the somatic variations recorded were adaptations to the special environment of the parent, or to a fixed characteristic of the strain employed. If the latter, the results were in accordance with the known rule that children of much varying parents tend to regress to the mean of the race.

Myzus whitei, Theobald, and Myzus dispar, Patch.

Two species closely resembling *M. ribis*, Linn., have been recorded from England and America respectively.

Theobald (27, p. 110) describes *M. whitei*, taken in the winged form only, on black and red currants. He says that it might be mistaken for *M. ribis*, but it is darker, the three to seven sensoria occur only on the proximal half of Joint V, the abdominal markings are distinctive, and the eyes are dark. A coloured plate of the two species is given, in which, however, both are figured with the red eyes of *M. ribis*. It has been shown already that the sensoria on Joint V are extremely variable, and therefore cannot alone be made the basis of a new species. As regards the abdominal markings, *M. ribis* is figured with three distinct transverse bands on segments 5–7, and four lateral spots on each side of segments 2–5, *M. whitei* is shown with a single large patch on segments 4–6, and two lateral spots on 2–3. Hitherto all the specimens that I have seen of *M. ribis* have had the square abdominal patch ascribed by Theobald to *M. whitei*. The lateral spots vary, but are generally three in number on segments 3–5, though some specimens have two (on 4–5) or four (on 2–5). I thought at first that the green leaf form might be identical with *M. whitei*; but this explanation will not cover the discrepancy of the colour of the eyes and abdominal markings. Flogel (11) does not mention the abdominal pattern in the parthenogenetic female, but he figures the winged male with a single large spot, exactly similar to that of *M. whitei*. Kaltenbach (16) says that *M. ribis* has a square patch in the middle of the abdomen, and three or four smaller ones at the sides. Koch's figure (19) resembles Theobald's *M. ribis* in having three distinct regular bands, but in the text he says: "On the back of the abdomen is a confluent black mark ('ein gemeinschaftlicher Fleck') on the 4th, 5th, and 6th segments; and in addition, at the side of the four anterior segments, a small mark also black." These descriptions apply better to the *M. whitei* than to the *M. ribis* of Theobald's figures; and I am inclined to think that *M. whitei* may prove to be only a variety of *M. ribis*, especially as Patch (23) remarks that in some collections of the latter species the

markings "extend across the abdomen in transverse bands instead of a solid patch."

M. dispar was first described by Miss Patch (23) from specimens taken together with *M. ribis* on gooseberry and currant in America. Dobrovliansky (9) also records it from black currant in Russia. The apterous form is green, and the alate form is very pale green, with three vivid longitudinal lines and a few blackish transverse markings on the caudal half of the abdomen. The pupa is pale green, and when nearly ready for the final moult, it has the thoracic lobes pellucid brown like the thorax of the winged female. I have already remarked that the colour of *M. ribis* varies with the character of the food, and therefore specific distinctions cannot be based on the colour of the wingless female; but the abdominal markings of the winged form do not correspond with those of *M. ribis*, nor does the dark thorax of the nymph. But for these differences, I should have been inclined to think that *M. dispar* was nothing but the green leaf form of *M. ribis*, for the author goes on to make a distinction between the antennal joints of the two species, which is precisely the difference which I have shown to exist between the two forms of *M. ribis*. She says: "In *ribis*, the terminal sensorium in Joint V and the sensorium at the base of the spur in Joint VI approximate the articulation between V and VI much more closely than is the case with *dispar*. Joint III is more slender in *ribis*, and the sensoria are proportionately larger. . . . The cornicles of *dispar* are relatively shorter and less slender than those of *ribis*."

It was this observation of Patch which led me to take the measurements between the permanent sensoria (fig. 3). Before, I had based the distinction between the green leaf and red blister forms only on the presence or absence of supplementary sensoria on Joint V. Patch does not mention the latter feature in the text, but she gives two excellent figures, in which *M. ribis* is shown with supplementary sensoria on this joint, and *M. dispar* with none at all. With regard to the cornicles—the cornicles of the green leaf form of *ribis* are proportionately shorter than those from the red blister.

In spite of the difference in colouring, which possibly may be accounted for by another climate, I incline to the view that *M. dispar* may prove to be identical with the green leaf form of *M. ribis*.

The Life-Cycle.

The life-cycle of *M. ribis* has been treated by several writers, notably by Kaltenbach, Koch, and Flogel in Germany, by Buckton and Theobald

in England, and by E. M. Patch in America, but until recently the fate of the aphis in the late summer has been a mystery. It was well known that this species appeared on the currant as the stem mother at the end of March or beginning of April, and reached its maximum abundance in June. During July it became scarcer, the red blisters were emptied, and in August it vanished altogether except for an occasional apterous female. It was an open question whether these remaining females were able to maintain the race until the autumn, when they gave birth to the winged males and wingless females, which, after copulation, produced the winter eggs; or whether there was an emigration to a second host plant in June, followed by a return migration in autumn of males and gynuparæ, such as Börner (1) has shown for *Rhopalosiphum ribis*.

On June 21st, in a cornfield near Cambridge, I found a plant of *Galeopsis tetrahit* infested with the apterous forms and nymphs of a species first described by Kaltenbach as *Aphis galeopsidis*, and later by Passerini and Buckton as *Phorodon galeopsidis*. In spite of their greater size, I was struck at once by their resemblance to *M. ribis*, and this impression was confirmed a few days later when the winged females appeared.

I took six apterous, and two alate, females of *M. ribis* from the generation series (A. I, 7) and transferred them to a potted plant of *Galeopsis*.* In two or three hours they had all settled down to feed, and by the next morning were producing young freely. Both *P. galeopsidis* and the transferred *M. ribis* attach themselves closely to the under side of the leaves, which when young have a tendency to curl round the aphides in the axis of the mid-rib. I continued to rear this stock of transferred *M. ribis* throughout the summer. During drought, when it became difficult to obtain fresh plants of *Galeopsis*, I substituted *Lamium purpureum* with complete success, but the aphides died within twenty-four hours when placed on such aromatic Labiatæ as *Mentha* or *Nepeta*.

About the same time, I collected plants of *Galeopsis* infested with *P. galeopsidis* in two other places near Cambridge, and likewise found this species on *Lamium purpurem* and *Lamium amplexicaule*. Early in July I observed the apterous forms swarming under the leaves of *Veronica*

* At this time I was not aware of the experiments of Gillette and Bragg (*Jour. Econ. Ent.*, Concord, vol. x, 1917, p. 388), who successfully transferred examples of *M. ribis* from *Ribes* to *Stachys* and *Leonurus*, and autumn forms back from these plants to currant. The writers suggest that Kaltenbach and Koch may have had this species under observation as *galeopsidis*, but do not discuss the discrepancies in the description of the two forms by the first-named observer. Kaltenbach (*Die Pflanzenfeinde*, p. 484) gives *Aphis lamii*, Koch, as a possible synonym of his own *galeopsidis*; but Koch cannot have referred to this species, since he describes his *lamii* with dark cornicles.

polita. They had already been severely parasitised by a hymenopteron, and a fortnight later had completely disappeared. During August and September I failed to find this species on any plant in the open.

P. galeopsidis was first described by Kaltenbach in his monograph (16, p. 35), and the description does not differ fundamentally from that of *M. ribis* in the same work. He says that in the apterous forms, *ribis* is citron-yellow with a very short cauda, while *galeopsidis* is green and the cauda is "small." Further on he mentions that *ribis* is found under swellings on the leaves, and the explanation suggests itself that he wrote his account of *ribis* from the form from the red blisters, which, as I have

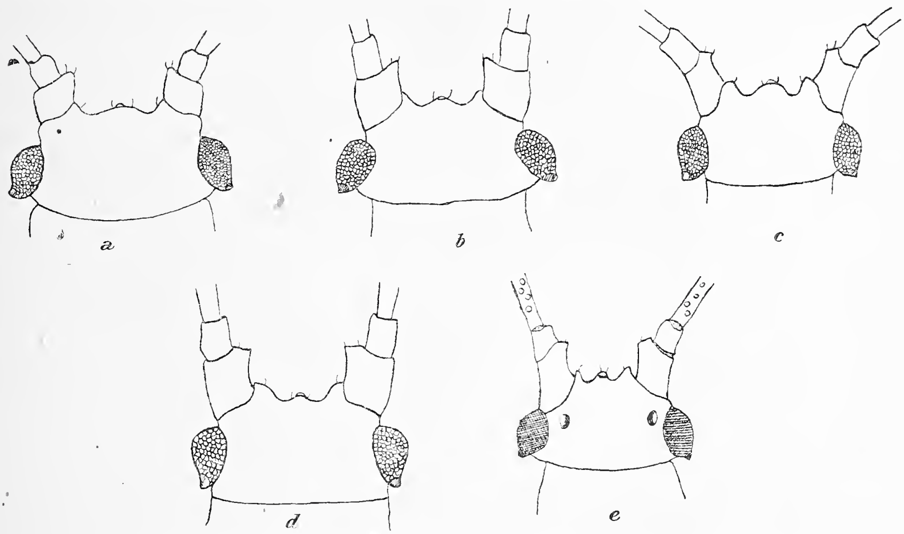

FIG. 6.—The development of the frontal tubercles. Semi-diagrammatic.

(*a*) *Myzus ribis*, winged ♀ from red blister.
(*b*) ,, ,, ,, ,, green leaf.
(*c*) ,, ,, ,, first generation after transference to *Galeopsis tetrahit*.
(*d*) *Phorodon galeopsidis*, Kalt., winged ♀ from *Galeopsis tetrahit*.
(*e*) *Myzus ribis*, ♂ from currant.

already shown, is yellow, and shorter in the body than the form from the green leaves. As regards the winged female, he says that the rostrum reaches to the second coxæ in *ribis*, whereas in *galeopsidis* it does not; but in all the examples that I found on *Galeopsis* the rostrum is indistinguishable from that of *ribis*. Moreover, he continues that the first antennal joint is drawn into a strong tooth. This accounts for the subsequent inclusion of the species by Passerini in his genus *Phorodon*, which, as Buckton points out, differs from *Myzus* only in the possession of a tooth on Joint I. It will be seen from fig. 6 that no distinction can be made between the forms in this respect. Furthermore, Kaltenbach describes *M. ribis* with a large dorsal patch on the abdomen, and three

or four smaller spots at the sides; and *galeopsidis* with a threefold spot in front of the cornicles, and two separate bands anterior to it. An examination of the twelve examples of each form measured in Table C shows that lateral spots may occur in *galeopsidis*, and bands on the anterior segments of the abdomen of *ribis*. The latter vary much, and in the red blister form often appear only as a narrow streak behind the metathorax, but in some specimens from green leaves they are nearly as pronounced

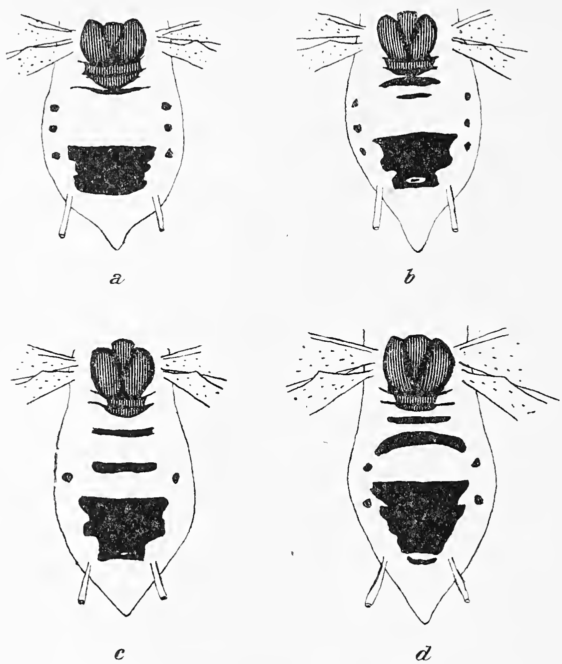

Fig. 7.—Diagrams of the abdominal pattern of

(*a*) *Myzus ribis*, winged ♀ from red blister.
(*b*) ,, ,, ,, ,, green leaf.
(*c*) ,, ,, ,, of the first generation after transference to *Galeopsis tetrahit.*
(*d*) *Phorodon galeopsidis*, Kalt., from *Galeopsis tetrahit.*

as in *galeopsidis*. In fact, when females from red-blistered currants and *Galeopsis* are compared, it would be a pardonable mistake to refer them to different species. It is only when a series is examined that the differences are found to be linked together completely by a chain of intermediate forms (fig. 7).

The first females of *M. ribis* transferred to *Galeopsis* did not show any appreciable effects of the change, but their progeny (A. I, 8 from the fundatrix on currant) were remarkable for an increase of size in both winged and wingless forms. The latter, breeding females and nymphs, were indistinguishable from similar examples of *P. galeopsidis*.

With regard to the winged females, circumstances prevented me from taking the average dimensions of a number when living; and as abdominal measurements of spirit specimens are unreliable, the lengths of the antenna, forewing, and cornicle alone are given in Table H, as these structures do not shrink in alcohol. However, I include the dimensions of a single female of this generation, mounted alive in balsam (Table G). The decrease in size of the later generation in Table H is perhaps due to malnutrition, for during drought in August it was sometimes difficult to procure sufficient fresh food.

It seems, then, that the summer host-plants of *M. ribis* are certain of the Labiatæ and other weeds, and that this aphis has been described previously during its second cycle as a different species, *P. galeopsidis*, Kaltenbach. The synonymy of this species is rather involved. Davis (6) records *P. galeopsidis* from *Polygonum* in America. His description closely resembles that of the form I have identified as *P. galeopsidis*, except that he says the pupa has a dorsal longitudinal red line, which is even more marked in the immature male. But Gillette (13) points out that Davis was actually describing *Rhopalosiphum hippophœs*, Koch, and he gives *P. galeopsidis* as a synonym of that species. In America *Hippophœ* is the winter host of two allied species—*R. hippophœs*, with clavate cornicles, migrating to *Polygonum*, and *M. braggii*, Gillette, with cylindrical cornicles, migrating to Compositæ. It is just possible that the latter may be identical with *M. carthusianus*, of which I described an apterous female taken from the thistle (15). Gillette believes that Buckton confused these two species under the name *galeopsidis*, figuring the apterous form of *braggii* and the alate form of *hippophœs*. Mordwilko (20) discusses the life-history of *R. hippophœs*, and says that Ferrari took the summer migrants from *Inula graveola*. Altogether the synonymy of this group is by no means clear; but it cannot be settled without further knowledge of the bionomics and of the sexual forms of these species, both in Europe and America. Until then, the best course seems to be to identify the summer migrants of *M. ribis* with the *P. galeopsidis* of Kaltenbach.

The following is a list of plants from which the latter species has been described by other writers:—

Galeopsis tetrahit . . .	Kaltenbach.	
„ *bifida*	„	
„ *versicolor* . . .	„	
Lamium album	„	and Buckton.
„ *purpureum* . . .	„	„
„ *amplexicaule* . .	„	

Stachys sylvatica	. . .	Kaltenbach and Buckton.
Polygonum persicaria	. .	Buckton.
" *hydropiper*	. .	Kaltenbach.
" *laxiflorum*	. .	"
" *lapathifolium*	. .	"

With regard to the distribution of the sensoria on Joint V, out of forty-one specimens collected in the field on June 25th, thirteen had the red blister and twenty-eight the green leaf type. From this it may be inferred that both forms are migratory from the currant, for the early date precludes the idea that they could have been established on the *Galeopsis* long enough for the change of host to have affected the antennal development. I was not able to determine whether the type remained constant through several generations on *Galeopsis*, because in the laboratory this stock produced a few apterous generations only and then died out. The green leaf form introduced from currant produced only the green leaf type of Joint V, though in the first generation on *Galeopsis*, nine examples out of thirty-eight had a number of small sensoria all over the shaft of the joint, but this did not appear in the later generations.

I made several attempts to transfer different generations of this stock back to currant, but they were invariably unsuccessful, as the following extracts from my notes show:—

July 18. Transferred two winged and five wingless females (second generation on *Galeopsis*) back to currant.

July 19. Feeding and produced two young.

July 21. All dead but two wingless forms which have lost their green translucent colour and become yellow and opaque.

July 25. Both females dead.

Later experiments always failed, and the aphides died in a few days without reproducing. Even in autumn, when it might have been supposed that a return migration to currant would take place, neither winged nor wingless forms re-established themselves, but crawled about the cage until they died of exhaustion. This inability to live on the first host after transference to the second is curious. At first I thought it might be explained by a structural change, such as is suggested by Kaltenbach's statement of a shortened rostrum in *galeopsidis* as compared with *ribis*, but on examination both of these, and of transferred forms, I can find no difference at all.

Table G.

Dimensions of alate female of *Myzus ribis*. First generation on *Galeopsis tetrahit*.

	Absolute Dimensions.	Dimensions expressed as per cent. of Length of Forewing.
	mm.	per cent.
Length of forewing	3·13	100
Length of body	2·26	7
Breadth across abdomen	·72	23
Breadth between cornicles . . .	·30	9
Total length of antenna	2·80	89
Length of cornicles	·20	6

Table H.

Average dimensions of twelve alate females of *Myzus ribis*. First generation on *Galeopsis tetrahit*.

	Average Absolute Dimensions.	Dimensions expressed as per cent. of Length of Forewing.
	mm.	per cent.
Length of forewing	3·43	100
Total length of antenna	2·91	84
Length of cornicles	·25	7

The Generation Series.

The generations descended from two stem mothers, A and B, were followed throughout the summer. Four lines of descent were chosen from each stock:—

A (B) I = eldest-born of eldest-born.
A (B) II = } eldest- and youngest-born chosen alternately from
A (B) III = } each generation.
A (B) IV = youngest-born of youngest-born.

The second and third lines were intended only as controls in case it should prove desirable to obtain a mean between the eldest and youngest series.

These observations were made on aphides kept in the laboratory. I intended to have identical series, founded from the laboratory stock, "sleeved" on currant bushes in the garden under entirely natural conditions; but in wet, windy weather it was found impossible to make accurate records each day of the number of young, etc. Details were therefore discontinued, but the garden series were maintained as controls for the

laboratory observations, the eldest-born of each generation being chosen as the parent of the next. The three garden series became extinct in August. In the nine weeks from June 2nd to August 10th they passed through five and four and five generations respectively, while during approximately the same time the A. I and B. I series in the laboratory passed through seven and five generations. I am inclined to account for their survival when the controls in the open disappeared, by the fact that they were better protected against enemies; for the well-known disappearance of the red-currant aphis in the late summer seems to be partly natural, and due to the diminished birth-rate, and partly accidental, owing to the increase of insect enemies.

The laboratory series passed altogether through fifteen of the eldest, and five of the youngest, generations. Eight of the A. I, 7 generation were transferred to *Galeopsis tetrahit*, and (including the cycle on the second host) this line of descent passed through eighteen generations. The B. I series passed through only eleven generations. Throughout the summer it was noticeably less productive than the other, and at the B. I, 5 generation there was a long period of inactivity. The parent—the apterous progeny of a winged female—delayed the fourth moult for over three weeks, and then died after producing three young at irregular intervals.

Winged females may occur in the first generation, but my records show that the maximum of these forms appeared in the fourth generation in each eldest line of descent (fig. 8), but in A. IV and B. IV it occurred in the third and second respectively. From this it may be inferred that the appearance of winged individuals is not governed by position in the line of descent. On the observation bushes in the garden, the maximum appearance of the winged forms was some days earlier on green leaves than on red blisters.

In mixed generations, the earlier births seem to be composed as a rule of both alate and apterous forms, while the later ones are nearly all alate. This rather supports the view that exhaustion of the parent may induce their appearance, but this is not borne out by the control lines (A. II, A. III, B. II, B. III), in which eldest and youngest were taken alternately in each generation. Mordwilko (22, p. 82) says that the appearance of winged females is accelerated by lack of food; and this view is held by Davis (5, p. 132) for *Aphis maidi-radicis*, by Webster and Phillips (28, p. 82) for *Toxoptera graminum*, and by Buckton. As much evidence can be found against as for this theory, which rests on general statements and not on facts established by experiment. For instance, in *M. ribis* the maximum swarm of winged forms occurs just when food is most plentiful.

Börner (1) points out that in autumn *Rhopalosiphum lactucæ* produces a host of apterous females, whose attack causes the plant to sicken and die, and yet no winged forms appear. He also records some observations on the "hop louse," where a healthy plant produced more alate females than a sickly one, but more evidence is needed on this subject. It is most probable that a change in the metabolic products of the host plant, such

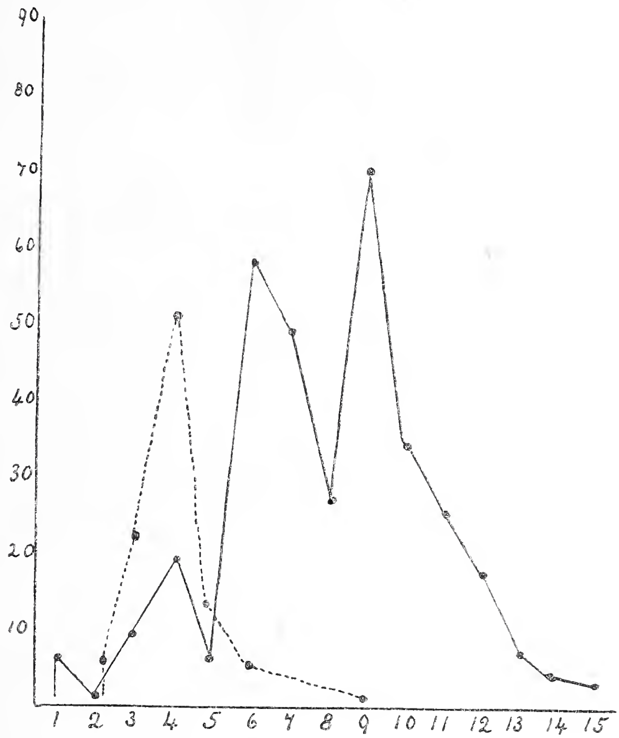

Fig. 8.—Curves showing the proportion of alate to apterous forms in the generation series, A.I.

——— = Apterous forms.
········ = Alate forms.

The abscissæ = the generations.

as a seasonal increase of tannins, may tend to the development of winged forms, but at present we are quite in the dark as to what this change may be. It is certain only that insufficiency of food alone cannot produce this result.

The average number of young per day per aphis throughout the productive period was 2·8; and the largest number of births from a single aphis in twenty-four hours was 12. Fertility diminished towards the end of the life of the female until there was sometimes an interval of several

days between the births. Fig. 9 shows the total number of progeny in each of the A. I generations. The maxima occur in the third, fifth, and eighth generations, and are succeeded by a gradual decline as autumn approaches. The curve formed by the total length of life of the parent

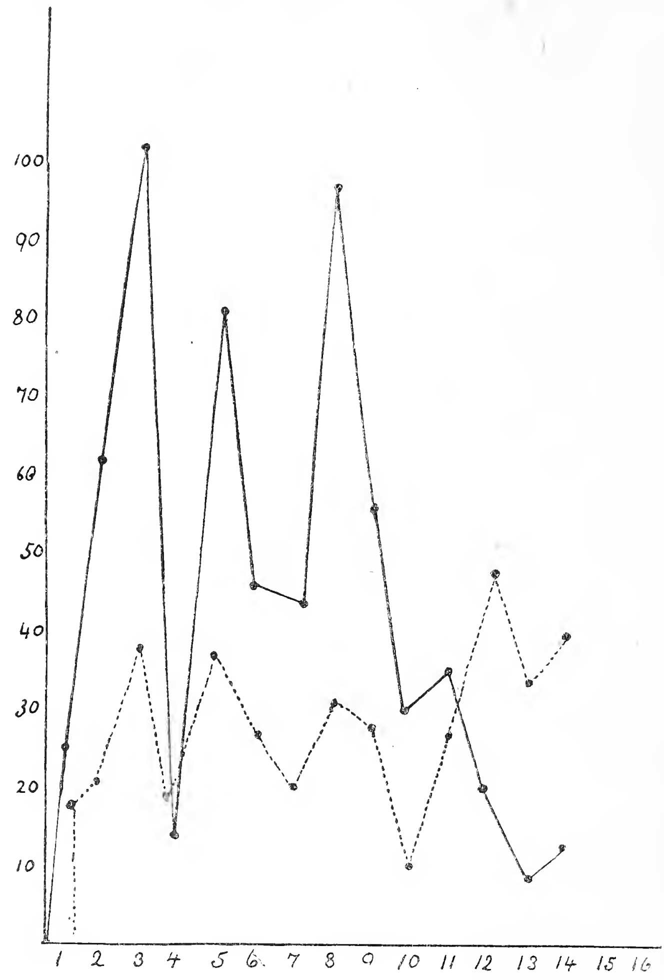

Fig. 9.—Curves to show the total number of young produced in each of the A. I generation series, and the longevity of the mother.

——————— = Total number of young of each ♀.
··················· = Length of life of ♀.

The abscissæ = the number of the generations.

is superimposed, and shows that the diminution in the birth-curve is a real decrease in fertility, and not a shortening of the life of the mother. This agrees with the observations of Davis on *Macrosiphum pisi* and *Aphis maidi-radicis*, and of Webster on *Toxoptera graminum*.

Fig. 9 represents as nearly as possible the total number of young born

in each generation. Fig. 8 shows only those identified as alate or apterous. Thus the numbers are not the same for the two curves, because the forms cannot be distinguished from one another until the third instar. As I have remarked elsewhere, there is a large mortality in the early stages, and therefore only a proportion of the actual number born were available in making fig. 8. From a comparison of the figures this mortality seems to have been about 25–35 per cent.

M. ribis is prolific, and an apterous female may give birth to over a hundred young. The generation records show no difference in fertility between the eldest and youngest lines of descent. In fact, the highest number was 128, born of a female of the B. IV, 1 generation. The average number of young per aphis in the two eldest lines was 33·2, and in the two youngest 48·7. The B. I generations were less prolific than the A. I, although they were kept under similar conditions, and this was also the case when the other lines of the two stocks were compared, except as regards B. IV, 1. To test the relative fertility and longevity of the red blister and green leaf mothers, I made a number of observations apart from the generation series, but there seemed to be no constant difference between them.

The winged forms are less productive than the wingless, and on the whole they are not so long-lived. In the former respect my observations agree with those of Webster on *Toxoptera* (28, p. 75). The alate females are more vagrant than those of most aphides and scatter their young promiscuously over the leaves. Their progeny may be either winged or wingless; but the only exact data I have as to the proportions in which the two forms occur in such a brood is from the generation series, where A. I, 5, B. I, 3, and B. I, 5 were derived from winged mothers, and the percentage of winged forms in these were 68, 40, and 13 respectively.

Reproduction by the winged forms does not begin for some hours, or even days, after emerging; but the insects are very sensitive to light, and after a period of sterility may sometimes be induced to give birth by removal to bright sunshine. In the case of A. I, 4, the aphis, after producing two young, remained inactive for four days. She was then put in the sun, and within ten minutes had dropped three larvæ. The winged aphides are distinctly phototropic and invariably fly to the window or other source of light. The apterous forms seem much less sensitive in this respect.

After transference to *Galeopsis*, the fertility of the females does not change much. During July and August five females under observation produced on an average 38·3 young each, with a maximum of eight in the twenty-four hours. On the other hand, the proportion of winged forms was high and appeared in each generation, although exact numbers

were not taken. These females, soon after emerging, began to reproduce on their birth plant.

It is worth remarking that apterous females of *M. ribis* were transferred to *Galeopsis*, for this disposes of the view sometimes held that only certain forms are morphologically or physiologically suited to live on the second host plant, and that the winged migrants leave the first host because they are unfitted to feed there. Last summer I proved that the apterous females of *Rhopalosiphum capreæ*, Fab. can live equally well upon *Salix* or Umbelliferæ. Mordwilko (20) showed that *Aphis sambuci*, Linn., from the elder tree, could live and reproduce upon *Lychnis*, and I have repeated his experiment, using the apterous forms with complete success.

It is the more remarkable that attempts to transfer *M. ribis* back from Labiatæ to currant invariably failed, although no structural difference beyond an increase in size could be detected between forms reared on the two plants. A similar and even more striking case is that of *Aphis grossulariæ*, Kalt. (= *viburni*, Schrank). Here, after the first generation from the migrants, it is impossible artificially to establish the currant form back on the parent *viburnum*, although the *viburnum* form can, with some difficulty, be cultivated on the currant. Meanwhile two stocks, both founded by natural migrants from *viburnum*, breed throughout the summer on currant and *viburnum* respectively, and are structurally indistinguishable.*

TABLE I.

Control lines of descent showing per cent. of winged forms in each generation.

Generation.	Parent.	Per cent. of Winged Forms.	Generation.	Parent.	Per cent. of Winged Forms.
A. II, 2	Youngest	40	B. II, 2	Eldest	74
A. II, 3	Eldest	33	B II, 3	Youngest	11
A. II, 4	Youngest	16	B. II, 4	Eldest	52
A. II, 5	Eldest	17	B. II, 5	Youngest	0
A. II, 6	Youngest	7	B. II, 6	Eldest	0
A. II, 7	Eldest	0	B. II, 7	Eldest	0
A. III, 2	Youngest	51	B. III, 2	Youngest	27
A. III, 3	Eldest	48	B. III, 3	Eldest	27
A. III, 4	Youngest	29	B. III, 4	Youngest	7
A. III, 5	Eldest	7	B. III, 5	Eldest	38
A. III, 6	Youngest	9	B. III, 6	Youngest	0
A. III, 7	Eldest	0	B. III, 7	Eldest	0

* Since the above went to press, I have found young fundatrices of *Aphis grossulariæ* on red currant. It is possible, therefore, that this species may undergo a complete cycle on *Ribes*.

Table J.

The Generation Series.

Generation Number.	Date of Birth.	Age at Birth of First Young.	Repro-ductive Period.	Number of Young.	Average Number of Young per Day of Pro-ductive Period.	Largest Number of Young per Day.	Date of Death.	Total Length of Life.
A.	...	...	?17 days	77	4·9	7	20/v/18	...
B.	...	...	+17 ,,	44	...	4	20/v/18	...
A. I, 1	3/v/18	13 days	5 ,,	24	4·4	7	20/v/18	17 days
B. I, 1	3/v/18	14 ,,	3 ,,	11	3·2	6	20/v/18	17 ,,
A. I, 2	15/v/18	5 ,,	13 ,,	61	4·9	9	8/vi/18	24 ,,
*B. I, 2	16/v/18	12 ,,	5 ,,	5	1	4	3/vi/18	18 ,,
A. I, 3	19/v/18	2 ,,	33 ,,	102	3·3	9	27/vi/18	39 ,,
B. I, 3	27/v/18	11 ,,	12 ,,	40	3·4	6	18/vi/18	22 ,,
*A. I, 4	21/v/18	11 ,,	6 ,,	24	4·0	12	8/vi/18	18 ,,
*B. I, 4	6/vi/18	15 ,,	10 ,,	20	2·0	5	30/vi/18	24 ,,
A. I, 5	1/vi/18	11 ,,	25 ,,	80	3·5	8	6/vii/18	36 ,,
B. I, 5	20/vi/18	14 ,,	8 ,,	7	·7	1	13/vii/18	23 ,,
A. I, 6	11/vi/18	8 ,,	17 ,,	41	2·7	6	6/vii/18	25 ,,
B. I, 6	3/vii/18	24 ,,	21 ,,	19	·9	6	16/viii/18	44 ,,
A. I, 7	18/vi/18	11 ,,	8 ,,	39	4·9	9	6/vii/18	18 ,,
B. I, 7	26/vii/18	17 ,,	13 ,,	32	2·6	7	26/viii/18	31 ,,
A. I, 8	29/vi/18	11 ,,	18 ,,	97	5·7	10	27/vii/18	28 ,,
B. I, 8	10/viii/18	10 ,,	5 ,,	16	3·1	7	26/viii/18	16 ,,
A. I, 9	9/vii/18	15 ,,	12 ,,	50	4·2	10	6/viii/18	28 ,,
B. I, 9	20/viii/18	12 ,,	9 ,,	7	·7	2	9/ix/18	21 ,,
A. I, 10	23/vii/18	6 ,,	3 ,,	30	10	10	1/viii/18	9 ,,
B. I, 10	31/viii/18	10 ,,	9 ,,	3	·3	1	19/ix/18	19 ,,
A. I, 11	28/vii/18	6 ,,	18 ,,	35	1·17	7	23/viii/18	26 ,,
A. I, 12	3/viii/18	21 ,,	16 ,,	20	1·4	4	18/ix/18	46 ,,
A. I, 13	23/viii/18	23 ,,	2 ,,	9	4·1	6	24/ix/18	32 ,,
A. I, 14	14/ix/18	11 ,,	4 ,,	6	1·2	3	23/x/18	40 ,,
A. I, 15	25/ix/18			Sexual	forms			
A. IV, 1	19/v/18	4 ,,	19 ,,	50	2·12	7	12/vi/18	24 ,,
B. IV, 1	13/v/18	7 ,,	28 ,,	120	4·8	10	18/vi/18	36 ,,
A. IV, 2	10/vi/18	15 ,,	37 ,,	104	2·30	6	6/vii/18	26 ,,
B. IV, 2	18/vi/18	12 ,,	16 ,,	11	·6	3	18/vii/18	30 ,,
A. IV, 3	31/vii/18	3 ,,	23 ,,	62	2·16	5	25/viii/18	25 ,,
B. IV, 3	15/vii/18	29 ,,	17 ,,	13	·7	3	31/viii/18	47 ,,
A. IV, 4	24/viii/18	4 ,,	7 ,,	28	4·0	9	8/ix/18	15 ,,
B. IV, 4	28/viii/18	11 ,,	2 ,,	2	1·0	1	22/ix/18	25 ,,

* The asterisk denotes a winged female.

Ecdysis.

M. ribis has a large mortality in the first and second instars, and during observations on the moulting of sixteen individuals, only nine reached maturity. The aphides were frequently found dead in the first and second moults. Mr Brindley tells me that he has observed an appreciable mortality among cockroaches from entanglement with the tracheal tubes during ecdysis, and it is possible that this may hold for aphides also.

This aphis, whether alate or apterous, like all others whose post-embryonic development is known, normally moults four times. The period of immaturity is variable and may be anything between six and nineteen days.

Table K.

Variation in the duration of the different instars.

Date of Birth.	Period between Birth and First Moult.	Period between First and Second Moults.	Period between Second and Third Moults.	Period between Third and Fourth Moults.	Period between Fourth and Fifth Moults.	Period between last Moult and Birth of Young.	Total Length of Immaturity.
June 22, 3 p.m. . .	5 days+ 19 hours.	4 days.	3 days.	2 days.	...	5 days.	19+days.
July 2, 6 p.m. . . .	2 days+ 16 hours.	1 day.	2 "	1 day.	8 days.	6 "	20+ "
June 9, noon (winged female).	1 day+ 22 hours.	4 days.	3 "	3 days.	...	...	12 "
May 29, 3.30 p.m. .	1 day+ 19 hours.	3 "	2 "	6 "	...	3 "	15+ "
June 21, 6.30 p.m. .	15 hours.	3 "	4 "	1 day.	...	1 day.	10 "
June 9	2 days.	3 "	2 "	3 days.	...	3 days.	13 "
July 2, 3 p.m. . . .	1 day+ 19 hours.	1 day.	1 day.	1 day.	1 day.	...	5+ "
July 10	2 days.	2 days.	4 days.	4 days.	...	4 "	16 "
July 11, 5 p.m. . .	Two moults in first 24 hours.	...	6 "	4 "	...	5 "	16 "
Averages	2 days+ 4 hours.	2 "	3 "	2 "	...	4 "	14+ "

Only one of the nine aphides included in Table K was winged, and she became mature in twelve days. Subsequent observations on the generation series showed that protracted development does not necessarily

result in winged forms. For instance, five apterous females of the fifth generation took twenty-three days to complete their ecdyses, while alate forms of the second and fourth generations were respectively eleven and ten days in attaining maturity.

Moreover, these observations on *M. ribis* show that in certain cases the female may have a fifth moult subsequent to the birth of young. It is claimed that a fifth moult has likewise been noticed exceptionally in *Macrosiphum pisi*; and Davis records a case where an apterous female of *Aphis maidi-radicis* produced six young, then moulted, became winged, and gave birth to twenty-one more.

As two of the nine examples of *M. ribis* under observation thus underwent a fifth moult, it seems likely that this is not an uncommon occurrence; but unless looked for, it is easily missed. Moreover, on May 25th, an apterous female of the generation series, which had already produced twelve young, was found in ecdysis.

The Sexual Forms.

The sexual forms have been already figured and minutely described by Flogel (11), and further description is unnecessary here except as regards the frontal tubercles in the male. Flogel remarks that they are almost absent in this sex, and his figure and that of Buckton (2, Plate xxxii, fig. 6) represent the head nearly as flat as in Aphis. In the males from my collection, including individuals from both first and second host plants, the tubercles are quite well developed, although less conspicuous than in the viviparous females. Moreover, a winged female occurred in the A. IV, 3 generation, whose head much resembled that of Flogel's figure. Hence the presence or absence of frontal tubercles is not always a sexual character.

Sexual forms first appeared in the generation series on August 22nd. The apterous mother, A. I, 11, produced thirty-five young, of which the eight eldest were males. At this time no oviparous females had appeared and the males died without mating. A few days later (September 5th) three oviparous females occurred in the B. II, 9 generation, which was the progeny of an apterous female, while the rest of the brood were wingless viviparous forms. After this oviparous females appeared in generations A. I, 14, A. II, 9, A. III, 8, A. IV, 5, and B. II, 10. In every case the mother was apterous, and the rest of the brood were wingless and viviparous. In mixed broods the sexual forms occurred among the earlier births. At the end of September an apterous female, A. I, 14, produced thirteen young. Nine died soon after birth; of the remaining four, two

were males and two were females. From the first these larvæ, which were reared on the same leaf, were distinct in colour—the males being bright green and the females pale yellow. Subsequently mating took place and eggs were laid.

The period of development of the oviparous females varied between sixteen and forty-five days. That of the males was twenty to twenty-five days. The total length of life ranged from twenty-nine to sixty days for females, and from thirty to thirty-five days for males. Oviposition did not take place until two or three days after copulation. Dissections of eight females yielded on an average five large eggs each, with others at various stages in the ovary. The eggs were occasionally deposited on the leaves, but for the most part the aphides crawled to the stem or into the axils of the buds for the purpose. One male may fertilise two, or probably more, females.

Thus it is clear that migration to the second host plant is not obligatory in this species, since the sexual forms can be produced after an unbroken cycle on currant. Mordwilko (20) writing of *A. sambuci*, Linn., says: "If this species is proved to migrate from elder, we should have the interesting problem of the sexuparæ produced on the first host plant; and this does not occur in any other of the migratory *Aphidinæ*." As the foregoing observations show, this is the case with *M. ribis*, a form unique in this respect in the sub-family. Moreover, the males and oviparous females may be produced on the second host plant also, a condition which, as far as I know, does not obtain with any other of the *Aphidinæ*.

Winged males appeared at the beginning of October upon *Galeopsis*. They were the progeny of two winged females of the A. I, 17 generation (*ribis* stock transferred to *Galeopsis*) and were indistinguishable from the males born on currant. Buckton (2, p. 172) says that the nymph of the male of *Phorodon galeopsidis* is very small and has a broad head, but these nymphs were identical with those of the winged viviparous females. The males from Labiatæ mated when placed with oviparous females from currant, and eggs were laid after such unions, but the females were unable to live or oviposit on Labiatæ.

Until the end of October, no oviparous females appeared among the *Galeopsis* stock. The researches of Mordwilko and others show that in other species of migratory *Aphidinæ* the male alone is born on the second host, and that the eggs are never laid there. The observations of Van der Goot (*Beiträge zur Kenntnis des Holländischen Blattlause*) seemed to confirm this for *M. ribis*. He records that migrants of *P. galeopsidis* appeared on currant in the autumn, and that oviposition took place on

that plant; but he doubted whether the species is identical with *M. ribis*, because he failed to establish *M. ribis* upon Labiatæ in the spring. Buckton (2, p. 172) remarks that he failed to find the oviparous female of *P. galeopsidis*, and as all observers are agreed that this form appears only from July to September, it seemed to me probable that the males and viviparous mothers of the sexual females returned to currant, and that the eggs were laid there. In the hope of verifying this, I tried repeatedly to transfer the last two viviparous generations to currant, but in every case they died in a day or two without reproducing. It is worth remarking that the males transferred from *Galeopsis* to *Ribes* lived only three or four days, although they fed readily, while on their birth plant they survived for a week or even longer.

On October 26th, an oviparous female (A. I, 18 generation) was found upon *Galeopsis*. Unfortunately it is not certain whether the parent was winged or wingless. This female differed from the oviparous females on *ribis* only by a vivid green and somewhat interrupted line extending along the dorsum. I put her into a tube with a newly-emerged male from *Galeopsis* and copulation took place within two hours. The following day she laid three eggs upon *Lamium* leaves, and died twenty-four hours later.*

It must not be overlooked that this female came of captive stock, and it is quite conceivable that her parent may have been a migrant, who, kept in confinement, was unable to reach the first host plant, and so of necessity produced her young on the second. At the same time it is clear that the oviparous forms of this species may be born, and can oviposit upon Labiatæ in the laboratory, and there seems no reason why this should not happen in the open, if from any cause *Ribes* is not attainable. The migratory stock from currant might thus under certain circumstances continue indefinitely upon Labiatæ. The difficulty in the way of this view is that *Galeopsis* and *Lamium* are both annuals, and therefore, even if the egg survived the winter, the fundatrix would be faced with starvation when she appeared in spring. Against this must be put the fact that *Lamium* at any rate is a very common weed, and in sheltered situations seems to flourish all the year round. On the whole, I incline to the view that while permanent colonisation of the second host plant is not usual, it is by no means impossible under certain circumstances.

* Since the above was written, these eggs, which were kept over the winter, have shrivelled up without hatching. Steven ("Biology of the Chermes of Spruce and Larch, and their Relation to Forestry," *Proc. Roy. Soc. Edin.*, vol. xxxvii, 1917, pp. 356–381) remarks that the eggs of *Gallicolæ migrantes* laid on spruce, and of *Gallicolæ non-migrantes* laid on larch, do not hatch. The failure of aphid eggs to hatch on certain food-plants requires further investigation.

The fate of the viviparous forms that remain on Labiatæ after the appearance of the sexuales is uncertain, but granted mild weather and sufficient food, there is no reason why they should not continue to reproduce viviparously through the winter. Several *Aphidinæ* are known to be able to do this. For instance, Webster and Phillips (28, p. 93) have shown that in the mild winter of the Southern States of America, *Toxoptera graminum* seldom produces sexual forms, but multiplies parthenogenetically throughout the year. Börner (1) has shown that the viviparous generations of *Aphis pruni*, *Hyalopterus trirhodus*, *Rhopalosiphum lactucæ*, etc., may live far into, or even survive, the winter; and Mordwilko (20) says the same of *Pemphigus affinis*. Probably this is rare in our climate, for although aphides can stand a good deal of cold, prolonged freezing kills them, as is shown by the experiments of Webster and Phillips (28, p. 92).

Thus the persistence of the viviparous line after the production of the sexuales is really dependent on accidents of temperature. The whole viviparous cycle indeed may be compared biologically to the soma of the individual animal. It exists merely as a vehicle of the germ plasm, and when the future of the species has been ensured by the production of the zygote, it may persist for a time under favourable conditions, but sooner or later disappears.

Natural Enemies.

More than a hundred and fifty years ago, Bonnet wrote: "Like as we sow grain to provide for our own subsistence, so it appears that nature sows plant-lice on all kinds of trees and plants for the nourishment of multitudes of different insects."

M. ribis has a large share of natural enemies, both predaceous and parasitic. To the former class belong the larvæ of certain Cecidomyiidæ, which devour numbers of the aphides from June onwards. Theobald (27) points out that Syrphid and Coccinellid larvæ seldom attack this species; but I once took two larvæ of the lacewing (*Chrysopa*) from a red blister. Individuals of all ages are persecuted by a mite, *Anystus cornigera*, Koch, though there is no evidence that it ever causes the death of its host. In July, the fungus *Empusa aphidis*, introduced into the laboratory on some aphides taken from *Galeopsis* in the field, was very prevalent for a time on the generation series on Labiatæ. Those on currant suffered very little, though kept under identical conditions, and I have seldom found aphides on currant killed by this fungus in the open.

But the principal enemy of *M. ribis* is a small parasitic Braconid,

Aphidius ribis, Haliday. Kaltenbach (16) remarks that out of no aphid did he rear so many Aphidiidæ. My observations on the embryonic and post-embryonic development of this parasite are incomplete, and it is intended here to offer only a few preliminary remarks on a subject which I hope to investigate more fully in the future.

Aphides struck by the *Aphidius* were first found on July 3rd, and were common until the end of August. The female *Aphidius* oviposits within twelve hours after emerging from the pupa, but I never induced her to do so without a previous meal of either "honey-dew" from the aphides, or sugar syrup. If a male is present, mating then takes place and oviposition begins within a few minutes. If no male is at hand, she lays a number of eggs parthenogenetically, varying in number from four to twenty. The number of ovipositions after mating was difficult to determine, but seems to be between thirty and forty. Aphides parasitised in the second and third instars frequently die with the larval Braconid within them. Hence the number of dried and bloated aphid skins found on the leaves cannot be taken as representing by any means the whole control of this pest by the *Aphidius*, for there is undoubtedly a large mortality of the hosts, and incidentally of the parasites within them, which is not recognised as due to this cause. An alate *Myzus* was invariably ignored by the *Aphidius*, and I have never found a winged female or nymph containing the parasite; but whether these forms are not attacked in the early stages, or whether they are unable to complete their development after parasitisation, I am unable to say. The apterous females of *M. ribis* are nearly, if not quite, sterile after the attack. On July 16th I isolated seven mature females which had been exposed to an *Aphidius* two days previously. After eight days, one which seemed healthy had produced twenty-five young, four had died, and three contained the pupæ of the parasite.

The larval life of individuals from the parthenogenetic eggs is about fifteen days, and the pupation is between seven and fifteen days. The larval life of individuals from fertilised eggs is about ten days (though in one exceptional case it was twenty-five days), and pupation is six or seven days. Hence the forms from parthenogenetic eggs develop rather more slowly than the others. Both sexes may arise from unfertilised eggs. From thirty-three parthenogenetic ovipositions I was able to rear only five males and two females. The life of the male is three to four days; that of the female is longer and she may live for a week. I have reared *Aphidius ribis* through three generations in the summer.

The *Myzus* is attacked both in the red blisters and on the green leaves, but my observations go to show a greater number of victims on the latter.

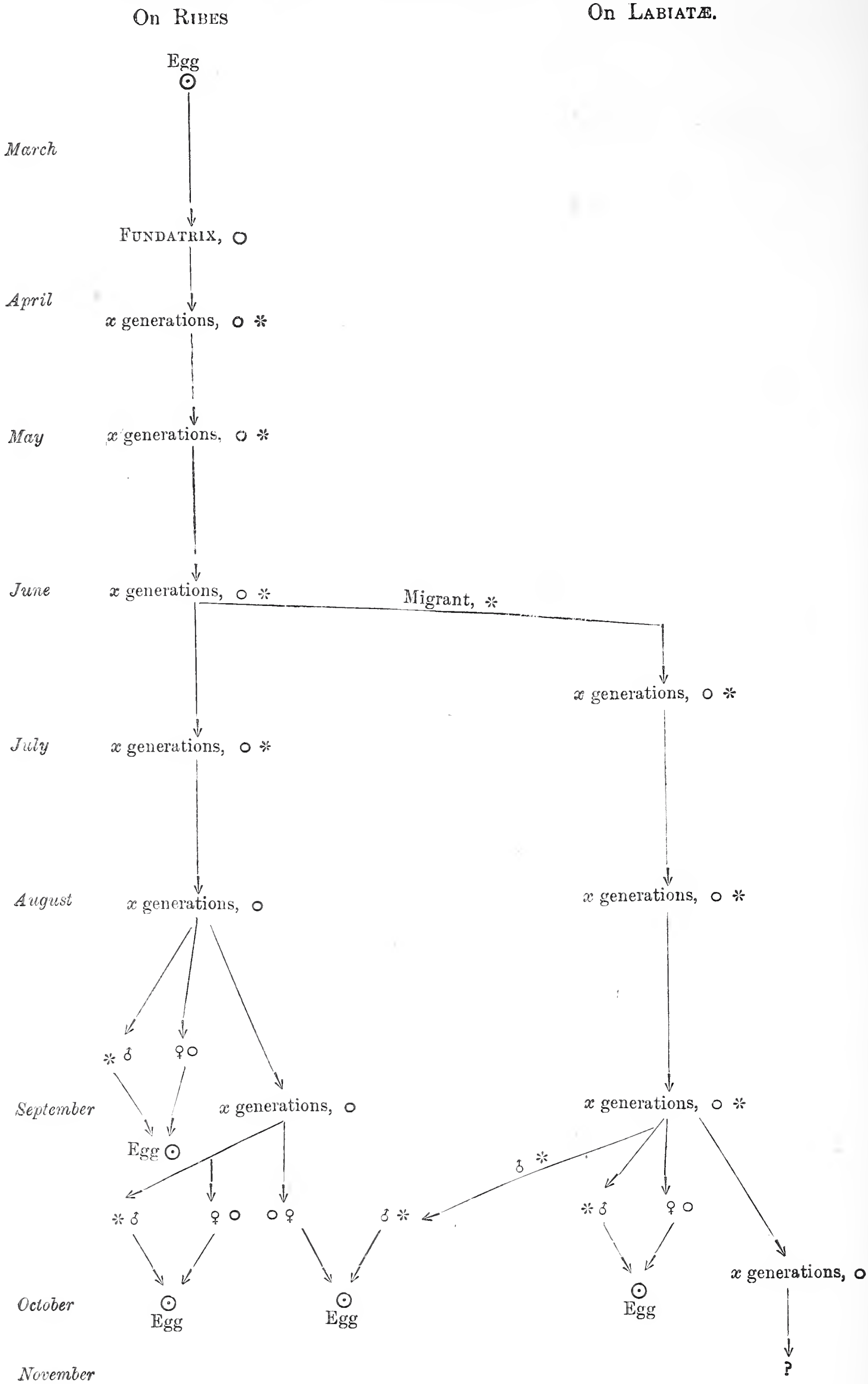

Diagram of the life-cycle of *Myzus ribis*, Linn., from observations on the A.I line in the Laboratory.

○ = apterous form. ✳ = alate form.

Frequently a parasitised aphis is found clinging to the edge of a leaf—a position it seldom chooses when alive; but it is not known whether aphides who have placed themselves thus are more liable to be struck, or whether they crawl thither at the approach of death. The female *Aphidius* is very undiscriminating, and will not only attack the cast skins of the *Myzus*, but will also try to oviposit in a spot where an aphid has been sucking. She also attacks the same victim repeatedly.

I obtained an *Aphidius* indistinguishable from *A. ribis* from *P. galeopsidis* collected in the field in July, and these oviposited in *M. ribis*, both from currant and Labiatæ. I also obtained a single example of a species nearly allied to, if not identical with, *A. rosæ*, Hal.

Summary.

M. ribis, Linn. (red-currant aphis), on *Ribes rubrum* is dimorphic in respect of certain features of the antenna and of abdominal and wing dimensions.

The nature of the food, whether healthy or blistered by the attack of the fundatrix, seems to be the determining factor of this dimorphism.

The form from healthy leaves is probably identical with *M. whitei*, Theobald, and *M. dispar*, Patch.

M. ribis is migratory, and in summer colonises certain Labiatæ and other weeds; but this migration is not obligatory, and the entire life-cycle may be passed on currant.

On its summer host plant this species has been previously described as *Phorodon galeopsidis*, Kaltenbach.

There is a decline in fertility in the later part of the summer among the forms remaining on currant.

This is caused by a lower birth-rate, and not by the shortening of the life of the parent.

This decline, together with the attacks of predaceous and parasitic enemies, accounts for the frequent disappearance of the species from currant in August and September.

Both sexual forms may be produced, and eggs may be laid, on either host plant. Males transferred from Labiatæ to *Ribes* can fertilise the females on the latter plant.

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[*Continued on next page.*

IX.—On Hamilton's Principle and the Modified Function in Analytical Dynamics. By G. H. Livens, M.A. *Communicated by* Professor E. T. Whittaker, F.R.S.

(MS. received February 1, 1919. Read March 3, 1919.)

1. The following note may be of some interest as helping in the elucidation of the rather complex analytical questions involved in the derivation of the modified Lagrangian function for a dynamical system. The results derived also have some bearing on the various questions involved in the transformation theory based on Hamilton's equations of motion. The discussion is given for the simplest type of system, but it can be easily generalised to the less restricted cases covered by the results.

We suppose that the configuration of the system is completely defined by n-generalised co-ordinates $q_1, q_2, \ldots q_n$, the velocities in which may be denoted by $\dot{q}_1, \dot{q}_2, \ldots \dot{q}_n$. If then L denotes the complete Lagrangian function for the system expressed directly in terms of these co-ordinates and velocities, then we know that the motion is completely determined by the conditions that the integral

$$\int_{t_1}^{t_2} \mathsf{L}\,dt$$

taken between fixed time-limits is stationary.

This is the ordinary form of Hamilton's principle, but it involves in any case a complete knowledge of the constitution of the system, because, before it can be applied, it is necessary to know the exact values of the kinetic and potential energies expressed properly in terms of the chosen co-ordinates and their velocities. As, however, we have frequently to deal with systems whose ultimate constitution is either partly or wholly unknown, it is necessary to establish, along the lines laid down by Routh, a modified form of the principles allowing for this ignorance of the constitution of the systems with which we have to deal. The modification was effected by Routh himself for the Lagrangian equations, and by Larmor for the Hamiltonian principle, the result obtained in both cases being practically equivalent to the statement that the ordinary equations may be used if the energy in all ignored co-ordinates is treated as potential energy.

2. In forming the variation of the integral

$$\int_{t_1}^{t_2} \mathsf{L}\,dt$$

we proceed by varying the co-ordinates arbitrarily and then calculating

therefrom the variations of the velocities: this makes the velocity variations dependent on the variations of the co-ordinates. We can, however, formulate the principle in such a way that the velocities and co-ordinates may all be treated as independent variables in forming the variation, latitude being allowed for the ultimate relations which must hold between them by the introduction of a number of undetermined multipliers. We form the variation of the integral

$$\int_{t_1}^{t_2} \mathsf{L}\,dt$$

wherein the function L is considered as a function of the $2n$ variables $q_1, q_2, \ldots q_n, \dot{q}_1, \dot{q}_2, \ldots \dot{q}_n$, these being, however, subject to n equations of the type

$$\dot{q}_r = \frac{dq_r}{dt} \qquad r = 1, 2, \ldots n.$$

The usual method is to introduce n-arbitrary functions of the time $\lambda_1, \lambda_2, \ldots \lambda_n$, then to express the conditions that the integral

$$\int_{t_1}^{t_2} \left[\mathsf{L} - \sum \lambda_r \left(\dot{q}_r - \frac{dq_r}{dt} \right) \right] dt$$

is stationary when the $2n$-quantities $q_1, q_2, \ldots q_n, \dot{q}_1, \dot{q}_2, \ldots \dot{q}_n$ are all independently variable, and finally to choose the functions λ_r so as to make the solutions of the derived equations satisfy the conditions which necessitated their introduction. The equations obtained for the vanishing of the variation are of the type

$$\frac{\partial \mathsf{L}}{\partial \dot{q}_r} - \lambda_r = 0 \qquad \frac{\partial \mathsf{L}}{\partial q_r} - \frac{d\lambda_r}{dt} = 0$$

which are equivalent to the ordinary Lagrangian equations of motion for the system. The undetermined functions introduced are seen to be the momenta corresponding to the different co-ordinates; denoting these by $p_1, p_2, \ldots p_n$ respectively, we see that our result is equivalent to the statement that the equations of motion of the system can be derived by varying the integral

$$\int_{t_1}^{t_2} \left[\mathsf{L} - \sum_{r=1}^{n} p_r \dot{q}_r + \sum_{r=1}^{n} p_r \frac{dq_r}{dt} \right] dt$$

wherein the co-ordinates q_r and the velocities $\dot{q}_r$ are all independent, and the momenta p_r are functions of the time.

3. The result derived in the last paragraph enables us to proceed immediately to the question of the ignoration of some of the co-ordinates of the system. We have obtained the integral of an explicit function of $2n$ independent variables whose variation vanishes when the motion of the

system is determined by the usual equations. We can therefore now alter these co-ordinates by substituting for them others whose values of course depend in some way on those replaced, but which will in themselves still be sufficient to determine the configuration and motion of the system: this is the intrinsic advantage of the variational principle. Let us, for example, replace the first m velocities by their corresponding momenta and regard these latter, with the remaining velocities and all the geometrical co-ordinates, as the independent co-ordinates of the system. The substitution is effected by solving the m equations

$$\frac{\partial L}{\partial \dot{q}_r} = p_r \qquad r = 1, 2, \ldots m$$

for the velocities $\dot{q}_1, \dot{q}_2, \ldots \dot{q}_m$, thereby determining them as functions explicitly of the other velocities, all the co-ordinates and the momenta $p_1, p_2, \ldots p_m$, and then substituting these values in L and the first m terms of the series $\sum\limits_{r=1}^{n} p_r \dot{q}_r$.

The variation of the integral, modified in the manner specified, can now be obtained in the usual way, but it leads to a different set of equations. Firstly, as regards the co-ordinates representing the momenta, we have for each of them two equations of the type

$$\frac{\partial}{\partial p_s}\left(L - \sum_{r=1}^{m} p_r \dot{q}_r\right) + \frac{dq_s}{dt} = 0$$

$$\frac{\partial}{\partial q_s}\left(L - \sum_{r=1}^{m} p_r \dot{q}_r\right) - \frac{dp_s}{dt} = 0$$

The summation in both cases need only be taken as far as the m^{th} term, for all the terms beyond are explicitly independent of the independent co-ordinates and momenta.

For the remaining co-ordinates the equations are of the type

$$\frac{\partial}{\partial \dot{q}_s}\left(L - \sum_{r=1}^{m} p_r \dot{q}_r\right) - p_s = 0$$

$$\frac{\partial}{\partial q_s}\left(L - \sum_{r=1}^{m} p_r \dot{q}_r\right) - \frac{dp_s}{dt} = 0$$

in which the sum Σ *must* now be restricted to the first m terms as given.

The four equations thus derived are equivalent to those usually given for this type of system, and are identical with those given by Routh. The function

$$L' = L - \sum p_r \dot{q}_r$$

is usually called Routh's modified function, and so far as the motion in the

non-modified co-ordinates is concerned it entirely replaces the ordinary Lagrangian function.

4. It is of importance to notice that the process employed above to render the velocities and co-ordinates independent as regards the initial variational problem need not be carried out in its entirety, it being only necessary to carry it to the extent of rendering it valid for the co-ordinates to be modified. Thus, for example, for the purposes of the last paragraph, we need only consider the variation of the integral

$$\int_{t_1}^{t_2}\left[\mathsf{L} - \sum_{r=1}^{m} p_r\dot{q}_r + \sum_{r=1}^{m} p_r \frac{dq_r}{dt}\right]dt$$

the variation with respect to all but the first m co-ordinates being effected in the usual manner.

This remark leads us to our next point. The complete equations of motion are equivalent to the conditions for the vanishing of the variation of this integral taken as if the velocities (or their momenta) and the co-ordinates are independent if they are represented in the sum $\sum$. Now, in the special case when the momenta are all constant in time this integral may be replaced by the integral

$$\int_{t_1}^{t_2}\left(\mathsf{L} - \sum_{r=1}^{m} p_r\dot{q}_r\right)dt = \int_{t_1}^{t_2}\mathsf{L}'dt$$

for the outstanding terms

$$\sum \int p_r \frac{dq_r}{dt}dt = \sum p_r \Big/ q_r \Big/_{t_1}^{t_2}$$

reduce to a set of constants depending only on the initial and final configurations, and cannot therefore contribute anything to the general expression for the variation. This is the result derived by Larmor, that it is in this special case that the integral

$$\int_{t_1}^{t_2}\mathsf{L}'dt$$

possesses the minimum property usually associated with the Hamiltonian integral.

Larmor's result has a still more general significance, for in all cases, whether the momenta in the modified co-ordinates are constant or not, the variation of the integral

$$\int_{t_1}^{t_2}\mathsf{L}'dt$$

with respect to the non-modified co-ordinates leads to the proper equations for the motion in those co-ordinates, for the remaining part of the complete

integral does not involve explicitly either the co-ordinates or the velocities in the unmodified part of the system.

5. So far our discussion has centred round the question of ignoration of co-ordinates, but the result obtained in paragraph 2 enables us to approach some of the most important results in the transformation theory associated with Hamilton's form of the equations of motion. We first write

$$H = \sum_{r=1}^{n} p_r \dot{q}_r - L$$

for the Hamiltonian function, and then, if we regard the co-ordinates q_r and the momenta p_r as the independent variables, we see that the equations of motion of the dynamical system are of the Hamiltonian type

$$\frac{\partial H}{\partial p_r} = \frac{dq_r}{dt}, \qquad \frac{\partial H}{\partial q_r} = -\frac{dp_r}{dt},$$

these being the conditions that the integral

$$\int_{t_1}^{t_2}\left(-H + \sum p_r \frac{dq_r}{dt}\right)dt$$

is stationary, the integrand being regarded as a function of the $2n$ independent variables $q_1, q_2, \ldots q_n, p_1, p_2, \ldots p_n$ and the time.

Suppose we now write

$$I = -H + \sum_{r=1}^{n} p_r \frac{dq_r}{dt}$$

then using δ to denote variations in which the time is maintained constant we have

$$\delta I = \sum_{r=1}^{n}\left(-\frac{\partial H}{\partial q_r}\delta q_r - \frac{\partial H}{\partial p_r}\delta p_r + \frac{dq_r}{dt}\delta p_r + p_r d\frac{\delta q_r}{dt}\right)$$

$$= \frac{d}{dt}\sum_{r=1}^{n} p_r \delta q_r$$

It follows that

$$\int \sum_{r=1}^{n} p_r \delta q_r$$

taken round a closed curve in the $2n$-dimensional space $(q_1, q_2, \ldots q_n, p_1, p_2, \ldots p_n)$ is an integral invariant of the dynamical system.

Conversely, if

$$\int \sum_{r=1}^{n} p_r \delta q_r$$

is an integral invariant in the above sense of a dynamical system, we must have

$$\int \frac{d}{dt}\left(\sum_{r=1}^{n} p_r \delta q_r\right) = 0$$

so that, on account of the arbitrariness of the curve to which the invariantive property relates,

$$\frac{d}{dt}\sum_{r=1}^{n} p_r \delta q_r \equiv \sum_{r=1}^{n}\left(\dot{p}_r \delta q_r + p_r \delta \dot{q}_r\right)$$

is a complete differential of some function I of the variables $(q_1, q_2, \ldots q_n, p_1, p_2, \ldots p_n)$ which may also contain the time as a parameter.

We have then

$$\delta \mathrm{I} \equiv \sum_{r=1}^{n} (\dot{p}_r \delta q_r + p_r \delta \dot{q}_r)$$

so that

$$\delta(\mathrm{I} - \sum_{r=1}^{n} p_r \dot{q}_r) = \sum_{r=1}^{n} (\dot{p}_r \delta q_r - \dot{q} \delta p_r)$$

and therefore using

$$-\mathrm{H} = \mathrm{I} - \sum_{r=1}^{n} p_r \dot{q}_r$$

we see that the equations of motion of the system are of the Hamiltonian type

$$\frac{\partial \mathrm{H}}{\partial q_r} = -\dot{p}_r \ , \qquad \frac{\partial \mathrm{H}}{\partial p_r} = \dot{q}_r$$

From this last result or by the same argument it is concluded that if a new set of variables $(\mathrm{Q}_1, \mathrm{Q}_2, \ldots \mathrm{Q}_n, \mathrm{P}_1, \mathrm{P}_2, \ldots \mathrm{P}_n)$, functions of $(q_1, q_2, \ldots q_n, p_1, p_2, \ldots p_n, t)$, be chosen as co-ordinates for the dynamical system, and if

$$\sum_{r=1}^{n} \mathrm{P}_r \delta \mathrm{Q}_r$$

is an integral invariant of the original system, then the new equations of motion are still of the type

$$\frac{\partial \mathrm{K}}{\partial \mathrm{Q}_r} = -\dot{\mathrm{P}}_r \ , \qquad \frac{\partial \mathrm{K}}{\partial \mathrm{P}_r} = \dot{\mathrm{Q}}_r$$

The transformations from the variables (p, q) to the variables (P, Q) are in general special to the problem considered, but they include all contact transformations, which are of a less special type. Let us consider the contact transformation defined by the relations

$$\Omega_s = 0 \qquad s = 1, 2, \ldots k$$

$$\mathrm{P}_r = \frac{\partial \mathrm{W}}{\partial \mathrm{Q}_r} + \sum_{s=1}^{k} \lambda_s \frac{\partial \Omega_s}{\partial \mathrm{Q}_r}, \qquad p_r = -\frac{\partial \mathrm{W}}{\partial q_r} - \sum_{s=1}^{k} \lambda_s \frac{\partial \Omega_s}{\partial q_r}$$

where (Ω_1, Ω_2, . . . W) are functions of the variables (q_1, q_2, . . . q_n, Q_1, Q_2, . . . Q_n, t). From these equations we have immediately

$$\sum_{r=1}^{n} p_r \frac{dq_r}{dt} = \sum_{r=1}^{n} P_r \frac{dQ_r}{dt} + \frac{\partial W}{\partial t} + \sum \lambda_s \frac{\partial \Omega_s}{\partial t} - \frac{dW}{dt}$$

so that we have

$$-H + \sum_{r=1}^{n} p_r \frac{dq_r}{dt} \equiv -K + \sum_{r=1}^{n} P_r \frac{dQ_r}{dt} - \frac{dW}{dt}$$

where

$$K \equiv H - \frac{\partial W}{\partial t} - \sum_{s=1}^{k} \lambda_s \frac{\partial \Omega_s}{\partial t}$$

and the equations of motion are thus derived by varying the integral

$$\int_{t_1}^{t_2} \left(-K + \sum_{r=1}^{n} P_r \frac{dQ_r}{dt} - \frac{dW}{dt} \right) dt$$

The last term in this integral is irrelevant to the problem, as it integrates out to terms at the time-limits, and therefore the equations of motion are

$$\frac{\partial K}{\partial P_r} = \frac{dQ_r}{dt}, \qquad \frac{\partial K}{\partial Q_r} = -\frac{dP_r}{dt}$$

Thus, if the transformation of co-ordinates is a contact transformation, the Hamiltonian form of the equations of motion for any system is conserved.

These are a few of the more important results of the transformation theory in dynamics: the remainder can be derived equally readily if the principles underlying the above discussion are kept in view; but it does not seem necessary to develop the discussion any further in the present place. It may, however, serve some purpose to conclude by emphasising the fact that the two types of equation, the Lagrangian and the Hamiltonian, can *both* be derived from the same integral by the variational method, using as the modified Lagrangian function of the system the expression

$$L - \sum p_r \left(\dot{q}_r - \frac{dq_r}{dt} \right)$$

and treating as independent variables the co-ordinates q_r and either the velocities $\dot{q}_r$ or their momenta p_r, or any suitable functions of these.

(*Issued separately July* 3, 1919.)

X.—The Cooling of the Soil at Night, with Special Reference to late Spring Frosts. By Captain T. Bedford Franklin, B.A. (Cantab.). *Communicated by* The General Secretary.

(MS. received April 7, 1919. Read May 5, 1919.)

SUMMARY.

I. Introduction.

In this paper I am mainly concerned with the surface soil temperature, but as we are accustomed by long usage to think of the grass minimum temperature as determining the occurrence or non-occurrence of frost, a few words of explanation on this point is perhaps necessary at the outset.

The grass minimum on nights of rapid radiation certainly does fall considerably below the surface soil minimum, but this is due to the very fact that it is the grass minimum, *i.e.* the air temperature just on the grass.

Now grass is a notoriously bad conductor, and Dr Aitken, in his paper "On Dew,"* mentions an instance when the grass minimum was 18·5° F. lower than the temperature of the surface of the soil beneath it. I myself, on February 10, 1919, observed a grass minimum of 15° F. when the surface of the soil beneath was 33° F., and a single primrose was flowering with

* "On Dew," by John Aitken, LL.D., F.R.S., *Transactions of the Royal Society of Edinburgh*, vol. xxxiii.

its flowers and roots in temperatures differing by 18° F. The air, as it were, rests on a cushion of non-conducting grass, and fails to make intimate contact with the soil, and so gains little or no heat from it.

No such large differences have existed during the past winter (1918–19) between the temperature of the air resting in close contact with open soil and the surface soil itself—in fact, the maximum difference only reached 5° F. on two occasions, and the average over many clear nights was only 2·4° F. Dr Aitken mentions a grass minimum 7° F. lower than the air minimum over open soil, and the following figures for nights during the past winter are from my own observations:—

Table I.

Date.	Surface soil minimum.	Air over open soil minimum.	Air over ashes minimum.	Grass minimum.
October 1, 1918	38·0° F.	38·0° F.	32·0° F.	28·5° F.
November 13, 1918	31·0°	28·5°	27·0°	24·0°
February 10, 1919	22·0°	19·0°	17·5°	15·0°

Thus the average difference was only 1·8° F. between the minima of soil and air over it, while the average difference between the soil minima and grass minima was as much as 7·8° F.

It would appear, therefore, that the grass minimum gives us a totally wrong impression of the fall in temperature of the air over open soil. In large fields, or even in gardens where lawns or ash or gravel paths do not cut up the cultivated plots, the air minimum will not differ greatly from the surface soil minimum, and the determination of the soil minimum will go a long way towards solving the question of whether there will be a frost or not.

The three main causes of the cooling of the soil are radiation, evaporation, and the fall of cold rain, sleet, or snow.

Radiation to be effective presupposes a clear sky and a dry atmosphere; we should therefore expect there to be a relation between the rate of radiation and the relative humidity. The existence of such a relation is discussed in Section III (*a*).

But on equally clear nights of the same average relative humidity, and even the same sunset temperature, we find very different surface soil minima—these being notably low when the soil surface is dry, and lower in a spring month when the underground layers have not warmed up after winter, than in an autumn month of the same length of night when the

underground layers are still warm from the summer. This is clearly shown in fig. 1.

Again, radiation is peculiarly hampered in its efforts to reduce the temperature of the soil; not only has it to overcome the up-flowing heat conduction from the lower and warmer layers, but in addition, by the very action of freezing the surface, it stores up against itself a reserve of latent

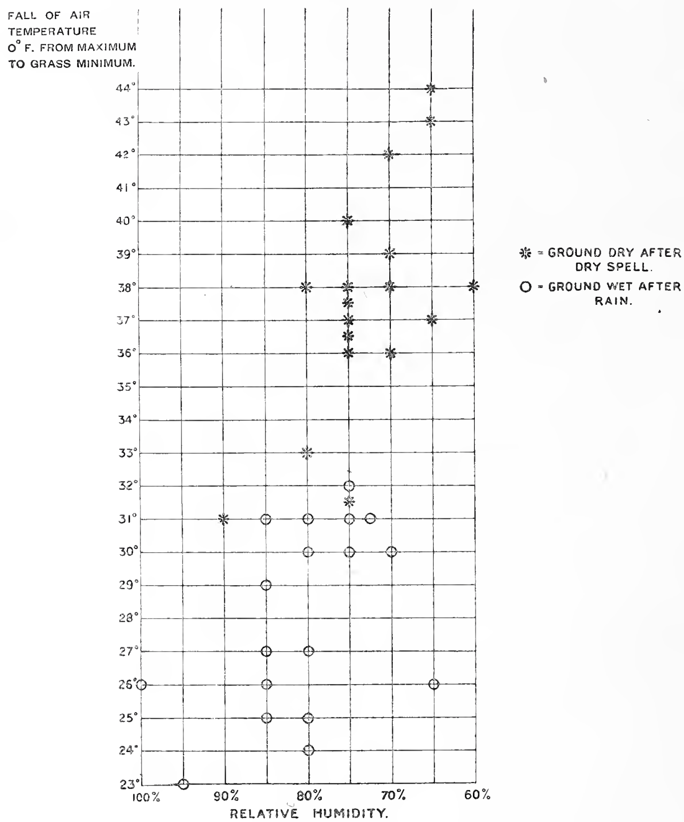

Fig. 1.—The above table is compiled from the observations at the Radcliffe Observatory, Oxford, during clear nights of April 1906–1915.

heat which has to be counterbalanced before any further fall in temperature of the soil can take place. The balance only of the radiation, after overcoming both these counter-influences, is available for lowering the temperature of the soil.

Thus when the surface is dry and conductivity is reduced, or when the temperature of the underground layers is low, the radiation having less to counteract has a larger balance left over for cooling the surface. The temperature of the surface soil thus depends on :—

(*a*) The relative humidity.
(*b*) The dryness of the surface layers.
(*c*) The temperature of the underground layers.

And the radiation balances the conduction, the latent heat, and the heat given up in cooling the surface layers; in other words, $R=C+L+CSL$, and if any three of these quantities are known, the fourth may be found.

During the past winter I have made observations of the quantities

R = Radiation,
C = Conduction,
L = Latent heat,
CSL = Heat given up in cooling surface layers,

all in calories per square centimetre, on every available clear night, and the first part of this paper shows that the relation given above between these quantities does hold in practice as well as in theory. If the relation can be shown to hold good, then by observing R, C and L, or R and C only if the surface does not freeze, we can calculate the possible fall of the surface temperature and so find the minimum surface soil temperature.

Since radiation is a surface phenomenon, we may reduce its effect by screening the surface, or by covering the soil with a layer of some poor conductor. The practical results of this are shown in Section VII.

Evaporation requires wind and a dry atmosphere, with or without a clear sky. It is very local in its effect, for as soon as a dry layer is formed on the surface, evaporation practically ceases.

Rain, sleet, or snow can only fall from a clouded sky; in comparison to the time during which they act, these varied forms of precipitation are very effective in cooling the soil. A heavy fall of melting snow will reduce the temperature of the soil to 32° F. to a depth of several inches, and will do more in a few hours towards cooling the lower layers of the soil than several nights of rapid radiation.

The practical results of screening the soil from evaporation and rain, sleet, and snow are given in Section VII.

II. The Physical Constants of the Soil under Consideration, and the Notation employed.

The soil in which the observations given in this paper have been made is a garden soil consisting of a layer of loam, rich in humus, of about 6 in. depth, resting on a stony subsoil of quite different nature. The constants here given refer only to the surface layer of made soil, in which all the observations have been taken; they are the mean results of many

experiments in the laboratory and in the soil itself. For comparison, I have given alongside them the values obtained in similar soils by well-known authorities.

TABLE II.

Constant.	The author.	E. J. Russell, *Soil Conditions and Plant Growth.*	Ingersoll and Zobel, *Theory of Heat Conduction.*	A. D. Hall, *The Soil.*	Mosier and Gustafson, *Soil Physics and Management.*
True density of dry soil	D=2·5	2·31	...	2·65	2·62
Apparent density of dry soil	D=1·25	1·46	...	1·22	1·36
Density of normal wet soil	D=1·7	...	1·65	...	1·55 (20% moisture)
Percentage volume:					
(1) Solid	50%	52·7%	...	50·8%	52·9%
(2) Water *	45%	40·0%	...	49·2%	47·1%
(3) Air	5%	7·3%	...	...	...
Specific heat wet soil	S=·46	...	·45	...	·36 (20% moisture)
Calories required to raise 1 c.c. 1° C.	C=·8	...	·74	...	·55 (20% moisture)
Latent heat of freezing 1 c.c.*	L=36 calories				
Conductivity *	K=·004	...	·0037		
Diffusivity *	H^2=·005	...	·0049	...	·005

* During the past winter the soil has remained almost uniformly damp up to the beginning of April. If, however, the surface soil becomes dry, new values for these must be found by experiment.

Throughout the paper the following notation will be used:—

ϕ_m = mean of surface temperatures °F. during observations.

ϕ'_m = mean of 4 in. depth " " "

$\theta_m = \phi'_m - \phi_m$ = mean difference of surface and 4 in. depth temperatures °F. during observations.

θ' = fall of surface temperature °F. during observations.

θ'' = fall of 4 in. depth " " "

θ'_F = fall of surface temperature °F. below 32° F.

x = depth of soil frozen (in centimetres) at beginning of observations.

x' = depth of soil " " at end of observations.

x_m = mean depth " " during observations.

h = hours during period of observations.

R = mean rate of radiation in calories per square centimetre per minute during period of observations.

The temperatures all through this paper are in degrees Fahrenheit.

III. The Method of taking Observations.

(a) *Radiation and Relative Humidity.*

According to Angström, "The cooling of a body, exposed to radiate to a clear night sky, is almost independent of the temperature of the surroundings, provided that the relative humidity keeps a constant value." *

From the observations made by Angström in Algeria and California in 1912 and 1913, I have worked out the relation between the rate of radiation and the relative humidity at each of these observations. The curve obtained is given in fig. 2, and gives the rate of radiation in calories per square centimetre per minute for any given value of the relative humidity.

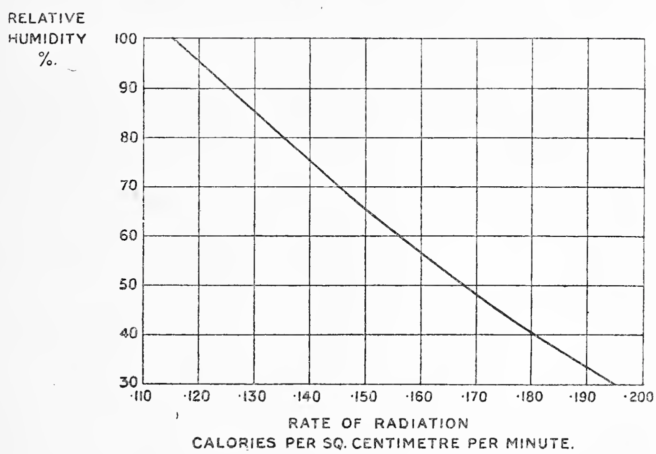

Fig. 2.

By taking several observations of the value of the relative humidity during the night, a mean value for the whole night is obtained, and the corresponding value of the rate of radiation is read off this curve.

I have employed the magnitude of the stars visible as a test of the complete clearness of the night; and I have only given the results of observations on nights of little or no wind, when stars of the 5th magnitude were clearly visible.

The total radiation in calories for the period h is 60 Rh.

(b) *Latent Heat of Freezing.*

From the table of constants the latent heat liberated on freezing 1 cubic centimetre of soil is 36 calories. Therefore the latent heat liberated when

* "A Study of the Radiation of the Atmosphere," by Anders Angström, *Smithsonian Miscellaneous Collections*, vol. lxv, No. 3.

the soil is frozen to a depth x centimetres is $36x$ calories per square centimetre.

If therefore the soil is frozen to a depth x at the beginning of the observations and x' at the end, the latent heat liberated $= 36(x' - x)$ calories.

(c) *Upward Conduction.*

In calculating the upward conduction from the warmer and lower strata, the question of the depth at which it would be best to take our observations has to be settled at the outset.

I have chosen a depth of 4 in. (10 centimetres) for the following reasons:—

(1) The surface soil only extends to a depth of 6 in.; below this the subsoil is of quite a different character. The conductivity at depths of 4 in. and 6 in. was found to be in each case equal to ·004; below 6 in. depth it varied from place to place.

(2) The lag of the maximum temperature at a depth of 4 in. is about $3\frac{1}{2}$ hours, and so roughly coincided with sunset during the greater part of the winter. This much simplified the calculation of the average temperature (ϕ'_m) at that depth during the night.

(3) Since 4 in. $=$ 10 centimetres, the computations were simplified.

To obtain a measure of the upward conduction, the difference between the temperature at the surface and 4 in. depth at various times during the night seemed necessary; but it also appeared reasonable to suppose that the mean difference would not vary much from the difference between the mean temperatures at the surface and at a depth of 4 in.

If this were so, then the conduction upwards equals

$$\frac{\mathrm{K}(\phi'_m - \phi_m)}{10} \times \frac{5}{9} \text{ calories per sq. centimetre per second}$$

$$= \frac{\cdot 004}{10} \times \frac{5}{9} \times \theta_m \times 60 \times 60h \text{ calories during the night}$$

$$= \cdot 8\theta_m h \text{ calories} \qquad (1)$$

I therefore tested, by taking hourly readings of the temperature at the surface and 4 in. depth, on two nights, November 21, 1918, and February 26, 1919, whether there was likely to be any serious discrepancy in taking $\phi'_m - \phi_m$ as the mean difference of the surface and 4 in. temperatures during the night.

It would appear that $\phi'_m - \phi_m$ is a very close approximation to the mean difference of the surface and 4 in. temperatures. Thus on November 21, $\phi'_m - \phi_m$ was equal to 6·0° F., whilst the average of the hourly differences of surface and 4 in. temperatures was 6·1° F. Similarly, on

February 28, $\phi'_m - \phi_m$ was 3° F. as against 2·95° F. Formula (1) holds as long as the surface does not freeze; when the soil is frozen, the bottom of the frozen layer is always at 32° F. Conduction from the 4 in. depth is now

$$\frac{K(\phi'_m - 32)}{10 - x_m} \times \frac{5}{9} \times 60 \times 60 \times h \text{ calories}$$

$$= \frac{8(\phi'_m - 32)}{10 - x_m} h \text{ calories} \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

(d) The Cooling of the Soil.

Since the soil freezes at 32° F., so long as the surface does not fall to 32° F. the temperature gradient is more or less uniform between the 4 in.

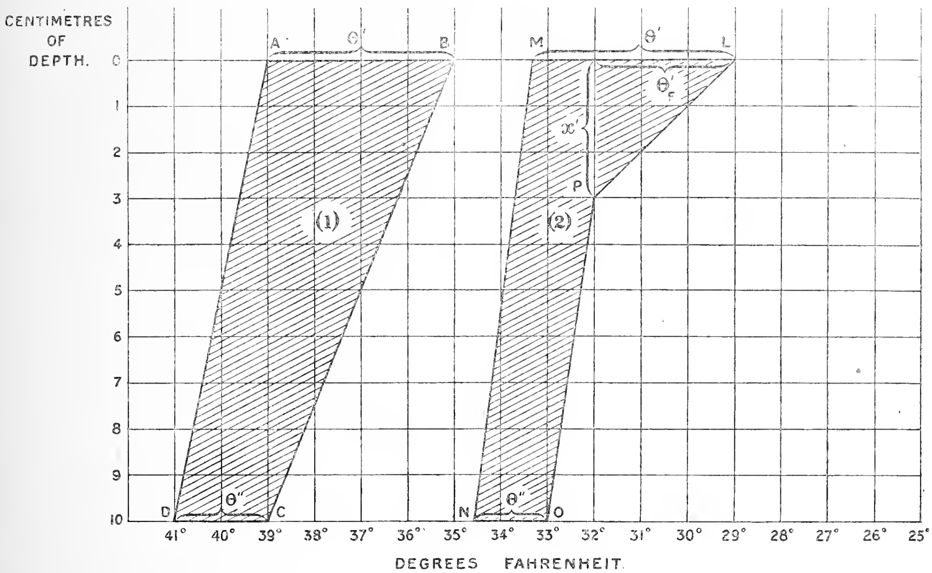

Fig. 3.

depth and the surface. This uniformity must obviously cease as soon as freezing sets in with its liberation of latent heat.

Fig. 3 shows graphically the cooling of the soil between the 4 in. depth and the surface:—

(1) When the surface does not freeze.

(2) When the surface freezes.

When the surface does not freeze, the number of calories used up in cooling the soil is—

$$\cdot 8 \times \frac{5}{9} \times \text{area ABCD}$$

$$= \cdot 44 \times \frac{10}{2}(\theta' + \theta'')$$

$$= \cdot 22[10(\theta' + \theta'')] \quad . \quad . \quad . \quad . \quad . \quad . \quad (3)$$

When the surface freezes to a depth x', the number of calories used up is—

$$\cdot 8 \times \frac{5}{9} \times \text{area LMNOP}$$

$$= \cdot 44 \times (\text{area LMNO} - \text{area LPO})$$

$$= \cdot 44 \times \left(\frac{10}{2}(\theta' + \theta'') - \frac{10 - x'}{2}\theta'_{\mathrm{F}}\right)$$

$$= \cdot 22[10(\theta' + \theta'') - (10 - x')\theta'_{\mathrm{F}}] \quad . \quad . \quad . \quad . \quad . \quad (4)$$

The accuracy of formula (4) is only approximate, and is greater the nearer the surface and 4 in. temperatures are to freezing-point at the beginning of the period of observations.

Formula (4) becomes the same as (3) where the surface does not freeze, *i.e.* when x' and θ'_{F} both become zero.

IV. Available Data.

The results of observations on calm clear nights during the winter of 1918–19 are given in Table III. The observations are grouped into two separate divisions—the nights when the temperature of the surface was almost wholly above 32° F. and the nights when the surface temperature was almost wholly below 32° F. being grouped separately.

If we analyse the values given in column 7 of the table we see that, in all cases when the surface did not fall below 32° F., the whole of the radiation has not quite been accounted for. This is to be expected, since we have not taken into account the cooling of those layers of soil below the 4 in. depth.

But choosing the nights on which the temperature of the 4 in. depth was close to 32° F. during the period of observations, and so could fall little lower without freezing, we see that the radiation on these nights has been accounted for to a close degree of approximation.

Thus on the following nights on which ϕ'_m was about 33° F. the differences between the actual computed radiation and the amount of that radiation accounted for by conduction, latent heat, and cooling of the soil are as follows:—

	Calories.
January 19, 1919	+2·7
February 9, „	+0·5
„ 24, „	+2·9
March 3, „	−5·6
„ 4, „	−2·8

TABLE III.

1 Date.	2 Conduction upward. Calories.	3 Latent heat liberated on surface freezing. Calories.	4 Cooling of surface layers. Calories.	5 Total calories to balance radiation.	6 Radiation. Calories.	7 Difference, cols. 5 and 6. Calories.	Remarks.
1918.							
November 22	50·0	72·0	5·7	127·7	129·6	−1·9	Nights when surface temperature was below freezing-point almost all night.
1919.							
January 19	27·3	82·8	14·1	124·2	121·5	+2·7	
February 8	47·0	72·0	9·2	128·2	130·5	−2·3	
„ 9	35·0	82·8	13·2	131·0	130·5	+0·5	
„ 24	36·3	64·8	11·0	112·1	109·2	+2·9	
March 3	28·0	79·2	9·0	116·2	121·8	−5·6	
„ 4	25·0	72·0	4·6	101·6	104·4	−2·8	
1918.							
November 13	76·8	21·6	22·0	120·4	124·8	−4·4	Nights when surface temperature was above freezing-point almost all night.
December 4	72·8	...	24·2	97·0	105·0	−8·0	
„ 8	84·0	...	22·0	106·0	109·2	−3·2	
„ 15	102·4	...	22·0	124·4	134·4	−10·0	
1919.							
February 27	28·0	36·0	18·0	82·0	78·0	+4·0	
March 2	28·8	36·0	24·4	89·2	90·0	−0·8	

It would appear that, during the period November 1918–March 1919, the following points were borne out by actual observations:—

(1) That the radiation from the soil may be accounted for in counterbalancing the upward conduction and the latent heat of freezing—the residue only cooling the soil.

(2) That the rate of radiation of the soil on calm clear nights, when 5th magnitude stars are visible, is a function of the relative humidity.

(3) That other causes such as condensation, evaporation, etc., have little effect on the temperature of the soil on calm clear nights.

(4) That the surface tends to fall rapidly such a number of degrees below the temperature of the 4 in. depth as will make the conduction from this depth balance the radiation; after this takes place the surface temperature can fall no faster than that of the 4 in. depth, and a sufficiently high temperature underground will obviously render a frost unlikely.

(5) That this temperature difference between the surface and the 4 in. depth, which makes the upward conduction balance the radiation, is probably about 10° F. during the winter, when the soil is almost invariably wet, and of uniform maximum conductivity from day to day, but may be as much as 20° F. after a dry spell in spring or early summer.

(6) That the prediction of frost on any given night depends on the possibility of assessing the value of the following:—

(*a*) Average relative humidity during the night.

(*b*) The temperature of an assigned depth—say 4 in.—at the time of surface minimum.

(*c*) The conductivity of the layer between the assigned depth and the surface.

(*d*) The difference between the surface soil minimum and that of the air above it.

(7) That it might be possible, after an extended series of observations with a set of electrical resistance thermometers, to forecast the minimum temperature on calm clear nights from observations taken in the early afternoon.

With a view to thoroughly investigating this question, I have recently had a set of four electrical resistance thermometers installed, and have taken hourly readings through the day and on clear nights during the last seven weeks (March and April 1919).

In so short a period one could not expect to arrive at any definite conclusions, but the observations so taken clearly point to the fact that the difficulties of forecasting the minimum surface soil temperature may be overcome successfully in the near future.

V. The Thermal Effect of a Layer of Poor Conducting Material on the Surface of The Soil.

To obtain a measure of the efficiency of various poor conductors in checking the loss of heat from the surface of the soil on clear nights, I

Table IV.

Date.	Open firmed soil minimum.	Minimum under ½ in. loose raked soil.	Minimum under ½ in. ashes.	Minimum under ½ in. manure.	Air just over soil minimum.	Weather.
1918.						
September 6 .	40·0	...	46·0	46·5	...	Clear
„ 8 .	38·5	...	44·0	45·0	...	„
„ 29 .	33·5	...	36·0	36·5	...	„
„ 30 .	36·0	...	39·0	39·0	...	„
October 13 .	33·5	34·5	36·5	37·0	...	„
„ 16 .	34·5	35·5	38·5	39·0	...	„
November 8 .	36·0	37·5	39·0	39·5	36·0	„
„ 9 .	32·5	34·0	36·5	37·5	33·0	„
„ 11 .	34·5	35·5	38·0	39·0	34·0	„
„ 12 .	28·0	31·0	33·0	34·0	24·0	„
„ 13 .	28·5	30·0	33·0	34·0	27·0	„
„ 15 .	29·0	31·0	33·0	34·0	27·0	„
Average increase in temperature over open firmed soil		1·5	4·0	4·7		
Greatest increase on any night		3·0	6·0	6·5		

have made a series of observations during the past winter on the minimum temperature of the surface when covered with layers (½ in. thick) of—

(1) Loose soil, well raked,
(2) Ashes,
(3) Manure,
(4) Fallen leaves,
(5) Grass and moss growing naturally,

as compared with the minimum temperature of open firmed soil. These results are given in Tables IV and V. Three very suitable periods for such observations occurred from December 15–22, December 23–26,

1918, and February 8–11, 1919. During these periods hard frost took place each night, and the day temperature seldom rose above 32° F.

The soil under the moss and grass has never frozen all winter, and it is interesting to note that the only occasion when it was near it was on February 5, 1919, after snow and rain, with a relatively high grass minimum of 28° F.

TABLE V.

Date.	Grass minimum.	Open soil.	Soil covered ½ in. fallen leaves.	Soil covered ½ in. moss and longish grass.	Remarks.
1918. December 15 ,, 16 ,, 17 ,, 18 ,, 19 ,, 20 ,, 21 ,, 22	 26·5 26·0 26·0 26·5 25·0 24·5 27·5 25·0	 Frozen to depth of 3 in.	 *Not* frozen	 *Not* frozen Temperatures— (*a*) Just under moss 33·0 (*b*) 2 in. depth 34·5 (*c*) 4 in. ,, 36·0	 Minimum surface temperature of open soil, 28·5
December 23 ,, 24 ,, 25 ,, 26	32·0 29·0 30·0 25·5	Frozen to depth of 2 in.	*Not* frozen	*Not* frozen Temperatures— (*a*) Just under moss 34·0 (*b*) 2 in. depth 36·0 (*c*) 4 in. ,, 37·5	
1919. February 5	 28·0	 Frozen to depth of ½ in.	 Frozen	 *Not* frozen but on point of freezing	 After snow and rain on previous afternoon
February 17	26·0	Frozen to depth of ¾ in.	*Not* frozen	*Not* frozen	Very cold N.E. wind —a freeze
February 8 ,, 9 ,, 10 ,, 11	26·0 17·5 15·0 22·0	Frozen to depth of 4 in.	Frozen to depth of ½ in., but not till open surface fell to 25·0	*Not* frozen Temperatures— (*a*) Just under moss 32·25 (*b*) 2 in. depth 33·0 (*c*) 4 in. ,, 34·0	Minimum surface temperature of open soil, 22·0

The maximum thermal efficiency of the various coverings seems to be as follows:—

	°F.
(1) Loose raked soil	3·0
(2) Ashes	6·0
(3) Manure	6·5
(4) Fallen leaves	7·0
(5) Natural moss and grass . . .	10·0

VI. The Effect of Re-aeration of the Soil after Rain.

A remarkable sudden alteration in underground temperature—due, it would appear, to the re-aeration of saturated soil after rain—may be noted here. On December 5, 1918, heavy rain began at 10 a.m., when the surface was at 39° F. and the 6 in. depth 44° F. As the rain percolated through the colder surface layers the 6 in. temperature fell to 42° F. By 12.30 p.m. the ground was saturated with water standing in pools on the surface. Then the clouds cleared very rapidly, the sun shone brilliantly, and the air temperature rose to 52° F.

As the warm air was drawn into the soil after the receding water the 6 in. temperature rose to 46·5° F.—a rise of 4·5° F. in two hours.

The reverse took place on December 10, 1918, when heavy rain began to fall at 2 p.m., when the surface was at 43° F. and the 6 in. temperature at 41·5° F. The rain percolating through the warm surface soon raised the 6 in. temperature to 43° F. also. By 6.30 p.m. the ground was flooded as before, by 7 p.m. the sky was clear and the air temperature had fallen to 33° F.

The cold air drawn into the soil by the receding water cooled the 6 in. temperature to 39° F.—a fall of 4·0° F. in three hours.

I venture to think that similar results on a smaller scale must always take place when a sudden rise in pressure is associated with a sudden fall in temperature; more especially when the rise in pressure and fall in temperature are in conjunction with the dropping of a strong wind, which would have tended to draw air out of the soil, and so make it necessary for more air than usual to enter to restore equilibrium.

VII. The Thermal Effect of screening the Soil from Radiation, Evaporation, and Cold Precipitation.

During the last four months I have made a series of observations on the minimum temperature of soil over which I had erected a canvas shelter, in the form of a small ridge tent, the sides of which could be opened and rolled up at will.

The shelter has been kept open during the day, except in times of cold rain, sleet, or snow, or strong winds; it has been shut regularly at night.

The soil underneath has thus been more or less effectively sheltered from radiation, evaporation, and cold precipitation, whilst it has received insolation during the day and warmth from warm rains and wind.

The differences of the minimum temperatures of the sheltered and open soil are given in Tables VI, VII, and VIII as follows:—

Table VI. Differences on nights of rapid radiation.
„ VII. „ „ of strong wind.
„ VIII. „ during periods of cold rain, sleet, or snow.

These results make it clear that in practically all weathers such a shelter has a marked effect on the temperature of the soil.

TABLE VI.—RADIATION.

Date.	Air just over soil minimum.	Open soil minimum.	Sheltered soil minimum.	Difference in favour of shelter.	Weather.
1918.					
December 27 . .	25·5	27·0	32·5	5·5	Calm, clear
„ 31 . .	29·0	30·0	35·0	5·0	„
1919.					
January 11 . . .	29·0	30·0	34·5	4·5	„
„ 12 . . .	30·5	30·5	35·0	4·5	„
„ 15 . . .	28·5	30·0	35·0	5·0	„
„ 18 . . .	26·0	28·0	32·5	4·5	„
„ 19 . . .	23·0	24·5	31·0	6·5	„
February 5 . . .	27·0	28·0	32·0	4·0	„
„ 8 . . .	21·0	24·0	29·5	5·5	„
„ 9 . . .	19·0	23·0	29·5	6·5	„
„ 10 . . .	22·0	24·0	31·0	7·0	„
„ 12 . . .	23·0	27·0	32·0	5·0	„
„ 14 . . .	24·0	29·0	34·0	5·0	„
„ 24 . . .	27·5	28·0	32·5	4·5	„
„ 28 . . .	28·0	28·5	32·5	4·0	„
March 2 . . .	30·0	30·0	35·0	5·0	„
„ 3 . . .	21·0	26·0	31·0	5·0	„
„ 4 . . .	24·0	27·0	31·0	4·0	„

Average difference in favour of shelter, 5·1° F.

Some of the advantages of the shelter are as follows:—

(1) During the period February 1, 1919–March 10, 1919, the mean nightly temperature of the sheltered soil was 3·9° F. higher than the open soil.

(2) During this same period the open soil froze on twenty-three occasions—falling as low as 23° F. on February 9, and 24° F. on February 8 and 10. The sheltered soil froze on four occasions only, with a minimum of 29·5° F. on February 8 and 9.

(3) As on most nights the sheltered soil did not freeze, as soon as

insolation started its temperature rose rapidly, often standing at about 40° F. when the open soil was still at 32° F.

TABLE VII.—EVAPORATION.

Date.	Air just over soil minimum.	Open soil minimum.	Sheltered soil minimum.	Difference in favour of shelter.	Weather.
1918. December 28 . . .	37·0	35·5	38·0	2·5	Overcast, windy
1919. January 10 . .	36·5	34·5	37·5	3·0	,, ,,
,, 26 . .	32·0	31·5	34·5	3·0	,, ,,
,, 31 . .	33·0	32·0	35·0	3·0	,, ,,
February 1 . .	35·0	33·5	37·0	3·5	,, ,,
,, 7 . .	32·0	30·0	33·0	3·0	,, ,,
,, 18 . .	26·0	29·0	32·0	3·0	,, ,,
March 1 . .	38·0	36·0	38·0	2·0	,, ,,
,, 8 . .	32·0	31·0	34·0	3·0	,, ,,
,, 10 . .	35·0	34·0	38·0	4·0	,, ,,

Average difference in favour of shelter, 3·0° F.

TABLE VIII.—COLD PRECIPITATION.

Date.	Air just over soil minimum.	Open soil minimum.	Sheltered soil minimum.	Difference in favour of shelter.	Weather.
1918. December 30 . .	30·0	32·0	35·0	3·0	Cold rain
1919. January 2 . .	32·0	33·0	35·0	2·0	,,
,, 16 . .	31·5	31·5	35·0	3·5	,,
,, 21 . .	34·0	34·0	37·5	3·5	,,
,, 22 . .	32·0	33·0	36·0	3·0	,,
,, 24 . .	35·0	35·0	38·0	3·0	,,
,, 27 . .	31·0	30·0	34·0	4·0	Snow
,, 28 . .	27·0	29·5	32·5	3·0	,,
,, 29 . .	30·0	31·5	33·5	2·0	,,
February 2 . .	32·0	32·5	35·0	2·5	Sleet and rain
,, 4 . .	31·0	30·0	33·0	3·0	Snow
,, 16 . .	33·0	33·0	35·0	2·0	Cold rain
,, 22 . .	35·0	35·0	40·0	5·0	Sleet
,, 26 . .	34·0	34·0	37·0	3·0	,,
,, 27 . .	31·0	32·0	35·0	3·0	,,
March 7 . .	30·0	31·0	34·0	3·0	Snow

Average difference in favour of shelter, 3·1° F.

(4) Two geranium plants weathered the winter under the shelter; those in the open were killed by frost in December.

(5) Potatoes planted on February 1 had developed long shoots by February 28, and were showing in the surface by March 22. Others planted in the open on the same date showed no signs of life by March 22, and even the shoots they had on them at the time of planting had died and rotted by that date.

VIII. Conclusions.

By both covering the earth with a layer of ashes and putting a shelter over it, I have on one occasion kept the soil 10° F. warmer than in the open. Even so, I have not surpassed Nature in this respect, for we have seen that under a cover of grass and moss the temperature of the soil was 10° F. higher than open soil. On the banks of ditches, under the lee of hedgerows, and in the woods, protected from the wind and from the effects of radiation by a covering of fallen leaves, grass, and moss, the roots of spring flowers lie untouched by frost the whole winter through. A spell of warm rain as early in the winter as December will add just that necessary warmth to the soil to make them put forth their leaves, and we find them in flower when as likely as not winter has returned with all its rigour.

Encouraged by the mild weather of December 1918, a single primrose had forced its way through the mossy turf I have had under observation, and on February 10, 1919, was in flower—its leaves and flowers in an air temperature of 15° F., its roots at 33° F., a difference of 18·0° F. Snow and cold rain in January and February have deterred its companions, and it stands there alone still—as it were a vision of a spring that might have been.

As far as wild spring flowers are concerned, I believe the temperature of the air plays but a very secondary part to the underground temperature in determining an early or late spring.

(*Issued separately August 5, 1919.*)

XI.—On the Presence of Formic Acid in the Stinging Hairs of the Nettle. By Leonard Dobbin, Ph.D.

(MS. received May 16, 1919. Read June 2, 1919.)

It is well known that when the stinging hairs of the common nettle (*Urtica dioica* or *U. urens*) are caused to discharge their contents upon blue litmus paper, intensely red spots are produced. On the subsequent exposure to the air of the paper thus spotted, the red colour gradually diminishes in intensity, and in a day or two is scarcely distinguishable, although it does not entirely disappear even after several weeks' exposure. This behaviour indicates that the reddening is due, in the main at least, to a volatile acid, and the range of acids probably present is thereby very strictly limited.

Although the statement is made quite definitely in many text-books and elsewhere that formic acid occurs in the stinging hairs of the nettle, an examination of the original literature bearing upon the subject shows that the evidence upon which the statement seems to be based is not at all convincing in light of our present-day knowledge. It appears that the earliest evidence for the statement is contained in a paper by Gorup-Besanez,* who distilled finely-cut and crushed nettles with four times their weight of water, with and without the addition of sulphuric acid, and obtained slightly acid distillates. He submitted these distillates to subsequent treatment designed to collect the acid, or the calcium salt prepared from the acid, into a small volume of liquid, and in the solutions so obtained he satisfied himself, by the application of a series of tests, as to the presence of formic acid or of calcium formate. In view, however, of the facts, first, that a number of observers have reported the presence of formic acid in the distillates obtained either by boiling various plant parts with water (with or without the addition of sulphuric or other acid) or by passing a current of steam through tubes packed with such material,† and, secondly, that distillates obtained by these methods are known frequently to afford reactions resembling some of those of formic acid (reduction of silver and mercury salts, for example), although it was not found possible to separate from them and to identify this acid,‡ the question of formic

* *J. Prakt. Chem.*, vol. xlviii (1849), p. 191.

† See, in particular, investigations by Bergmann (*Bot. Zeit.*, vol. xl (1882), p. 731, etc.), who gives a review of the earlier literature.

‡ Compare Shannon, *Journ. Indust. Engin. Chem.*, vol. iv (1912), p. 526.

acid having come from the stinging hairs of the nettles, as distinguished from the general plant tissue, cannot be regarded as settled by the experiments of Gorup-Besanez, even were it admitted that this acid had been definitely proved to be present in the distillates which this investigator obtained.

In a paper by Haberlandt,* in which the poison of the stinging hairs of the nettle is dealt with at considerable length, the author simply accepts, without making any attempt to prove it, the view that the strongly acid reaction of the liquid ejected on breaking the tip of a stinging hair may be due to formic acid, but holds that the quantity of this acid which could be present is insufficient to account for the degree of irritation produced by the sting; and he supports this opinion by pointing out that when the contents of a stinging hair were permitted to become quite dry on the point of a needle, whereby he assumes that any formic acid which was present would be volatilised, the subsequent pricking of the skin with the needle point produced, after a few seconds, the characteristic stinging sensation as well as reddening of the skin at the spot. Haberlandt's investigations, in which the contents of the stinging hairs were submitted to an elaborate micro-chemical examination, led him to the conclusion that the active poison is most probably a substance of the nature of an enzyme.

Harvey Gibson and Warham † did not find any evidence of formic acid in the stinging hairs of the nettle, and as the result of their experiments they were at first inclined to consider that tartaric acid is the irritant substance which these hairs contain. Towards the close of their short note, however, they state that they hazard no conjecture as to the chemical nature of the substance, but reserve their conclusions for a further note. Endeavours to trace any further note by these authors have not been successful.

In a paper "On the Stinging Property of the Giant Nettle Tree" (*Laportea gigas*), Petrie,‡ when comparing the amount of acid obtained by distilling 100 grams of the fresh leaves of this plant in a current of steam in presence of phosphoric acid with the amount obtained by the same process from 100 grams of the fresh young leaves of the common nettle (*Urtica urens*), estimated the latter at 0·002 per cent., but apparently assumed, without applying any test to the distillate, that the acid obtained in the case of the nettle was formic acid.

* *Sitzungsber. der Akad. der Wissenschaft. Wien*, vol. xciii, 1. (1886), p. 130.

† *Proc. Liverpool Biolog. Soc.*, vol. iv (1890), p. 93.

‡ *Proc. Linn. Soc. N.S. Wales*, vol. xxxi (1906), p. 530.

In view of the uncertainty thus attaching to the matter, it seemed desirable to seek for some quite conclusive evidence as to the presence or absence of formic acid in the stinging hairs of the nettle. The attempt to procure such evidence necessitated, first, the collection and fixing of the acid contents of these hairs in sufficient quantity and wholly uncontaminated with cell contents or juices from any other part of the plant; and, secondly, the conversion of any formic acid which might exist in the contents of these hairs into a formate that could be identified as such beyond all doubt.

With a view to attaining the first of these ends, strips of the purest stout filter paper were impregnated with finely-divided barium carbonate by first soaking them in a 2 per cent. solution of barium hydroxide and then exposing them to the air for a sufficiently prolonged period. Two strips of this prepared paper, each measuring about 15 by 7 cm., were taken in gloved hands, on a fine day, when the foliage was dry, and were somewhat firmly pressed against the upper and under surfaces of a large number of the leaves of growing nettles (the species being exclusively *U. dioica*). In this manner, by dealing with some hundreds of leaves, the liquid expelled from many thousands of the stinging hairs could be collected in a comparatively short time and under conditions which ensured the fixing of the free acid contents of the hairs while they precluded the contamination of these contents with extraneous matters. The quantity of liquid taken up by the filter papers during this operation was sufficient to impart to these a distinctly damp feel.

A rough estimate of the number of hairs whose contents had been collected was rendered possible by carrying out a collection under similar conditions when the papers employed in the operation had been prepared by soaking them in a solution of sodium carbonate instead of barium hydroxide, since such papers showed, after drying, a pale olive-green stain coinciding with the spot at which each stinging hair had discharged its contents. One estimate of the number of these stains indicated that about fifteen minutes' work had resulted in the collection of part, at least, of their contents from between 10,000 and 12,000 stinging hairs. Whilst this number seems large, the total quantity of acid collected in this instance could not amount to 1 milligram if the estimate of Haberlandt * be correct that there is expelled from each hair at most 0·00006 milligram of formic acid.

The filter papers containing any new barium salt produced by the interaction of the acid from the stinging hairs with the barium carbonate

* *Loc. cit.*, p. 132.

were extracted twice with cold distilled water, and the filtered extract was mixed with phosphoric acid and distilled until the liquid in the retort became syrupy. The distillate so obtained had a slight but distinct acid reaction. Since it was considered that if formic acid were present in this distillate its presence should be readily established by converting it into lead formate and submitting this to examination under the polarising microscope, the distillate was mixed with excess of moist lead hydroxide which had been precipitated from a solution of lead nitrate, in presence of phenol phthalein, by the addition of sodium hydroxide until a rose coloration was just distinguishable, and had been subsequently washed with water eight times by decantation. After standing for a short time, the mixture was filtered, and the filtrate was saturated with carbon dioxide and then evaporated to dryness on the steam bath. The residue was extracted with a few drops of hot water, and the filtered extract was evaporated to dryness over sulphuric acid, portions of it having been placed upon a number of microscope slides with a view to obtaining any crystalline residue in a form suitable for optical examination.

Several successive preparations were carried out, in the manner just described, with varying quantities of material from nettles, and a collection of slides was obtained. In some later experiments, preparations were made in which barium hydroxide was substituted for the lead hydroxide mentioned above, and additional slides were obtained carrying residues which should contain barium formate if formic acid were present in the acid distillates. A large number of slides of both descriptions were examined with great care by Mr David Balsillie, B.Sc., whose report upon them is included here :—

"The majority of the earlier slides, carrying lead salts derived from nettles, which were submitted for examination, exhibited only dendritic growths which were isotropic and of no value for determinative work. Several of the later preparations, however, were distinctly more satisfactory, and showed crystalline substance occurring as (*a*) stellate groupings of strongly bi-refringent needles, and (*b*) single crystals with well-developed faces and sharp edges. The characters of these, respectively, were briefly as follows :—

"(*a*) The needles invariably showed straight extinction, were of distinctly higher refractive index than methylene iodide, and always had the faster ray vibrating along their length. Their general resemblance in these (as well as other) particulars to similar preparations of known lead formate was exceedingly striking. Plathan * asserts that lead formate

* See Groth, *Chemische Krystallographie*, iii Teil (1910), p. 17.

is optically negative, and, further, that the crystal axis *c* is the first mean line. On the assumption that the needles in the preparations submitted have their elongation parallel to this direction, there is thus an immediate explanation of why the faster ray should constantly be polarised in the transverse plane.

"The slides of lead acetate furnished for comparative purposes show that this salt is altogether different in its optical properties. The indices of refraction are below that of methylene iodide. The symmetry is monoclinic, the faster ray is not always found to vibrate along the length of the needles, in which the substance crystallises, as in lead formate: further, the salt is optically positive. There is thus no difficulty in distinguishing between lead acetate and lead formate. The slides carrying the material derived from nettles did not show any determinable traces of the former salt.

"(*b*) The single crystals were of orthorhombic symmetry, and exhibited generally a combination of prisms and domes, with occasional pinacoids closely resembling the figures given by Groth * and by Gehlen-Bernhardi.† Occasionally, the development was equidimensional, conferring an octahedral appearance upon the individuals, and groups that had grown in parallel orientation were not uncommon. Like those of the needle-shaped crystals, the indices of refraction were here invariably high, and the double refraction similarly strong. So far, therefore, as optical comparison (without actual measurements, which were out of the question in view of the minute size of the crystals) affords sanction for definite assertion, one can have no hesitation in saying that lead formate is present in these preparations.

"The slides of the barium salt were entirely confirmatory of the foregoing conclusion, and were of additional interest on account of the fact that one preparation showed the substance crystallised in complete—though extremely small—bisphenoids. This habit does not seem to have been previously noted in barium formate, and, though several attempts were made to crystallise the known salt in this form, no success was attained. Very probably concentration and conditions of separation have a delicate bearing upon the ultimate form of the solid substance."

In the light of the foregoing report, it may fairly be asserted that the presence of free formic acid in the stinging hairs of the nettle has been

* *Chemische Krystallographie*, iii Teil (1910), p. 15.

† *Schweigger's Annalen*, vol. iv (1812), pp. 36, 38, and figure. (It is to be noted that Bernhardi figures barium formate, but that the habit of lead formate is in some cases identical with that of barium formate. See Groth, *loc. cit.*, p. 17.)

definitely established. The only apparent doubt attaching to this assertion is that the formic acid undoubtedly obtained might possibly have been formed during the distillation process by the action of phosphoric acid on some constituent of the cell contents of the stinging hairs.* There does not appear to be any method available at present whereby the occurrence of this possibility can be proved or disproved.

The conclusion arrived at here that formic acid is present in the stinging hairs of the nettle is not to be regarded as affecting the question as to whether or not this acid is the main cause of the intense irritation produced by nettle stings. This question, discussed at length by Haberlandt,† is not within the scope of the present paper.

The author wishes to acknowledge with grateful thanks his indebtedness to Professor Bayley Balfour, who kindly propagated for him, at the Royal Botanic Garden, a number of specimens of *Urera baccifera* for comparative purposes; and to Mr Balsillie for the time and care devoted to the optical examination of the numerous preparations that were submitted to him.

* Compare Lieben, *Monatshefte*, vol. xix (1898), 352.
† *Vide ante*, p. 138.

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(*Issued separately August* 5, 1919.)

XII.—On the Mode of Action of Metal Sols. By Professor C. R. Marshall.

(Read May 5, 1919. MS. received May 10, 1919.)

During the last twenty years colloidal solutions of various metals have been largely used in therapeutics. Few investigations, however, have been made on their mode of action, and most of these have been made on mammals. As a contribution to the subject some experiments made under simpler conditions seem worthy of record. The reaction employed was the effect of an electrolyte-free silver sol on bacteria. Somewhat similar experiments have been made by Henri and coadjutors. These, which are referred to later, were unknown to me at the time of the investigation.

The sol was prepared by Bredig's electrical method, using conductivity water made in Bourdillon's apparatus, pure silver wire 1·25 mm. thick, and a current of seven amperes. It was greenish brown in colour, and contained 0·004 per cent. Ag. The conductivity was the same as that of the conductivity water exposed under the same conditions, namely 1×10^6 (telephone method).

Preliminary tests on the antiseptic action of this and other silver sols showed that *Bacillus typhosus* was the organism most susceptible, and this was therefore chiefly used. The order of susceptibility of the strains of bacilli investigated was—*B. typhosus*, *B. pestis*, *B. paratyphosus* A, *B. paratyphosus* B, *B. enteritidis* (Gaertner), *B. coli communis*.

The bacillary emulsion was made by adding 1 to 3 c.c. of sterile conductivity water to a twenty-four hours' growth of *B. typhosus* on agar medium, agitating gently and pouring off. The number of bacilli in a known volume was then counted. As nearly as possible a definite number of bacilli was added to a known concentration of the sol, and at definite intervals a platinum loopful of the mixture was added to 5 c.c. of a slightly modified Douglas broth and incubated.

The minimum lethal concentration of silver in solution was determined with silver nitrate. For a ten-minutes action it was found to be thirty-thousandth normal.

Effect of Silver Nitrate on Bacillus typhosus.

Concentration of Silver.	10 Minutes' Exposure.	15 Minutes' Exposure.
1 Ag in 3,200,000 . . .	0	0
1 Ag in 3,400,000 . . .	+	0
1 Ag in 4,000,000 . . .	+	+

Compared with this powerful action of free silver ions, the action of silver sols is relatively slight.

Effect of Electrically Prepared Silver Sols on Bacillus typhosus.

Colloid.	Concentration of Silver.	15 Minutes' Exposure.	30 Minutes' Exposure.
Electrolyte-free (Bredig) . .	1 Ag in 25,000	+	0
Electragol *	1 Ag in 32,000	+	0

The question at issue is how silver in a particulate form can exert a bactericidal action. A settlement of the question would enable us to understand the mode of action of metal colloids in general; and it is convenient to consider it from the point of view of the general properties of colloids.

Brownian Movement.—It is conceivable, although improbable, that the bacilli are affected by the impact of the larger submicroscopic particles during their incessant movement. If it were permissible to compare the effects produced by these minute missiles on bacteria with those which would follow similar missiles of the same relative size and moving with the same relative velocity on, say, man, we should conclude that the bacteria would seriously suffer. No such effect, however, occurs. Bacteria may be seen to move actively for long periods in a moderately concentrated colloidal metal sol, and they can be seen to be frequently bombarded, although it must be confessed that the number of impacts is less than would have been expected. In a number of experiments the bacilli and the visible submicroscopic particles (above 15 $\mu\mu$) were counted, and it was found that the bacilli multiplied when the visible particles were in the proportion of several thousand to one bacillus. Moreover, if such a

* Electragol is a commercial electrically prepared silver sol, stabilised by the addition of a small quantity of protein, and made isotonic by the addition of sodium chloride. It was found to contain more amicrons than the electrolyte-free colloid.

mechanical action as the trajectory of the particles played an important part in the pharmacological effect, it would be expected that all suspensoids would be equally effective, which is not the case.

Surface Phenomena.—An attempt has been made to explain certain important pharmacological actions by alterations in surface energy, and in the case of certain emulsoids it is probable that such an action plays a part. But surface actions of this type, dependent on adsorption, questionably occur under similar conditions in the case of suspensoids, and ultramicroscopic observations show no adhesion at least of the larger particles of a silver sol to the bacilli.

Electric Charge.—Nor does the pharmacological action appear to depend on the kind of electric charge on the particles. The negative electrical charge of a colloidal silver sol was shown by Hardy's modification of Whetham's method,* and the quantity of an alum sol necessary to reverse the charge and convert the sol into an electro-positive sol was determined. It was found that electro-negative and electro-positive sols produced the same antiseptic action.

Bacillus typhosus.

Amount of Sol added to 5 c.c. of Culture Medium.	Result after Incubation for		
	24 hours.	48 hours.	72 hours.
Electro-negative :—			
0·8 c.c.	0	+	
1·0 ,, ...	0	0	0
Electro-positive :—			
0·8 c.c.	0	+	
1·0 ,, ...	0	0	0

Against *B. coli communis* the electro-positive sol seemed to be slightly the more active.

Catalytic Power.—This was not systematically determined, as preliminary experiments seemed to show that it was insufficient to explain the effects obtained in this class of experiment. Electragol, for example, was much more powerfully catalytic when compared with the electrolyte-free sol than the relative bactericidal actions would account for. In the absence of surface effects it is difficult to understand how catalysis alone could play a predominant part.

* *Journ. of Physiol.*, vol. xxxiii, p. 289 (1905).

Liberation of Ions.—A small concentration of free silver ions occurs in silver sols. The amount, however, appeared to be insufficient to affect the conductivity, as determined by the telephone method of the sol chiefly used. But in view of the much greater bactericidal power of solutions of silver salts as compared with those of colloidal silver, it seemed a plausible hypothesis to attribute the effects observed to the liberation of free ions from the ultramicroscopic particles. Such an explanation occurred to Cernovodeanu and Henri,* for they state that they filtered a colloidal silver sol through a collodion filter and found the filtrate to be free from bactericidal or antiseptic action. Their investigation was unknown to me at the time I made the following experiment. Electragol containing 0·044 per cent. of silver was filtered under a pressure of forty atmospheres through a gelatined Chamberland candle, which had been hardened in 10 per cent. alum sol and washed in distilled water for several days. A clear, colourless filtrate was obtained which was neither bactericidal nor antiseptic. When left in a beaker in the laboratory for a few days an abundant growth of bacteria developed.

Effect of Size of Particles.—The size of the particles in the sols used was not actually measured; but from their stability, and the absence of precipitate after standing for three years, we may conclude that they probably contained few particles much above 60 $\mu\mu$ in diameter. In view of the fact that the concentration of free silver ions was insufficient to explain the pharmacological action of silver sols, it was decided to investigate in some measure the influence of size of particles. It was found that a sol prepared with conductivity water containing five-thousandth normal sodium hydroxide was suitable from this point of view for comparison with the colloid previously used. The amount of silver and the number of visible submicroscopic particles above 15 $\mu\mu$ having been estimated, the light diminishing (scattering) power of the two sols was compared. This was done by means of a spectrophotometer, a beam of light from an incandescent mantle being passed through a column of 5 cm. of sol, and the absorption estimated in the middle violet, the middle green, and the middle red of the spectrum, by comparison with crossed nicols. The angle of rotation of the analysing nicol in the violet and green with the number of visible submicrons (above 15 $\mu\mu$) and the percentage of silver, is given for the two preparations in the appended table. For convenience the electrolyte-free colloid has been termed A; the colloid prepared with sodium hydroxide solution, B.

* *Compt. Rend. de la Soc. de Biol.*, vol. lxi, p. 123 (1906).

Colloid.	Percentage of Silver.	Rotation of analysing Nicol for Violet.	Rotation of analysing Nicol for Green.	Number of visible Submicrons.
A, 50 per cent.	0·002 per cent.	79·5°	73·0°	14×10^5
B, 50 ,,	0·0027 ,,	complete	73·2°	10×10^5
C, 25 ,,	0·0014 ,,	76·8°	61·6°	5×10^5

It is evident that colloid B contained a much larger number of so-called amicrons than colloid A. It was found to be more powerful as a bactericide and antiseptic.

Bacillus typhosus (10 millions per cb.mm.).

Colloid.	15 Minutes' Exposure.	30 Minutes' Exposure.
A, 0·004 per cent. Ag . .	+	0
B, 0·0027 ,, . .	+	0

The difference in their action was much more marked in the antiseptic series of experiments. Those with *B. paratyphosus* A, which was the most concentrated bacillary emulsion used (34 million organisms per cb.mm.), may be given as an example.

Bacillus paratyphosus A.

Colloid.	18 Hours' Incubation.	66 Hours' Incubation.
A, 0·002 per cent. Ag . .	+	
B, 0·0004 ,, . .	0	+
B, 0·0008 ,, . .	0	+
B, 0·001 ,, . .	0	0

Similar results were obtained with other organisms It would therefore seem that the chief, if not the whole, activity of silver colloids in antiseptic experiments *in vitro* is to be ascribed to the ultramicroscopic particles below 15 $\mu\mu$ in diameter. And since particles below 5 $\mu\mu$ in diameter have no influence on the polarisation and therefore probably the scattering of light, a pharmacological effect must be attributed to silver particles between 5 $\mu\mu$ and 15 $\mu\mu$ in diameter. Cernovodeanu and Henri also came to the conclusion that the smallest particles were the most active. They

found that a red-brown silver sol prepared by Bredig's method was more powerfully bactericidal than an olive-green sol containing coarser particles prepared by the same method. Whether these amicrons produce surface effects or are taken up by the bacilli and converted into a soluble product within the organism is at present difficult to determine. But in view of the fact that colloidal sols of silver act more slowly than ionised silver, the latter view seems most probable. And it seems to me to receive some support from the experiments of Gompel and Henri,* who found, after intravenous injection of colloidal silver sols into animals, silver (spectroscopically) in some of the secretions of the body.

I am indebted to Miss A. W. Andrew, M.A., M.B., and to Miss Elizabeth Gilchrist, M.A., B.Sc., for help in the performance of some of the experiments.

* *Compt. Rend. de la Soc. de Biol.*, vol. lxi, p. 488 (1906).

(*Issued separately October* 20, 1919.)

XIII.—Some Conditions influencing the Reaction Velocity of Sodium Nitrite on Blood. By Professor C. R. Marshall.

(MS. received May 5, 1919. Read May 5, 1919.)

When a moderately strong solution of sodium nitrite is added to blood a change of colour almost immediately occurs, mainly owing to the formation of methæmoglobin. The actual products of the reaction are not definitely known, and are not of importance to the present paper. The rapidity of the reaction is affected by the nature and concentration of the blood solution on the one hand, and by the concentration of the sodium nitrite solution on the other. Probably other factors, such as temperature, which has not been investigated, are of importance. The experiments referred to in this paper were made at room temperatures.

Considerable variability was found in the reactivity of blood towards nitrites. The blood of different animals of the same species sometimes reacted differently quantitatively, and the same blood kept in the laboratory also tended to show slight differences from fresh blood. This was most noticeable when minimal concentrations of sodium nitrite were used. It was also found that different specimens of sodium nitrite gave different results; the commercial sodium nitrite, which is decidedly alkaline, being much slower in effecting a change than the sodium nitrite employed medicinally. These preliminary observations were made with solutions of blood in a test-tube examined with a simple pocket spectroscope. In one series of experiments in which laked blood was diluted to contain definite concentrations of sodium nitrite, and examined every few seconds for the earliest appearance of the absorption band in the red, the results plotted in the lower curve in fig. 1 were obtained.

It is necessary to state that the curve, although indicating generally the action of sodium nitrite, is only true for the particular specimen of blood and the conditions under which the experiment was made. For comparison the curve of a similar series of experiments with washed blood corpuscles in place of blood is given. The bend in the curve of the blood experiments with increasing dilutions of sodium nitrite, and the fact that discrepancies in the action of sodium nitrite more commonly occurred when whole defibrinated blood was used, suggested a possible influence of the serum; and as the action and the variations were of interest in connection with another research, they were further investigated.

For the purpose Hüfner's spectrophotometer was employed. After plotting the curves of absorption of diluted blood and the product obtained by the action of sodium nitrite, it was decided to observe the effect in the neighbourhood of λ 574. Under the influence of sodium nitrite the absorption band of oxyhæmoglobin present in this region becomes much lighter. The appearance of the band in the red was found to be much less suitable for the purposes of the investigation owing to the low intensity of the absorption band of methæmoglobin. The source of light was an incandescent burner. The collimator and ocular slits were made as narrow

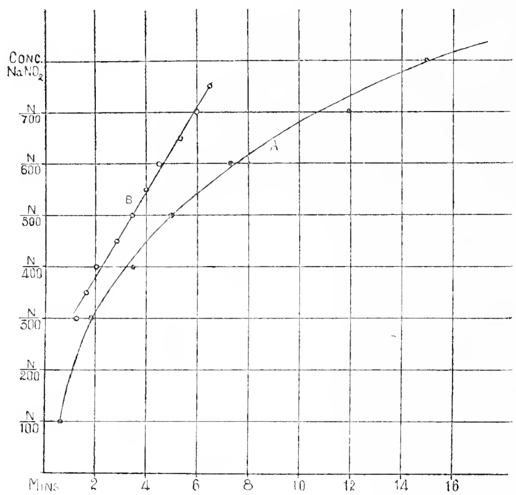


Fig. 1.—Curves showing time of appearance of absorption band in red (methæmoglobin) on adding different strengths of sodium nitrite solution to a solution of defibrinated blood and of washed blood corpuscles.

A, solution of defibrinated blood; B, solution of washed corpuscles; *Ordinates*, strengths of sodium nitrite in final product; *Abscissæ*, time in minutes

as was possible for accurate observation. The collimator slit, measured with a microscope, was 0·01 mm.; the ocular slit was 0·1 mm. The region of the spectrum observed was λ 571–577. The solution of ox blood (ox blood was used throughout the experiments in this paper) was put into one test-tube and the solution of sodium nitrite into another test-tube. At a definite time the two solutions were mixed by pouring from one test-tube to the other, at least three times, and then filling the Schulz absorption vessel. The first observation of the absorption could be made within twenty seconds of the commencement of the mixing. Further observations were taken at definite intervals, at first usually at half-minute intervals. The shortest time in which two observations could be made and recorded, with reasonable accuracy, was ten seconds. The extinction coefficient was then

calculated in the usual way and the results plotted. Owing to the rapidity with which the observations had frequently to be made, great accuracy was not possible; but, as the figures show, the changes were so marked that the results are not thereby invalidated. For the same reason it was only possible to employ one quadrant of the divided circle; but as only relative results were required, this was no disadvantage. All observations made during the change of absorption, with rare exceptions, are shown in the graphs. Those made prior to the commencement of any change and subsequent to its completion are, for the most part, omitted for the sake of simplicity. The sodium nitrite employed was that sold for medicinal use, neutralised and standardised against potassium permanganate.

As previously stated, different samples of blood may react differently to small concentrations of sodium nitrite. The differences are not so much in the form of the curve when the reaction has commenced, as in a delay in the commencement of the action. With four-hundredth normal sodium nitrite solution containing 1 per cent. of blood no obvious change occurs with average specimens of blood for about one and a half minutes; then a reaction begins, and proceeds very rapidly, almost reaching completion within another minute. In some samples of blood, although the reactive period has been practically the same, the delay in the commencement of the reaction has been shorter; in others it has been more prolonged. Thus in one experiment the reaction did not commence for eleven minutes. The cause of this delay in the appearance of the reaction has not been specifically investigated, and it is mentioned at this place chiefly to point out that the different graphs illustrating the paper, which are for the most part from different series of experiments, are not necessarily comparable with one another.

Effect of Concentration of Blood.—The effect of four-hundredth normal sodium nitrite on different concentrations of blood is shown in fig. 2. It will be observed that the action is delayed by increasing concentration of blood. The preliminary dip in the curve with 2 per cent. blood is unusual, and a broken line has been inserted to indicate the course more commonly followed. This was the only occasion on which a dip of this extent was observed. Not infrequently a slight initial fall in the extinction coefficient of nitrited 2 per cent. blood is seen, but its significance has not been determined. With smaller concentrations of blood it is less common. The observations with 0·5 per cent. blood will be noted to be somewhat irregular. This is due to the difficulty of matching the hue of the blood owing to the relatively slight absorption, and to the fact that from zero up to 40° the analysing nicol has to be moved through a considerable arc to

induce any change readily perceptible to the eye. The general trend of the experiment shown in the graph, however, is clear. It is probable, nevertheless, that the course of the reaction would be more truly indicated by greater steepness of the curve.

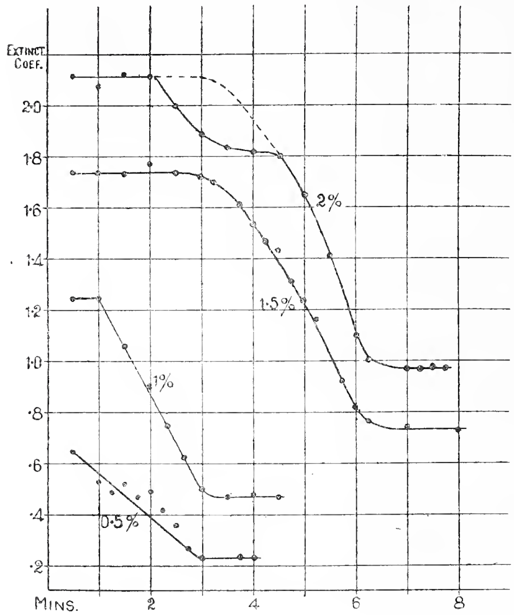


FIG. 2.—Effect of $\frac{N}{400}$ sodium nitrite on the different concentrations of blood indicated against the curves.

Ordinates, extinction coefficients ; *Abscissæ*, time in minutes.

Effect of Different Concentrations of Sodium Nitrite.—Although this is of the same order as the class of experiment just described, it is worthy of separate mention. In the previous case the concentration of sodium nitrite was kept constant; in these experiments the concentration of blood was kept constant, or as constant as the measuring of small volumes of solutions allows. With the method of investigation adopted, higher concentrations of sodium nitrite than centinormal could not be used owing to the rapidity of their action on the blood, unless by some means the action was delayed. Even with centinormal solutions the time of commencement of the reaction could not be observed. In the curve given (fig. 3) it was probably not before ten seconds from the time of commencement of the mixing. If this was the induction period, it is interesting to note that the two-hundredth normal solution took about seven times as long, and that the four-hundredth normal solution took about seven times as long as the two-hundredth normal solution. Once the reaction had

commenced it proceeded with about the same rapidity in the case of the two stronger solutions, and only slightly more slowly in the case of the weakest solution.

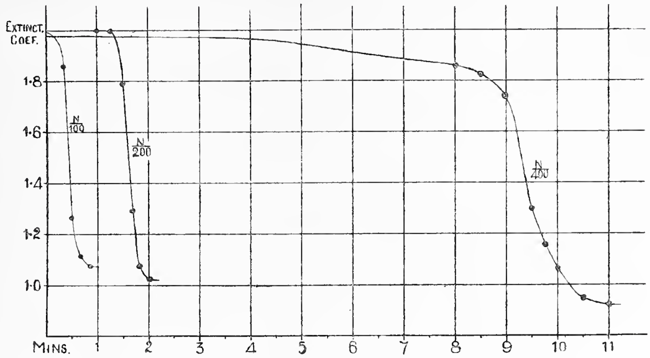


FIG. 3.—Effect of the different concentrations of sodium nitrite indicated against the curves on 2 per cent. blood solution.
Ordinates, extinction coefficients; *Abscissæ*, time in minutes.

Influence of Serum.—It has been mentioned that with dilute solutions of sodium nitrite the reaction on the blood, as determined by the appearance of the methæmoglobin band in the red, was delayed when compared with the action on a solution of washed blood corpuscles. Examined by the more delicate spectrophotometric method, an earlier appearance of the reaction with washed blood corpuscles invariably occurred. The influence of the serum was determined by adding definite proportions of the separated serum, diluted when necessary with 0·9 per cent. sodium chloride, to a solution of washed blood corpuscles. The results of experiments with four-hundredth normal sodium nitrite solution are shown in the graph (fig. 4). The presence of 25 per cent. of serum completely inhibited the reaction with this strength of nitrite within the time of observation; 2·5 per cent. of serum markedly retarded it; and even an amount of serum less than that in defibrinated blood had a delaying action. This retardation was not so marked in some experiments, but with fresh serum it was always considerable. It was immaterial whether the serum was added to the solution of blood corpuscles or to the solution of the nitrite before mixing. Heating the serum to 67° C. alone or previously mixed with the nitrite solution, even boiling the mixed solution, or the addition of a salt of quinine, did not influence the reaction. If the serum and nitrite solutions were mixed and allowed to stand at room temperature for some hours, less retardation of the reaction on adding to a solution of blood corpuscles occurred. And it was further noticed that blood serum some days old had less retarding effect than fresh serum, and that the same serum kept in the ice-chest

caused greater retardation than when kept at room temperature. These facts seem to point to the inorganic constituents of the serum as the active factors, and suggested that the reaction might be similar to that of nitrous acid on many organic compounds.

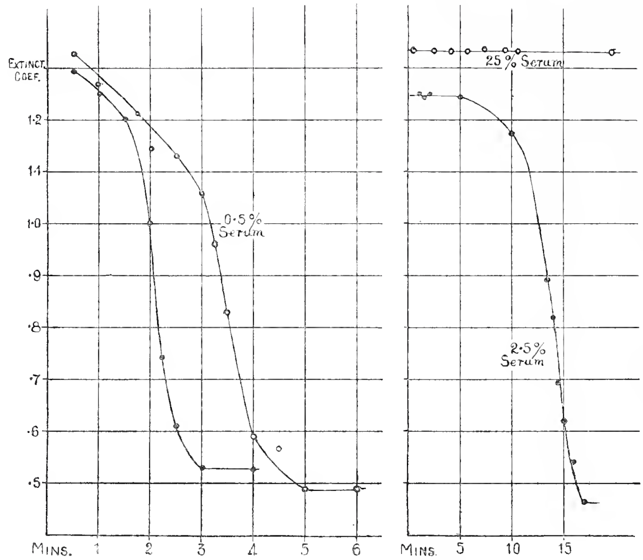


FIG. 4.—Influence of serum on the reaction between $\frac{N}{400}$ sodium nitrite and 0·5 per cent. solution of washed blood corpuscles. The amount of serum is indicated against the curves. The unmarked curve shows the course of the reaction without serum.

Ordinates, extinction coefficients; *Abscissæ*, time in minutes.

Effect of Sodium Hydroxide.—Alkali, when added to the solution of blood or to the solution of nitrite, also caused marked delay in the appearance of the reaction. The effect of different concentrations of sodium hydroxide on the reaction is seen in figs. 5 and 7. Fig. 5 shows the effect of one-thousandth, two-thousandth, and four-thousandth normal sodium hydroxide on the reaction between two-hundredth normal sodium nitrite and a 2 per cent. solution of blood. Five-hundredth normal sodium hydroxide tested under the same conditions prevented any change for two and a half hours, when the experiment was discontinued. The delay in the reaction is dependent, however, not only on the concentration of sodium hydroxide, but also on that of the sodium nitrite. This is seen in fig. 6, which illustrates the effect of twentieth normal and of two-hundredth normal sodium nitrite on a solution of 0·5 per cent. blood corpuscles in thousandth normal sodium hydroxide. With the same strength of blood corpuscles in distilled water the reaction with four-hundredth normal sodium nitrite was completed in little more than a minute. The slight

effect of millinormal caustic soda on the reaction of twentieth normal sodium nitrite on blood is also seen in fig. 7. In this particular series of experiments, which were made on washed blood corpuscles that had been

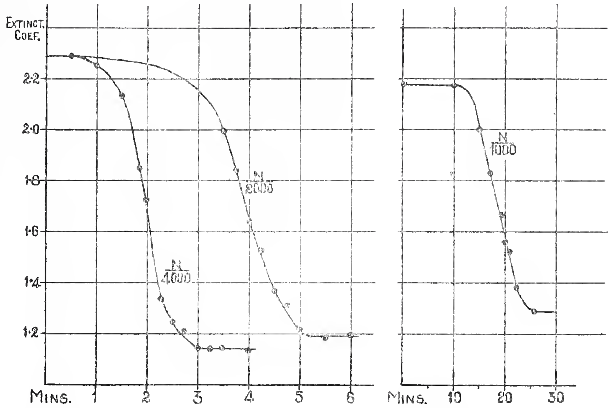


FIG. 5.—Effect of concentration of sodium hydroxide indicated against the curves on the reaction between $\frac{N}{200}$ sodium nitrite and 2 per cent. blood solution.

Ordinates, extinction coefficients; *Abscissæ*, time in seconds.

stored in the laboratory for six days, twentieth normal sodium nitrite completed the change on the blood in fifteen seconds, and the course of the

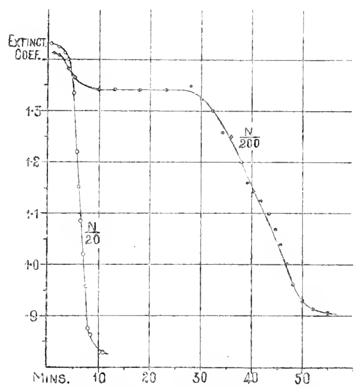


FIG. 6.—Effect of concentration of sodium nitrite indicated against the curves on a solution of 0·5 per cent. washed blood corpuscles in millinormal sodium hydroxide.

Ordinates, extinction coefficients; *Abscissæ*, time in minutes.

reaction could not consequently be followed. Even in the presence of two-thousandth normal caustic soda the reaction was completed in half a minute. The addition of millinormal sodium hydroxide caused an induction period of half a minute and a reactionary period of one and a half

minutes. The effect of larger concentrations of sodium hydroxide is seen in the figure.

The experiments made seem to show that the minimal amount of sodium hydroxide necessary to lengthen the induction period of the reaction bears some proportion to the amount of sodium nitrite. For four-hundredth normal nitrite it is about ten-thousandth normal sodium hydroxide, and for two-hundredth normal nitrite about four-thousandth normal sodium

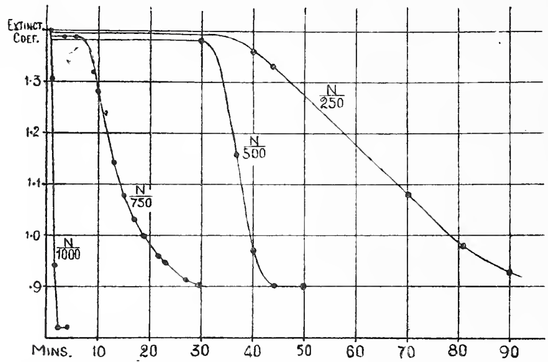


Fig. 7.—Effect of concentration of sodium hydroxide indicated against the curves on the reaction between twentieth normal sodium nitrite and 0·5 per cent. solution of washed blood corpuscles.

Ordinates, extinction coefficients; *Abscissæ*, time in minutes.

hydroxide; for twentieth normal nitrite it is about two-thousandth normal sodium hydroxide. With concentrations below this minimal strength no effect is produced. As the strength of alkali is increased above the minimum, the result, at first, is one of prolongation of the induction period, and it is only after the concentration has been considerably increased that the reactionary period itself is influenced. Even relatively concentrated alkali does not inhibit the reaction provided a concentrated solution of sodium nitrite is also used. Thus twentieth normal sodium nitrite reacts with a solution of blood in the presence of fortieth normal sodium hydroxide. The reaction, however, is so slow that its course has not been followed. This fact would seem to dispose of the idea that the reaction is dependent on the formation of nitrous acid. The presence of acid undoubtedly powerfully accelerates the reaction: the acceleration is distinctly noticeable with solutions of blood in four-thousandth normal acetic acid. But urea has no influence on the reaction. This proceeds in the presence of 25 per cent. urea, both in acid and alkaline solutions, as if the urea were not there. The mode of action of sodium nitrite on blood will be dealt with in a future communication.

(*Issued separately October* 20, 1919.)

XIV.—The Propagation of Earthquake Waves through the Earth, and connected Problems. By Professor C. G. Knott, D.Sc., LL.D.

(MS. received July 10, 1919. Read November 4, 1918, and January 20, 1919.)

This paper is a continuation of two papers on Seismic Radiations published in the *Proceedings of the Royal Society of Edinburgh*, vol. xxviii, pp. 217–230 (1907–8) and vol. xxx, pp. 23–37 (1909). The object of the present communication is to place on record a new determination of the laws of propagation of seismic waves based upon a method of calculation in which no assumptions are made as to the functional relation between velocity of propagation and distance from the earth's centre. References to the work of others will be given incidentally as occasion arises.*

To make the present discussion intelligible in itself, it is necessary to reproduce from my earlier paper the mathematical investigation on which the calculations are based, along with the fruitful transformation given by Dr Bateman in a paper published in the *Philosophical Magazine* for April 1910.

The earth is assumed to be a sphere, the elastic properties at any point being a function only of the distance from the earth's centre.

The disturbance, assumed to originate near the surface, will be propagated in a succession of waves, each trajectory or ray having the property of a brachistochrone meeting the surface at a point depending on the initial direction of the ray, and lying wholly in the plane containing the centre, the source, and the point of emergence. The position of any point may therefore be determined by the polar co-ordinates r and θ, referred to the earth's centre and to any convenient line in the plane of the ray passing through the point.

If v is the velocity of propagation at any point, then Hamilton's general method applied to brachistochronic problems gives

$$\left(\frac{\partial \mathrm{T}}{\partial r}\right)^2+\left(\frac{\partial \mathrm{T}}{r\partial \theta}\right)^2=\frac{1}{v^2} \qquad . \quad . \quad . \quad . \quad . \quad (1)$$

T being the time.

The discussion is simplified when the earth's radius is chosen as the unit length, so that r is a fraction and v is expressed in the unit earth-radius per second.

* See, however, the closing paragraph of the second paper referred to above for an account of the important work of Wiechert and Zöppritz in this connection.

Equation (1) becomes in the usual way

$$\frac{r^2}{v^2} - r^2\left(\frac{\partial \mathrm{T}}{\partial r}\right)^2 = \left(\frac{\partial \mathrm{T}}{\partial \theta}\right)^2 = p^2,$$

where p is independent of r and θ along any one ray.

Thus

$$\frac{\partial \mathrm{T}}{\partial \theta} = p, \quad \frac{\partial \mathrm{T}}{\partial r} = \pm \frac{1}{r}\sqrt{\left(\frac{r^2}{v^2} - p^2\right)} \quad . \quad . \quad . \quad . \quad . \quad (2)$$

leading to the integral

$$\mathrm{T} = p\theta \pm \int \frac{dr}{r}\sqrt{\left(\frac{r^2}{v^2} - p^2\right)} \quad . \quad . \quad . \quad . \quad . \quad . \quad (3)$$

The equation of the ray is obtained by equating $\partial \mathrm{T}/\partial p$ to an arbitrary constant, or

$$\theta \mp p\int \frac{dr}{r\left(\frac{r^2}{v^2} - p^2\right)^{\frac{1}{2}}} = \text{constant} \quad . \quad . \quad . \quad . \quad . \quad (4)$$

In every case the ray is symmetrical with reference to the radius which bisects the arc between the source and the point of emergence. This radius, which will for the moment be taken as the line of reference, meets the ray at its vertex distant from the centre by the stationary or turning value of r for each particular ray. Let this stationary radial distance be represented by z. Then, integrating from $r=z$ to $r=1$, and from $\theta=0$ to $\theta=a$, we find

$$a \mp p\int_z^1 \frac{dr}{r\left(\frac{r^2}{v^2} - p^2\right)^{\frac{1}{2}}} = 0 \quad . \quad . \quad . \quad . \quad . \quad . \quad (5)$$

where $2a$ is the arc between the source and the point of emergence.

Let ϕ, measured with reference to the radius of symmetry, be the angle at which the radius cuts the ray at any given point. Its cotangent, sine, and cosine are given by the following expressions—

$$\left.\begin{aligned} \cot\mathrm{an}\,\phi &= \frac{dr}{rd\theta} = \sqrt{\frac{r^2}{p^2v^2} - 1} \\ \sin\phi &= \frac{rd\theta}{ds} = \frac{pv}{r} \\ \cos\phi &= \frac{dr}{ds} = \sqrt{1 - \frac{p^2v^2}{r^2}} \end{aligned}\right\} \quad . \quad . \quad . \quad . \quad . \quad (6)$$

ds being the element of arc.

From the second of these relations by differentiation with regard to r there results

$$\cos\phi\frac{d\phi}{dr} = -\frac{pv}{r^2} + \frac{p}{r}\frac{dv}{dr},$$

or, by use of the third and second equations of (6),

$$\frac{d\phi}{ds} = -\frac{d\theta}{ds} + \frac{p}{r}\frac{dv}{dr},$$

giving the curvature *

$$\frac{1}{\rho} = \frac{d(\phi+\theta)}{ds} = \frac{p}{r}\frac{dv}{dr} \quad . \quad . \quad . \quad . \quad . \quad . \quad (7)$$

The parameter p is constant along each brachistochronic path or ray but varies from ray to ray, having by definition the value $\partial T/\partial\theta$. Thus along any ray the curvature depends on the expression $r^{-1}dv/dr$. It is conceivable that, through a limited interval from $r=r_1$ to $r=r_2$, this expression may be constant. This consideration, which is referred to by Rudzki in a note at the end of his paper, is made the foundation on which Wiechert and Zöppritz construct their numerical solution of the problem of finding the law connecting v and r. †

Assuming constancy, they put

$$\frac{1}{r}\frac{dv}{dr} = \mp f,$$

the sign being negative or positive according as v diminishes or increases with distance from the earth's centre. Integration gives

$$v = \tfrac{1}{2}f(R^2 \mp r^2),$$

either assumed law of variation carrying with it the circular or constant curvature form of path within each of a succession of concentric spherical layers of the earth. By taking three layers and properly piecing together the results for the successive layers, Wiechert and Zöppritz worked the problem out in full detail. It is to be noted, however, that this process of solution of equation (4) is a tentative one, being based on the assumption of an analytical form of v as a function of r.

Passing back now to equations (6), let us continue the analytical discussion of the integral equation. In the first place, if we put $\phi=\frac{\pi}{2}$, we find

$$1 = \frac{pv}{z} \quad \text{or} \quad z = pv \quad . \quad . \quad . \quad . \quad . \quad (8)$$

Now for any given ray $p=\partial T/\partial\theta$, a relation which holds for the special case when T and θ refer to the point on the surface at which the ray emerges. This value of T is determined by observation at stations where delicate seismographs are installed, and may be tabulated as a numerical

* This result, among others, was first given by Rudzki (see Gerland's *Beiträge zur Geophysik*, vol. iii, p. 496 (1898)).

† See their paper, "Ueber Erdbebenwellen," *Nachrichten von der König. Gesell. d. Wissenschaften zu Göttingen*, 1907, p. 491 *et seq.*

function of a, half the arcual distance from epicentre to point of emergence, or, symbolically,

$$p=\left(\frac{\partial T}{\partial \theta}\right)_{\theta=a} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (9)$$

In this expression p is given as a function of a; but the relationship may be supposed to be inverted so that a is given as a function of p, or, symbolically, $a=f(p)$. The form of this function cannot be expressed analytically, but by suitable treatment of the observations it may be represented by a series of tabulated numbers to any degree of closeness that may be found to be necessary.

The integral equation (5) then becomes

$$f(p)=p\int_z^1 \frac{dr}{r\left(\frac{r^2}{v^2}-p^2\right)^{\frac{1}{2}}} \quad . \quad . \quad . \quad . \quad . \quad . \quad (10)$$

from which it is required to determine the quantity v as a function of r.

Putting $r/v=\eta$, a new variable, so that

$$\frac{dr}{r}=d(\log r)=\frac{\partial}{\partial \eta}(\log r)d\eta,$$

we get the integral equation in the form

$$f(p)=p\int_{z/v=p}^{1/V} \frac{\frac{\partial}{\partial \eta}(\log r)d\eta}{\sqrt{\eta^2-p^2}},$$

where V is the value of v at the surface $r=1$.

As shown by Bateman, this may be identified with an integral equation solved by Abel. The solution is

$$\frac{\partial}{\partial \eta}(\log r)=-\frac{2}{\pi}\frac{\partial}{\partial \eta}\int_\eta^{1/V}\frac{f(p)dp}{\sqrt{p^2-\eta^2}},$$

or

$$\log r = C-\frac{2}{\pi}\int_\eta^{1/V}\frac{f(p)dp}{\sqrt{p^2-\eta^2}}.$$

At the upper limit $p=1/V$ and $\log r=0$. Hence C vanishes and

$$\log r=-\frac{2}{\pi}\int_\eta^{1/V}\frac{f(p)dp}{\sqrt{p^2-\eta^2}} \quad . \quad . \quad . \quad . \quad . \quad . \quad (11)$$

By means of equation (10) or its equivalent a solution may be obtained by assuming some convenient expression for v^2 which will make the integral immediately integrable. This was the method adopted in my earlier paper, and it was also essentially the method adopted by Wiechert and Zöppritz. Even Bateman, after developing the solution as given in

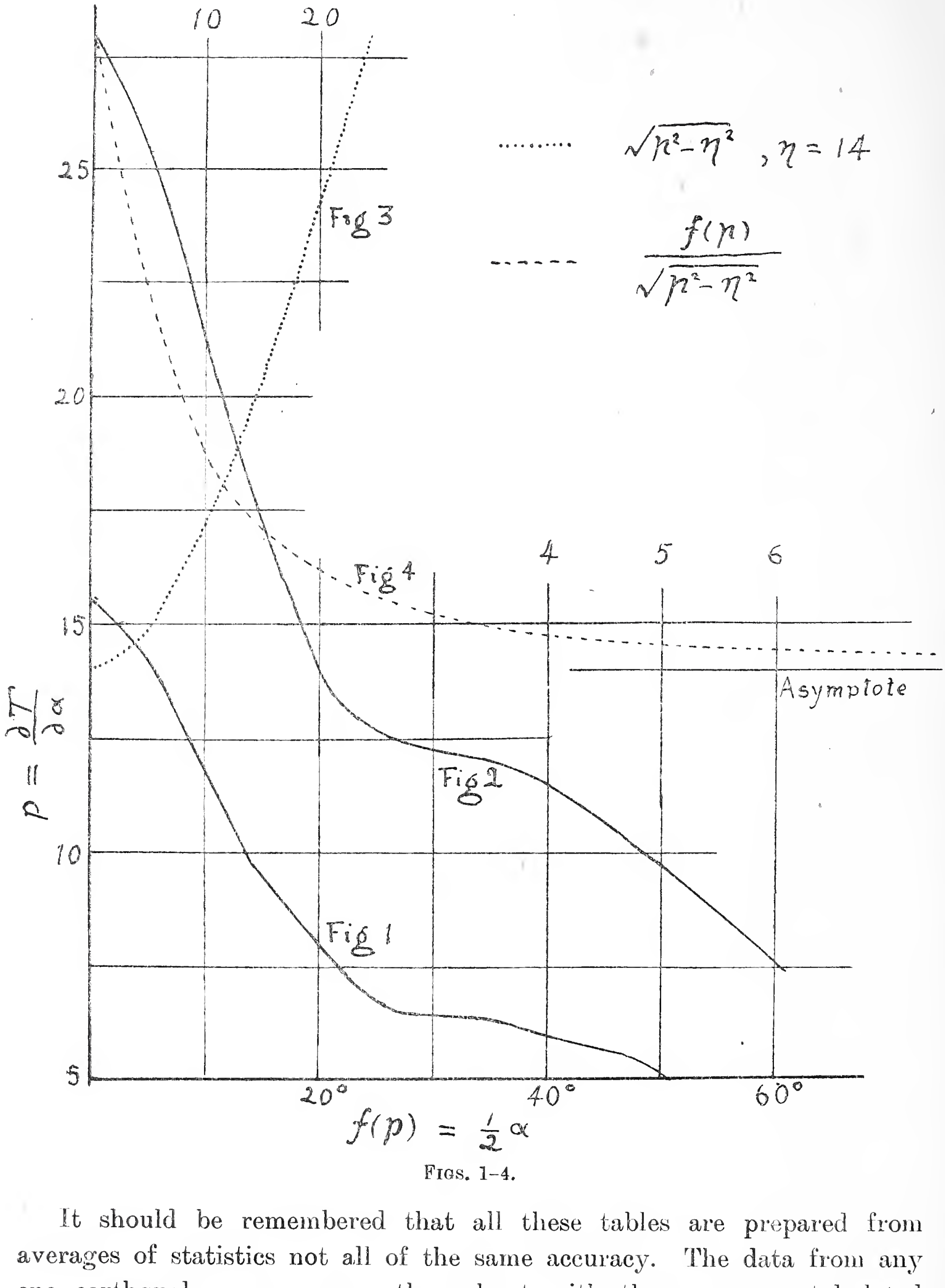
equation (11), did not seem prepared to make any better suggestion than to express T in ascending powers of θ, deduce therefrom the corresponding values for p and $f(p)$, and obtain an integrable form for the integral either as a whole or in bits.

What seemed to be most desirable was a direct way of evaluating the integral without any assumption of a functional relation between T and θ or p and θ. I hoped to hit upon some analytical method, but in this I was baffled. Fortunately I was able to discuss the problem with Professor Whittaker, who is thoroughly at home not only in the theory of integral equations, but also in all the best modes of numerical calculation in higher mathematics. He at once pointed out one general line of attack, the only objection being the length of time required to carry it out efficiently. After a few preliminary trials I decided to carry through the calculations in the manner now to be described. This description is given under three headings, namely, the data used, the reduction of these data to a form suitable for application to the integral equation (11), and the evaluation of the integral and of all the quantities involved.

(1) *The Data Used.*—These are given in the tables familiar to all seismologists, in which times of transit of the primary and secondary waves are expressed in terms of the arcual distances of the stations of observation from the source or (to be quite accurate) the epicentre. John Milne was the first to put these data of observation in tabular form and to draw an average curve giving the relation between time and distance (see B.A. Reports for 1899 and following years). With the accumulation of earthquake records this average curve underwent continuous corrections, and Milne's final values did not differ essentially from the values prepared by Wiechert and Geiger in 1907 from what they regarded as the best statistics then available. Milne tabulated the times of transit against the corresponding arcs measured in degrees; Wiechert and Geiger translated the degrees over the earth's surface into kilometres, 10,000 kilometres to the quadrant. For some years Professor H. H. Turner, chairman of the Seismological Committee of the British Association, has published Wiechert and Geiger's values in a modified form, following Milne's original method of expressing arcual distances. It is this modified table which I have used in the main calculations.

Lately Dr Klotz of Canada has collated a valuable set of tables for use by seismologists, adopting, however, somewhat smaller values of the times of transit of the Primary waves in accordance with a recent discussion by Dr Mohorovicié of Agram. The times of transit of the Secondary waves he obtains by simply adding to those for the Primary

waves the corresponding differences obtained from the values adopted by Wiechert and Geiger, a method which is open to criticism.



Figs. 1–4.

It should be remembered that all these tables are prepared from averages of statistics not all of the same accuracy. The data from any one earthquake never agree throughout with these average tabulated values, and no doubt as time goes on considerable corrections will have

to be applied. It was in the hope that Turner might be able to supply me with better data that I reserved the final calculations for a whole year after the method of attack had been planned and partly carried out. But the subject bristles with grave difficulties, and although in recent B.A. Reports Turner has indicated probable corrections at special parts of the tables, I felt that it would be sufficient meanwhile to adhere to the presently accepted values. These are reproduced in the Appendix, Table A.

(2) *The Reduction of the Data in a form suitable for the Evaluation of the Integral Equation* (11).—In the table in the Appendix the times of transit of both the Primary and Secondary waves are given for every integer number of degrees to the arc. That is, T is given in terms of $2a$. From these tabulated values appropriate mathematical methods lead to the determination of $\partial T/\partial a$ for every chosen value of a; and this is the quantity p. It is abundantly clear that the probable error in this quantity is considerable. Let the values of p be now plotted on a sufficiently large scale in terms of a, and through the points so obtained let a continuous graph be drawn. From this graph let a new table be constructed giving the values of a corresponding to successive equidistant values of p. The graphs are shown in figs. 1 and 2, and the new tabulations are given in Tables I and II, although not quite in the detail necessary for certain parts of the calculation.

Table I.—Primary Wave.

$p=\frac{\partial T}{\partial a}$.	$f(p)=a/2$.	$p=\frac{\partial T}{\partial a}$.	$f(p)=a/2$.	$p=\frac{\partial T}{\partial a}$.	$f(p)=a/2$.
15·5	0	8	19·9	5·3	48·9
15	2·5	7·5	21·5	5·2	49·3
14·5	4·15	7	23·2	5·1	49·9
14	5·5	6·5	26·5	5·0	50·5
13·5	6·7	6·4	35·5	4·9	51·0
13	7·7	6·3	36·3	4·8	51·9
12·5	8·7	6·2	37·0	4·7	53·5
12	9·7	6·1	37·6	4·6	56·5
11·5	10·7	6·0	38·5	4·5	59·5–62·5
11	11·7	5·9	40·0	4·4	65
10·5	12·6	5·8	41·5	4·3	68·5
10	13·7	5·7	43·0	4·2	69·4
9·5	15·0	5·6	44·5	4·1	70·5
9	16·5	5·5	47·5	4·0	71·5
8·5	18·2	5·4	48·4		

The next step towards building up the integrals is to tabulate the values of $f(p)$ at equal intervals dp and the corresponding values of $\sqrt{(p^2-\eta^2)}$, where η is the lower limit of the value of p. To this end the values of p^2 are first tabulated in column alongside the corresponding

values of $f(p)$. One of the quantities p^2 is then chosen as the limit η^2 and subtracted from all above it, and the square root of all these differences taken and tabulated. Each $f(p)$ is then divided by the corresponding

Table II.—Secondary Wave.

$p=\frac{\partial T}{\partial a}$.	$f(p)=a/2$.	$p=\frac{\partial T}{\partial a}$.	$f(p)=a/2$.	$p=\frac{\partial T}{\partial a}$.	$f(p)=a/2$.
28	0	20	11·4	12	35·5
27·5	1·25	19·5	12·0	11·5	39·8
27	2·35	19	12·6	11	43·0
26·5	3·3	18·5	13·25	10·5	46·0
26	4·2	18	13·95	10	48·5
25·5	5	17·5	14·7	9·5	51·0
25	5·65	17	15·3	9	53·5
24·5	6·3	16·5	15·85	8·5	56·0
24	6·85	16	16·35	8	58·5
23·5	7·4	15·5	17·1	7·5	61·0
23	8·0	15	18·2	7	63·5
22·5	8·6	14·5	19·1	6·5	66·0
22	9·1	14	20·0	6	68·5
21·5	9·6	13·5	21·8	5·5	61·0
21	10·2	13	23·1	5	63·5
20·5	10·75	12·5	26·5		

value $\sqrt{(p^2-\eta^2)}$, and this when multiplied by dp is one of the elements of the integral to be finally summed.

The process is shown in Table III, in which 14 is the lower limit value of p.

(3) *The Evaluation of the Integral and of all the other Quantities involved.*—In this table the quantity dp has the value 0·5 for all the intervals except the last five. For these it becomes 0·1, a diminution which is necessary, since the values of $f(p)/\sqrt{(p^2-\eta^2)}$ rise rapidly towards infinity as $\sqrt{(p^2-\eta^2)}$ diminishes towards zero. The curve giving the relation between $f(p)/\sqrt{(p^2-\eta^2)}$ and p is shown in fig. 4 (p. 162), fig. 3 being the graphical representation of $\sqrt{(p^2-14^2)}$ in terms of p. What is sought for is the area of the region between the curve of fig. 4, the vertical axis and the horizontal asymptote. The greater part of this area can be readily reckoned by the process of mechanical quadrature. Leaving out of consideration meanwhile the part from $p=14$ to $p=14·1$ which is bounded by the asymptote and the infinite branch of the curve, we first calculate the area of the portion from $p=14·1$ to $p=14·5$, using the formula

$$\text{area}=\frac{14(u_0+u_4)+64(u_1+u_3)+24u_2}{45}h,$$

where u_0, u_1, u_2, u_3, u_4 are the five values of the ordinates bounding the four elemental strips and h is the width of the strip, in this case 0·1.

The remaining elemental strips of width 0·5 are then grouped in sets of six, and the area of each set is calculated by Weddle's Rule, namely,

$$\text{area} = \frac{3\{u_0 + u_2 + u_3 + u_4 + u_6 + 5(u_1 + u_3 + u_5)\}}{10} h.$$

In the present case we apply Weddle's Rule from $p = 14·5$ to $p = 17·5$, from $p = 17·5$ to $p = 20·5$, from $p = 20·5$ to $p = 23·5$, and from $p = 23·5$ to $p = 26·5$.

Table III.—Showing the Calculations of the Elements of the Integral (11) from $p = 14$ to $p = 28$.

$f(p) = \frac{a}{2}$.	$\frac{\partial T}{\partial \theta} = p$.	$\sqrt{(p^2 - \eta^2)}$.	$f(p)/\sqrt{(p^2 - \eta^2)}$.
0	28	24·3	0·000
1·25	27·5	23·7	·053
2·35	27	23·1	·102
3·3	26·5	22·5	·147
4·2	26	21·9	·192
5	25·5	21·3	·234
5·65	25	20·7	·273
6·3	24·5	20·1	·313
6·85	24	19·5	·351
7·4	23·5	18·9	·392
8	23	18·3	·438
8·6	22·5	17·6	·489
9·1	22	17·0	·535
9·6	21·5	16·3	·588
10·2	21	15·7	·650
10·75	20·5	15·0	·718
11·4	20	14·3	·798
12	19·5	13·6	·885
12·6	19	12·8	·981
13·25	18·5	12·1	1·094
13·95	18	11·3	1·234
14·7	17·5	10·5	1·400
15·3	17	9·64	1·587
15·85	16·5	8·73	1·816
16·35	16	7·75	2·110
17·1	15·5	6·65	2·571
18·2	15	5·39	3·377
19·1	14·5	3·77	5·066
19·25	14·4	3·37	5·712
19·35	14·3	2·91	7·650
19·50	14·2	2·37	8·228
19·75	14·1	1·68	11·756
20	14	0	∞

This leaves three elemental strips still to be summed together, and for this the formula to be used is

$$\text{area} = \frac{3(u_0 + 3u_1 + 3u_2 + u_3)}{8} h.$$

The method is in all cases to begin at the lower end and mark off in sets of seven, so that Weddle's Rule may be applied. If there be left over

at the upper end of the column a number less than seven, then the quadrature may be effected by the appropriate formula. Although this is less accurate than Weddle's formula, it affects the evaluation of a very small part of the whole and the error involved is of no moment.

The following are the areas of the successive portions for $\eta = 14$:—

Limits of Values of p.	Area.
14·1 to 14·5	2·914
14·5 „ 17·5	7·250
17·5 „ 20·5	3·021
20·5 „ 23·5	1·626
23·5 „ 26·5	·816
26·5 „ 28·5	·115
14·1 to 28·5	15·742

The area of the part which passes off to infinity has now to be estimated, namely,

$$I' = \int \frac{f(p)dp}{\sqrt{(p^2 - \eta^2)}}$$

between the limits $p_1 = \eta(1 + e_1)$ and $p_0 = \eta$, where e_1 is a small quantity. Writing $p = \eta(1 + e)$ we, have

$$dp = \eta de$$

$$\sqrt{(p^2 - \eta^2)} = \eta(2e + e^2)^{\frac{1}{2}} = \eta\sqrt{2e}\left(1 + \frac{e}{2}\right)^{\frac{1}{2}}$$

and

$$f(p) = f(p)_1 - e\eta f'(p)_1 + \dots$$

Hence

$$I' = \int_0^{e_1} \frac{f(p)_1 - e\eta f'(p)_1 + \dots}{\sqrt{2e}}\left(1 - \frac{e}{4} + \frac{3}{8}\frac{e^2}{4} - \dots\right)de$$

$$= \frac{f(p)_1}{\sqrt{2}}\int_0^{e_1}(e^{-\frac{1}{2}} - \tfrac{1}{4}e^{+\frac{1}{2}} + \dots)de - \frac{\eta f'(p)_1}{\sqrt{2}}\int_0^{e_1}(e^{\frac{1}{2}} - \tfrac{1}{4}e^{\frac{3}{2}} + \dots)de$$

$$= \frac{f(p_1)}{\sqrt{2}}(2e_1^{\frac{1}{2}} - \tfrac{2}{12}e_1^{\frac{3}{2}} + \dots) - \frac{\eta f'(p)_1}{\sqrt{2}}(\tfrac{2}{3}e_1^{\frac{3}{2}} - \dots)$$

$$= f(p)_1\sqrt{2e_1}\left\{1 - \tfrac{1}{12}e_1 - \tfrac{1}{3}\frac{f'(p)_1\eta e_1}{f(p_1)}\right\} \quad \dots \dots \quad (12)$$

In the particular case chosen above

$$f(p_1) = 19{\cdot}75, \quad \frac{f'(p_1)\eta e}{f(p)} = -\frac{{\cdot}25}{19{\cdot}75}, \quad e = \frac{0{\cdot}1}{14}.$$

Hence

$$I' = \frac{19{\cdot}75}{\sqrt{70}}\left\{1 - \frac{1}{1680} + \frac{1}{3}\cdot\frac{25}{1975}\right\} = 2{\cdot}369.$$

Adding this to the previous result, we find for the whole evaluated integral the value

$$\mathrm{I} = \int_{14}^{28} \frac{f(p)dp}{\sqrt{(p^2 - 14^2)}} = 15{\cdot}742 + 2{\cdot}369 = 18{\cdot}111.$$

Equation (11) may now be written

$$\log_e r = -\frac{2}{\pi}\mathrm{I}_1,$$

where all the quantities entering into I_1 must be expressed in fundamental units. Now $f(p)$ has throughout been expressed in degrees of arc. The

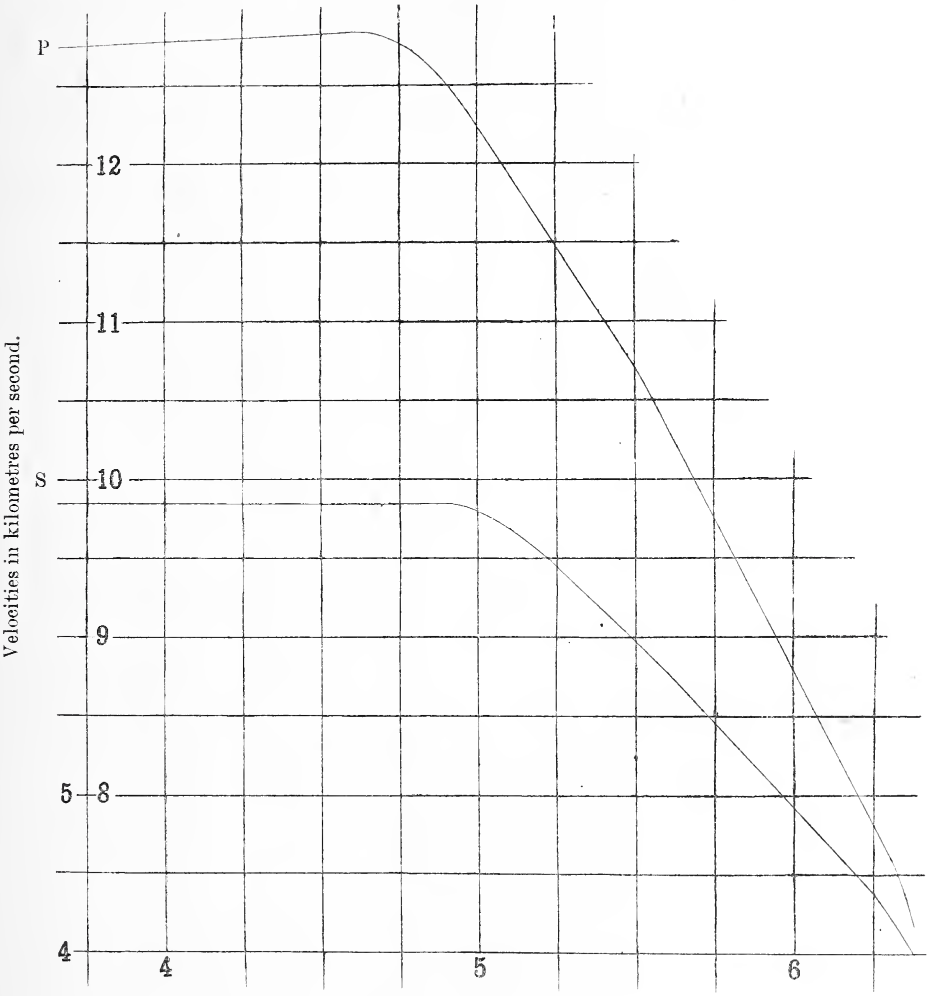


FIG. 5.

numbers obtained by summation must therefore be divided by $180/\pi$ so as to express the arcs in radians. It does not matter in what units p

is expressed, since $dp/\sqrt{(p^2-\eta^2)}$ is necessarily a ratio. Dividing each of the calculated integrals by

$$\frac{\pi}{2}\cdot\frac{180}{\pi}=90,$$

we find

$$\log_e r = -\frac{2}{\pi}\mathrm{I}_1 = -\mathrm{I}/90 = -0\cdot2012,$$

from which r may be immediately determined.

Thus for any chosen value of η, that is, of p, we can calculate the corresponding value of r. Knowing r and η, we find the corresponding value of $v=r/\eta$, after we have expressed η in radians. Since $p=\partial\mathrm{T}/\partial\theta$, the factor required is the reciprocal of the factor required to reduce degrees to radians, that is, $180/\pi$. This gives v in earth-radii per second. To reduce to kilometres per second, it is necessary to multiply r by the number of kilometres in the radius of the earth, namely, 6378.

The main results are tabulated in the following Tables IV and V and graphically shown in fig. 5. In these tables, $\mathrm{R}=6378r$, and the velocity is expressed in kilometres per second.

TABLE IV.—PRIMARY WAVE.

In Degrees.		In Radians.		Integral (11).	Log $\frac{1}{r}$ to Base 10.	Radial Distance, R. kilom.	Velocity $=\mathrm{R}/\eta$. kilom./sec.
2α.	η.	2α.	η.				
0	15·5	0	888	0	...	6378	7·18
5·0	15	·047	860	0·411	0·00198	6349	7·38
8·3	14·5	·072	831	1·093	·00487	6307	7·59
11·0	14	·096	802	1·807	·00872	6252	7·80
13·4	13·5	·117	774	2·641	·01274	6194	8·00
15·4	13	·134	745	3·483	·01640	6142	8·24
17·4	12·5	·152	716	4·386	·02116	6075	8·48
19·4	12	·169	688	5·347	·02580	6010	8·74
21·4	11·5	·187	659	6·402	·03089	5940	9·01
23·4	11	·204	630	7·523	·03630	5867	9·31
25·2	10·5	·220	602	8·815	·04253	5783	9·61
27·4	10	·239	573	10·047	·04848	5705	9·96
30·0	9·5	·262	544	11·528	·05563	5612	10·32
33·0	9	·288	516	13·32	·06428	5501	10·66
36·4	8·5	·318	487	15·17	·07322	5389	11·07
39·8	8	·347	458	17·30	·08347	5263	11·49
43·0	7·5	·375	430	19·86	·09584	5115	11·90
46·4	7	·405	401	22·50	·10856	4968	12·39
53·0	6·5	·463	372	25·70	·12401	4794	12·89
71·0	6·4	·620	367	27·03	·13041	4724	12·87
74·0	6·2	·646	355	30·73	·14827	4534	12·77
77·0	6	·672	344	33·32	·16079	4405	12·85
95·0	5·5	·829	315	41·01	·19790	4044	12·84
101·0	5	·881	287	49·60	·23936	3676	12·81
122·0	4·5	1·065	258	60·18	·29041	3268	12·67
143·0	4	1·248	227	74·67	·34416	2888	12·73

TABLE V.—SECONDARY WAVE.

In Degrees.		In Radians.		Integral (11).	$\text{Log}\,\frac{1}{r}$ to Base 10.	Radial Distance, R. kilom.	Velocity $=R/\eta$. $\frac{\text{kilom.}}{\text{sec.}}$
$2a$.	η.	$2a$.	η.				
0·0	28	0·000	1604	0·000	·00000	6378	3·98
2·5	27	0·022	1547	0·433	·00209	6348	4·10
8·4	26	·073	1490	1·141	·00551	6298	4·22
11·3	25	·097	1432	1·818	·00874	6251	4·37
13·7	24	·120	1375	2·799	·01351	6183	4·50
16·0	23	·140	1318	3·691	·01781	6122	4·65
18·2	22	·159	1261	4·824	·02328	6045	4·79
20·4	21	·178	1203	5·957	·02875	5970	4·97
22·8	20	·199	1146	7·211	·03480	5887	5·14
25·2	19	·220	1089	8·533	·04118	5788	5·32
27·9	18	·243	1031	10·044	·04847	5705	5·53
30·6	17	·267	974	11·761	·05675	5618	5·77
32·7	16	·285	917	13·533	·06530	5488	5·98
36·4	15	·318	859	15·631	·07543	5361	6·24
40·0	14	·349	802	18·111	·08739	5216	6·50
46·2	13	·403	745	21·17	·10214	5042	6·77
53·4	12·5	·463	716	23·21	·11198	4929	6·88
71	12	·620	688	27·17	·13113	4716	6·85
86	11	·750	630	35·36	·17062	4306	6·84
97	10	·846	573	43·82	·21143	3920	6·84
107	9	·933	516	53·10	·25625	3536	6·85
117	8	1·021	458	63·81	·30794	3139	6·85
127	7	1·108	401	77·27	·37285	2703	6·74
137	6	1·195	344	89·80	·43335	2352	6·84
147	5	1·202	287	107·62	·51932	1929	6·72

It will be noticed that each of the curves is almost exactly a straight line from R=6300 km. to R=5700 km. Between R=5700 and R=5600 there is a distinct bend, and then the graph is again approximately rectilinear to about R=5000 km. For values of R lower than 4700 km. v becomes fairly constant. The uncertainties in the observational data for arcs of transit greater than 100° prevent us attaching any importance to the variations in v in the deeper layers. The fourth figure in the values of v has not been retained throughout, for it is quite clear that the third figure may be in error by several units.

Working out the linear relation between the speed v and the distance R by the method of least squares for the two portions of each curve, we find as follows:

A. *The Primary Wave.*

From R=6250 to R=5710 the velocity in km. per sec. is given by

$$\text{velocity} = 32{\cdot}78 - {\cdot}0040\,R.$$

From R=5620 to R=4790

$$\text{velocity} = 28{\cdot}27 - {\cdot}0032\,R.$$

B. *The Secondary Wave.*

From R = 6300 to R = 5700

$$\text{velocity} = 17{\cdot}37 - {\cdot}00208\ \text{R}.$$

From R = 5620 to R = 5040

$$\text{velocity} = 15{\cdot}93 - {\cdot}00181\ \text{R}.$$

In the great majority of cases the values of the velocity calculated from these formulæ differ from the given values by less than 1 in 200. In the few cases in which the differences are of the order of 1 to 2 per cent., the differences are well within the errors of the data on which the original calculations are based.

For values of R less than about 5000 the velocities of both the Primary wave and the Secondary wave become practically constant, the former having the value 12·8 and the latter 6·85 km. per second. It should be remembered that the data for arcual distances greater than 110° or 120° are very uncertain.

Determination of the Rays.

In accordance with equation (4) the path or ray depends on the value of the integral

$$p\int \frac{dr}{r\left(\frac{r^3}{v^2} - p^2\right)^{\frac{1}{2}}} = p\int \frac{dr}{r\sqrt{\eta^2 - p^2}}.$$

This might be evaluated in parts by means of the empirical relations just given between the velocity and the radial distance; but difficulties would arise in passing properly from the one linear relation to the other. The only certain way is to work out a sufficient number of points on the paths by means of quadratures from the Tables IV and V suitably prepared.

For any one ray the quantity p is constant, and the other important quantities are the radius r and the ratio $(r/v)^2 = \eta^2$. The first step is obviously to tabulate corresponding values of r and η^2 from the data given in Tables IV and V. Let these then be plotted on a suitable scale, the relative scales of the ordinates being adjusted in the various parts so as to give well-conditioned graphs. From each of these graphs a new table is then to be constructed giving the values of η^2 for successive equal differences of r. The numerical values of p and of η depend on the unit of arc employed; and in Table VI, containing the data for both the Primary and Secondary waves, η^2 involves the radian as the unit of arc.

Table VI.—Primary Wave (P), Secondary Wave (S).

$\eta^2=(r/v)^2$ tabulated as a function of r.

r.	$10^{-4}\eta^2$. P.	$10^{-4}\eta^2$. S.	r.	$10^{-4}\eta^2$. P.	$10^{-4}\eta^2$. S.	r.	$10^{-4}\eta^2$. P.	$10^{-4}\eta^2$. S.
1	78·85	257·3	0·805	18·8	59·8	0·61	9·23	32·3
0·995	73·96	239·2	·8	18·3	58·3	·605	9·08	31·8
·990	69·8	228·5	·795	17·68	56·7	·6	8·93	31·3
·985	66·9	217·0	·79	17·15	55·2	·595	8·79	30·8
·98	64·3	205·1	·785	16·67	54·0	·59	8·64	30·3
·975	61·8	198·0	·78	16·2	52·8	·585	8·5	29·8
·97	59·3	190·7	·775	15·73	51·7	·58	8·35	29·3
·965	56·5	182·2	·77	15·29	50·8	·575	8·2	28·8
·96	54·3	173·7	·765	14·87	50·3	·57	8·08	28·3
·955	52·4	167·2	·76	14·48	49·7	·565	7·97	27·8
·95	50·5	160·5	·755	14·13	49·1	·56	7·85	27·3
·945	48·5	154 6	·75	13·8	48·5	·555	7·73	26·8
·94	46·4	149·1	·745	13·58	47·9	·55	7·6	26·3
·935	44·2	143·7	·74	13·41	47·3	·545	7·48	25·9
·93	42·5	138·4	·735	13·26	46·8	·54	7·36	25·4
·925	41·2	133·5	·73	13·12	46·2	·535	7·24	25·0
·92	39·7	128·7	·725	12·99	45·6	·53	7·12	24·5
·915	38·4	124·6	·72	12·86	45·0	·525	7·00	24·0
·91	37·1	120·5	·715	12·72	44·4	·52	6·88	23·6
·905	35·8	116·1	·71	12·57	43·8	·515	6·76	23·1
·900	34·4	111·8	·705	12·38	43·2	·51	6·64	22·7
·895	33·2	107·5	·7	12·2	42·6	·505	6·52	22·2
·89	31·9	103·3	·695	12·02	42·0	·5	6·39	21·8
·885	30·8	98·7	·69	11·83	41·4	·495	6·26	21·3
·88	29·6	94·2	·685	11·67	40·8	·49	6·13	20·9
·875	28·8	91·8	·68	11·50	40·2	·485	6·00	20·5
·87	28·0	89·2	·675	11·33	39·7	·48	5·88	20·2
·865	27·2	86·6	·67	11·17	39·0	·475	5·76	19·8
·86	26·3	84·1	·665	10·99	38·4	·47	5·63	19·4
·855	25·4	81·5	·66	10·82	37·9	·465	5·49	19·1
·85	24·6	78·7	·655	10·64	37·3	·46	5·37	18·7
·845	23·7	76·0	·65	10·47	36·8	·455	5·24	18·3
·84	23·0	73·3	·645	10·30	36·2			
·835	22·3	71·2	·64	10·13	35·6			
·83	21·6	69·0	·635	9·97	35·0			
·825	21·0	66·8	·63	9·82	34·5			
·82	20·4	64·7	·625	9·67	33·9			
·815	19·8	63·0	·62	9·52	33·3			
·81	19·3	61·4	·615	9·38	32·8			

These tabulations are now to be treated in exactly the same way as the former tabulations were. A particular η^2 is chosen as the p^2 of the ray whose course is to be traced point by point. This value of p^2 is subtracted from all the higher values of η^2, and for every value of r the corresponding value of the reciprocal of $r\sqrt{(\eta^2-p^2)}$ is calculated. The values of p^2 were chosen so that the summations could be made in groups of seven according to Weddle's Rule, and in this way each integral was evaluated; all except the last interval, in which infinite values had to be considered.

A glance down the columns of Table VI will show that the successive differences of the values of η^2 change very slowly, so that over the interval between any two we may assume a linear relation between η^2 and r. Suppose, for example, that $\eta^2=br-a$. Then b and a are to be determined from the equations

$$\left.\begin{aligned} br_1 - a &= \eta_1^2 \\ br_0 - a &= p^2 \end{aligned}\right\},$$

where r_0 is the value of r corresponding to p and r_1 the value of r corresponding to η_1, the value of η immediately above p.

Thus

$$b = \frac{\eta_1^2 - b^2}{r_1 - r_0} \quad \text{and} \quad r_0 = \frac{a+p^2}{b}.$$

The integral then becomes

$$\Gamma = \int_{r_0}^{r_1} \frac{pdr}{r\sqrt{(\eta^2-p^2)}} = p\int_{r_0}^{r_1} \frac{dr}{r\sqrt{br-a-p^2}} = \frac{p}{\sqrt{b}}\int_{r_0}^{r_1} \frac{dr}{r\sqrt{r-r_0}}$$

$$= \frac{p}{\sqrt{b}}\int_{r_0}^{r_1} \frac{dr}{\sqrt{r^3}\sqrt{\left(1-\frac{r_0}{r}\right)}} = -\frac{2p}{\sqrt{br_0}}\int_{r_0}^{r_1} \frac{d\sqrt{\frac{r_0}{r}}}{\sqrt{\left(1-\frac{r_0}{r}\right)}}$$

$$= \frac{2p}{\sqrt{br_0}}\cos^{-1}\sqrt{\frac{r_0}{r_1}} = 2\sqrt{\frac{r_1-r_0}{r_0}\cdot\frac{p^2}{\eta_1^2-p^2}}\cdot\cos^{-1}\sqrt{\frac{r_0}{r_1}} \quad . \quad . \quad . \quad (13)$$

This is the area of the last element which contains the infinite branch.

In order to be able to apply Weddle's Rule for integration systematically throughout, it is necessary to group the numbers in sets of seven, the seventh of any one set being the first of the succeeding set. The very last number which is the p^2 in the formula lies outside the last set of seven. Hence the positions of the numbers which are to be chosen as the different values of p^2 are to be represented numerically by any number of the form $6n+2$ where n is an integer. There will then be n groups of seven, for each of which the integral is to be summed. This summation, when properly worked out, will measure the angle between the radius corresponding to the first number and that corresponding to the last number of the set. Thus n points are obtained lying on the portion of the path or ray between the vertex and the surface, that is, on half the ray. The $(n+1)$th point will be given by taking into account the last element involving the infinite branch.

The method will be made clear from the details of one of the cases. Let $n=9$; then there will be nine points given by the Weddle summations, and the value of η^2 chosen as the p^2 will occupy the $(6n+2)$th position in the tabulated figures, that is, the 56th place. For this position $r=\cdot 725$,

$10^{-4}\eta^2 = 12{\cdot}99$. Subtract 12·99 from all the higher values of the η^2 quantities, extract the square root, multiply by the corresponding r, and take the reciprocal. Group these numbers in successive sets of seven, sum each set according to Weddle's Rule, and multiply by the value of p. The result is the radian measure of the angle between the initial and final radii of the set.

The angle between the final radius of the last set and the radius which is asymptotic to the infinite branch is calculated according to the formula (13).

The angular co-ordinates, whether referred to the position of $r=1$ or to the position of the asymptote, in the present case of $r = {\cdot}725$, are obtained obviously by simple addition of the successive inclinations. The following table gives the various values of the radii vectores r, the angles contained ($\Delta\theta$) etween successive radii vectores, the angular co-ordinates θ', referred to the radius through the vertex, or θ referred to the radius through the epicentre. Evidently $\theta + \theta' = a$.

r.	$\Delta\theta$.	θ.	θ'.	θ in Degrees.	θ' in Degrees.
1	...	0	0·645	0°	36° 56′
0·97	·0149	0·015	·630	0 51	36 5
·94	·0180	·033	·612	1 53	35 3
·91	·0220	·055	·590	3 9	33 47
·88	·0269	·082	·563	4 42	32 14
·85	·0334	·115	·530	6 36	30 20
·82	·0427	·158	·487	9 3	27 53
·79	·0565	·214	·431	12 17	24 39
·76	·0867	·301	·344	17 15	19 41
·73	·2065	·508	·137	29 5	7 51
·725	·1369	·645	·000	36 56	0

These co-ordinates r and θ', or r and θ, give a series of points on the one half of the ray, either the portion from the vertex to the surface or from the surface at the epicentre to the vertex. The other half is exactly similar, but drawn in the other direction, being a mirror reflection of the former half in the axis along the radius vector through the vertex.

In this way seventeen complete seismic rays have been worked out, ten for the Primary waves and seven for the Secondary. The results are embodied in Table B in the Appendix, and the forms of the paths or rays for the Primary waves are shown in fig. 6. The Primary rays are numbered successively from I to X. The Secondary rays are numbered II, III, V, and VII to X, each Secondary ray corresponding as regards depth of vertex with the Primary ray of the same number.

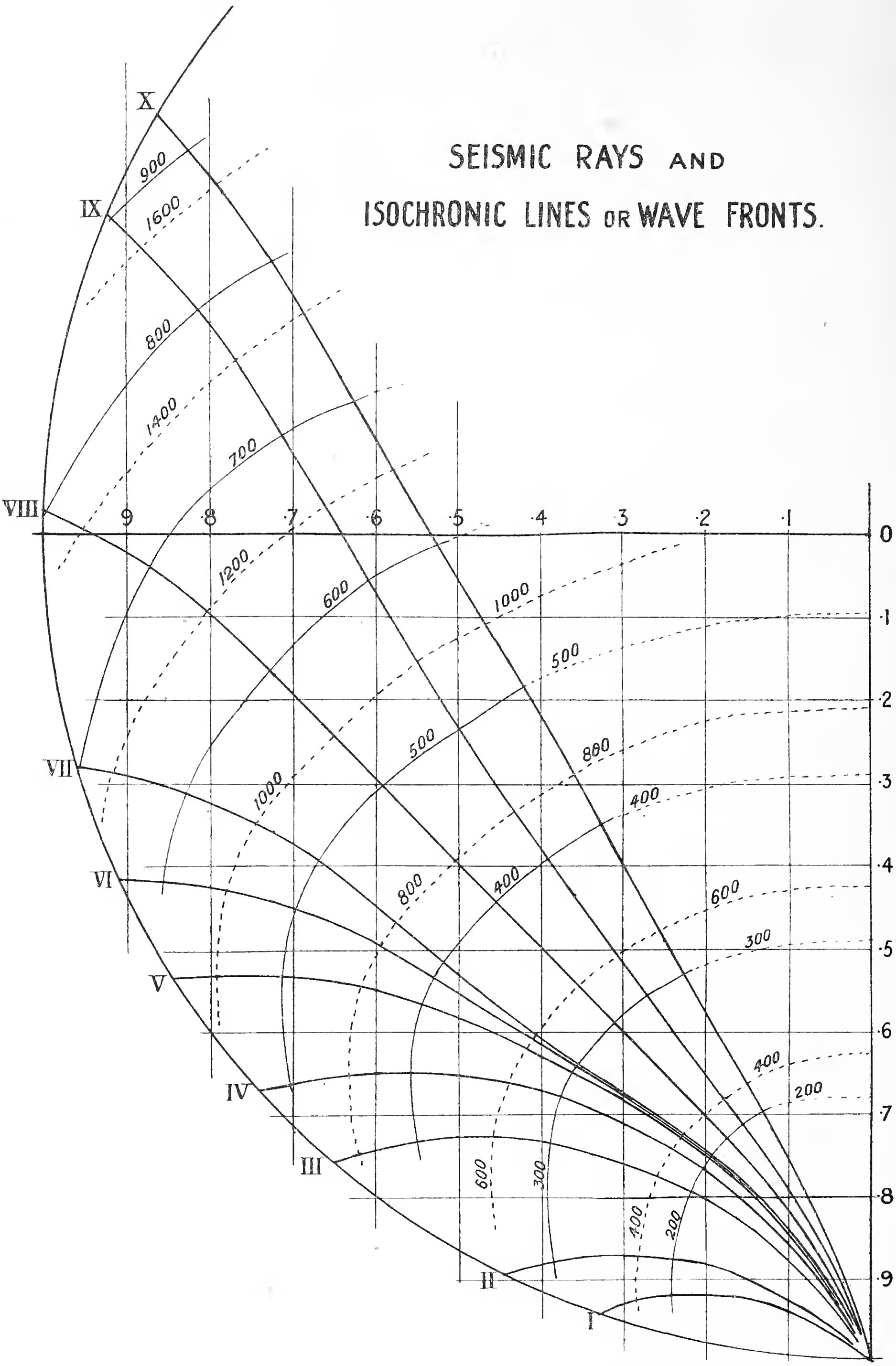

SEISMIC RAYS AND
ISOCHRONIC LINES OR WAVE FRONTS.
I
II
III
IV
V
VI
VII
VIII
IX
X
·9
·8
·7
·6
·5
·4
·3
·2
·1
0
900
1600
800
1400
700
1200
600
1000
500
400
300
200

Fig. 6.

For convenience of showing graphically the forms of these rays, the positions have been calculated also in Cartesian co-ordinates, the origin being at the earth's centre and the x-axis being drawn through the epicentre. In the figure the x-axis is drawn vertically downwards and the y-axis horizontally to the left. Only a little more than a quadrant of a diametral plane of the earth is shown.

In Table B the first column contains the name and number of the ray, the value of the parameter p, and the initial value ϕ_0 between the direction of the ray and the epicentral radius.

The second, third, and fourth columns give the polar co-ordinates, the angle being expressed both in radians and in degrees.

The fifth and sixth columns contain the co-ordinates, $x = r\cos\theta$, $y = r\sin\theta$, used in plotting the representative curves in fig. 6. In this figure the rays for the Primary waves only are drawn in. A simple comparison of the corresponding co-ordinates in the two types of wave shows that the Secondary rays differ very slightly from the corresponding Primary rays. To have drawn them in also would have unnecessarily confused the figure.

To complete the representation it is useful to indicate the form of the wave-front as well as of each ray. The wave-front is obviously an isochronic surface or surface of equal times of transmission from the source of the disturbance. It cuts the diametral plane in a curve whose equation is given by equation (3) above, namely,

$$T = p\theta + \int \frac{dr}{r}\sqrt{\left(\frac{r^2}{v^2} - p^2\right)}$$

when T is put equal to a constant. This curve will cut the rays orthogonally.

As a first step towards construction of the wave-front in various positions we calculate from the data now to hand the times of passage of the disturbance through the points already determined on each seismic ray. The expression for the time consists of two parts, $T = T_1 + T_2$, where

$$T_1 = \int \frac{dr}{r}\sqrt{\left(\frac{r^2}{v^2} - p^2\right)} \quad \text{and} \quad T_2 = p\theta.$$

The latter is found at once by multiplying the θ (in radians) of any point on a chosen ray by the value of p which belongs to that ray. The values obtained in this way are tabulated in the eighth column of Table B.

The value of T must be calculated by quadratures from the quantities r and $\eta^2 = (r/v)^2$ tabulated in Table VI.

The method is clear. For each ray, and therefore for the appropriate

value of p, the quantities $\sqrt{(\eta^2-p^2)}/r$ are calculated and summed together in successive groups of seven, in accordance with Weddle's Rule. For each case the range is from the value of η corresponding to r equal to unity to the value of η immediately above p. This leaves a final element to evaluate with zero as its limiting co-ordinate. The valuation is effected in much the same manner as in the case of the rays. Thus with

$$\left.\begin{aligned}\eta_1{}^2&=br_1-a\\ p^2&=br_0-a\end{aligned}\right\} \qquad \left.\begin{aligned}br_0&=a+p^2\\ b(r_1-r_0)&=\eta_1{}^2-p^2\end{aligned}\right\}$$

where r_1, η_1 relate to the upper limit, and r_0, η_0 or p to the lower limit, we easily find

$$\int_{r_0}^{r_1}\frac{dr}{r}\sqrt{\eta^2-p^2}=2\sqrt{\eta_1{}^2-p^2}\left\{1-\sqrt{\frac{r_1}{r_1-r_0}}\,.\,\cos^{-1}\sqrt{\frac{r_0}{r_1}}\right\}.$$

By appropriately summing together these several integrations in each case, we obtain the values of the integral as tabulated in the seventh column of Table B (Appendix). It is obviously unnecessary to go beyond the vertex of each ray.

The whole time T of propagation of the disturbance to each point is obtained as the sum of the corresponding numbers in columns 7 and 8, and is entered in column 9.

Thus the time of transit is found to every calculated point on each ray.

The next step is to obtain by interpolation the points on the rays which correspond to successive chosen equal intervals of time.

The times chosen for the Primary waves were eight in all, from 200 to 900 seconds inclusive at intervals of 100 seconds; and for the Secondary waves seven, from 400 seconds to 1600 seconds inclusive at intervals of 200 seconds.

Inspection of the simultaneous march of the values of T, θ, and y, or of the graphs giving T in terms of θ or of y, shows that the relation is approximately linear, with slight sinuosities. It was therefore deemed sufficiently accurate to calculate the θ and y in any case for the time $100n$ (n an integer) by simple interpolation between the sets of numbers corresponding to the times lying nearest to $100n$ on each side of it. When these interpolated values of θ and y are found, the values of x and of r follow in accordance with the formulæ

$$x=y\cot\mathrm{an}\,\theta, \quad r=y\,\mathrm{cosec}\,\theta.$$

These interpolated values are arranged in Table C (Appendix), tabulated in columns under the appropriate times, and in rows according to the associated seismic rays. The corresponding isochronous points along the

ray which coincides with the epicentral radius are also estimated and entered.

In fig. 6 the successive positions of the wave-front for the Primary waves are indicated by full lines cutting the rays orthogonally. The very similar curves drawn in broken line are the successive positions of the wave-front for the Secondary waves. They would intersect orthogonally the Secondary rays if these were drawn in.

As already stated, the Primary and Secondary waves were transmitted along paths which are of much the same general character. How comparatively slight the differences are may be indicated by a comparison of the two wave moduli on which the speeds of propagation of the two types of wave are believed to depend. For an elastic medium of density ρ, rigidity n, and incompressibility k, the speeds of propagation of the two waves are given by the formulæ

$$V=\sqrt{\frac{k+\frac{4}{3}n}{\rho}}, \quad U=\sqrt{\frac{n}{\rho}}.$$

Knowing the values of the two velocities for various values of the distance from the earth's centre, we may calculate the ratio k/n and also Poisson's ratio of transverse contraction to longitudinal elongation under the influence of a longitudinal pull.

These are given in the following table:—

r.	R.	V/U.	k/n.	Poisson's ratio.
1	6738	1·80	1·92	·272
·89	6000	1·80	1·89	·275
·816	5500	1·79	1·87	·273
·742	5000	1·81	1·93	·280
·668	4500	1·87	2·16	·3
·594	4000	1·88	2·19	·302
·445	3000	1·87	2·15	·298

For distances from the earth's centre greater than three-fourths of the radius the two elastic constants seem to vary according to the same law, so that the ratio of the velocities remains at the value 1·8 throughout. For distances less than this, however, the ratio increases to nearly 1·9, and this increase takes place fairly suddenly. Either the incompressibility has increased relatively to the rigidity, or the rigidity has diminished relatively to the compressibility. Now the curves showing the velocities in terms of the distances from the centre indicate that the Secondary wave velocity becomes constant at positions somewhat nearer the surface than where the

velocity of the Primary wave becomes constant. This constancy implies that the particular elastic constant is increasing at the same rate as the density, although at less depths in the earth it is increasing at a greater rate. Thus the elastic modulus of the Secondary wave-velocity shows signs of flagging at less depths than does that of the Primary wave-velocity. There is, in other words, a fall-off in the rigidity before the incompressibility shows a similar fall-off in rate of increase. At this critical depth of about 2000 km. below the surface of the earth the ratio of the incompressibility to the rigidity increases from 1·9 to 2·16.

Consider now the form of the seismic rays as shown in fig. 6. The manner in which they radiate from the epicentre is particularly interesting. The shallower rays—that is, those which penetrate to the smaller depths in the earth—are concave outward. With Primary Ray VII, however, a new feature comes to light. The ray although concave outwards for the shallower portions becomes during a certain part of its course convex outwards, suggesting somewhat the form of a Parthian bow. The same peculiarity appears also in the corresponding Secondary ray, but not quite so distinctly marked.

The rays, both Primary and Secondary, whose forms were first calculated were II, III, VII, VIII, IX, X, which correspond to vertex distances from the earth's centre equal to ·905, ·815, ·725, ·635, ·545, and ·455 of the radius of the earth. This series is in arithmetical progression, each distance differing by 0·09 from its neighbour. But this simple progressive variation does not hold either for the arcual distances between the epicentre and the points of emergence of the successive rays or for the corresponding angles of emergence. This is brought out in the following table for the Primary rays just named:—

Ray.	Radius of Vertex.		Arcual Distance.		Angle of Emergence.	
	r.	δr.	2α.	$2\delta\alpha$.	ϕ	$\delta\phi$.
II	·905	·09	26° 29′	14° 28′	42° 32′	12° 14′
III	·815	·09	40 57	32 54	30 18	6 15
VII	·725	·09	73 51	17 15	24 3	3 8
VIII	·635	·09	91 36	20 50	20 55	2 54
IX	·545	·09	112 26	7 42	18 1	3 1
X	·455		120 8		15 00	

The striking increase in the value of 2α with the transition from Ray III to Ray VII, taken in conjunction with the sinuous shape of Ray VII

at once suggested filling in the region between with the Rays IV, V, and VI. The table for the Primary Rays III to VII then becomes:—

Ray.	r.	δr.	2α.	$2\delta\alpha$.	$\frac{2\delta\alpha}{\delta r}$.	ϕ.	$\delta\phi$.	$\frac{\delta\phi}{\delta r}$.
III	·815		40° 57′			30° 18′		
		·045		6° 59′	1·55		3° 39′	0·811
IV	·77		47 56			26 39		
		·025		9 44	3·85		2 1	·808
V	·745		57 40			24 38		
		·010		7 52	7·87		0 20	·330
VI	·735		65 32			24 18		
		·010		8 19	8·32		0 15	·250
VII	·725		73 57			24 3		

On account of the inequality in the successive values of r, it is convenient to supply the two columns headed $2\delta\alpha/\delta r$ and $\delta\phi/\delta r$ so as to compare the relative rates of change. The numbers in column 6 show that the ratio $2\delta\alpha/\delta r$ increases rapidly with the transition from Ray IV to Ray VII.

Another feature of the peculiarity under discussion is the slight change in the emergence angles of the Rays IV to VII, although the change in the corresponding arcual distances is large. Thus in fig. 6 the four rays mentioned form at the start a very close bundle; but they ultimately separate out and reach the surface at wide intervals apart. This rapid dispersion of the rays is obviously associated with the advent of the change in the sign of the curvature, a change whose significance in accordance with equation (7) may be expressed in these words:—

The speed of propagation through the surface layers increases with the depth below the surface down to a certain depth, below which the speed of propagation begins to diminish as the depth increases.

It does not, however, appear that this condition continues or develops to any marked extent, for the sinuous form of still deeper rays seems to become less evident. As will be seen later, other causes begin to operate.

In lack of any definite information to the contrary, the energy radiated from an earthquake origin may be assumed to be, on the average, equally distributed in all directions. Contiguous cones of rays whose initial angles of direction differ by a definite chosen amount will emerge along circles on the surface enclosing between them a zone over which a definite amount of energy will be spread. It needs no calculation to conclude that the rapid dispersion of the rays whose arcual distances are in the neighbourhood of 65° implies a marked decrease of energy associated with unit surface at that distance. It may be of interest, nevertheless, to determine for the true law of velocity variation now obtained the energy distribution over the surface of the globe, the method being the

same as that used in my earlier paper (*Proc. Roy. Soc. Edin.*, vol. xxviii, pp. 228–9).

Let the figure represent a diametral section of the earth through the epicentre E, and let EP be a seismic ray emerging at the arcual distance EP. If Ep is the tangent to the ray at E, the straight line Ep represents the course of the ray if the speed of propagation of the seismic disturbance had been the same at all depths. The cone traced out by all lines Ep which make the same angle with the radius OE cuts the sphere in a small circle of which p may be taken as the representative point. The arc EPp will then represent that part of the spherical surface whose ratio to the

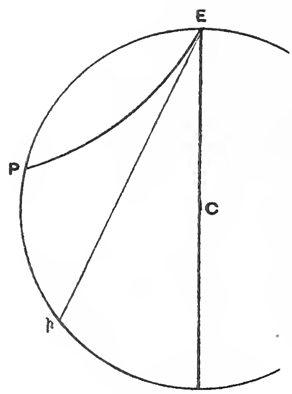


FIG. 7.

whole sphere gives the proportion of energy which falls on the surface represented by the arc EP.

Now the area of the spherical surface represented by the arc EPp is proportional to $1+\cos 2\phi$, where ϕ is the angle OEp. This divided by 2, the value when $\phi=0$, represents the fraction of energy which, radiating outwards from E, is finally distributed over the surface defined by the arc EP. Also the area of this surface in relation to the area of the whole spherical surface is $\frac{1}{2}(1-\cos 2\alpha)$.

These various connected ratios are laid down in the table on p. 181, where 2α is the arcual distance of the ray from epicentre to point of emergence, and ϕ the angle between the radius and the initial direction of the ray. The so-called emergence angle is equal to $90°-\phi$.

Each number in column E $(=\frac{1}{2}(1+\cos 2\phi))$ gives the fraction of the original energy which emerges over the spherical cap whose arcual radius measured from the epicentre has the value 2α; and the area of the spherical cap is given in the column A $(=\frac{1}{2}(1-\cos 2\alpha))$. The differences, δE, give the energies associated with the successive zones as we pass from

the margin of any one cap to the margin of the next following. The corresponding differences, δA, give the areas of these zones. The ratio $\delta E/\delta A$ measures the average value over each zone of the amount of energy associated with unit area of that zone.

2α.	2ϕ.	E. $\frac{1}{2}(1+\cos 2\phi)$.	δE.	A. $\frac{1}{2}(1-\cos 2\alpha)$.	δA.	$\delta E/\delta A$.
			PRIMARY WAVE.			
19° 25′	97° 28′	·435	·108	·0285	·024	15·25
26 29	85 4	·543	·203	·0525	·0705	4·5
40 57	60 36	·746	·053	·123	·042	2·88
47 56	53 18	·799	·028	·165	·065	1·26
57 40	49 16	·827	·004	·230	·063	·431
65 32	48 36	·831	·003	·293	·068	·064
73 51	48 6	·834	·039	·361	·153	·044
91 36	41 50	·873	·031	·514	·177	·255
112 26	36 2	·904	·029	·691	·06	·175
120 8	30	·933	·067	·751	·249	·483
180	0	1·000		1·000		·269
			SECONDARY WAVE.			
25 37	84 26	·549	·206	·049	·074	11·2
41 4	59 20	·755	·048	·123	·084	2·78
54 9	52 46	·803	·020	·207	·176	·571
76 27	49 48	·823	·041	·383	·139	·113
92 31	43 18	·864	·036	·522	·152	·295
110 20	37 0	·900	·029	·674	·118	·237
125 44	30 56	·929	·071	·792	·208	·246
180	0	1·000		1·000		·241

Each value of $\delta E/\delta A$ may be very approximately associated with the middle point of the corresponding zone, thus giving a fairly accurate relationship between the epicentral distance and the energy incident on unit area at that distance. This relationship is given in the tables on p. 182, along with the corresponding values of ϕ.

It will be seen at once that the value of $\delta E/\delta A$ passes through a well-marked minimum in the neighbourhood of 65°. Is there any evidence of such a minimum in the seismograms due to a given earthquake as recorded at different stations? Apparently not. In searching for it, however, we encounter the difficulty of comparing accurately, as regards the amplitude of displacement, the records obtained by quite different forms of instrument. Were all stations supplied, say, with the Galitzin seismographs, whose indications can be absolutely standardised, it might be possible to

make the necessary comparisons and obtain direct evidence of this minimum in the outcropping seismic energy. But there is another difficulty. The seismograms indicate displacement and not energy, and

PRIMARY WAVE.

Epicentral Distance, $2a$.	ϕ.	$\delta E/\delta A$ Energy per Unit Area.	$\sqrt{\frac{\delta E}{\delta A}}$.	$\sin\phi$.	Horizontal Displacement in Plane of Ray.
9° 43′	69° 22′	15·25	3·9	·936	3·65
22 57	45 38	4·50	2·03	·715	1·45
33 43	36 25	2·88	1·70	·594	1·01
44 23	28 6	1·26	1·12	·471	·528
52 48	25 30	·431	·66	·431	·284
61 36	24 28	·064	·25	·414	·104
69 42	24 15	·044	·21	·411	·096
82 43	22 21	·255	·505	·380	·192
102 1	19 28	·175	·43	·333	·140
116 17	16 31	·483	·69	·284	·197
150 4	7 30	·269	·52	·131	·068

SECONDARY WAVE.

$2a$.	ϕ.	$\delta E/\delta A$.	$\sqrt{\delta E/\delta A}$.	$\cos\phi$.	Average Horizontal Displacement.
12° 49′	60° 7′	11·2	3·35	·405	2·56
33 22	35 57	2·78	1·67	·799	1·51
47 37	27 21	·571	·756	·888	·715
65 18	25 51	·113	·336	·900	·320
84 29	23 24	·295	·543	·918	·521
101 26	19 57	·237	·487	·940	·473
118 2	18	·246	·496	·951	·484
152 52	7 44	·241	·491	·991	·489

the records would be comparable with the $\sqrt{\delta E/\delta A}$ and not with $\delta E/\delta A$. When the square roots are calculated and tabulated as in column 4 of the table just given, we may assume these to be proportional to the amplitudes of the vibratory motion existing just within the surface of the earth.

In the case of the Primary wave the to-and-fro motion in the direction of the ray is the most important feature of the vibration, and the component of this along the surface will be comparable with the record on the horizontal pendulum seismograph. This component is obtained by multiplying $\sqrt{\delta E/\delta A}$ by the sine of ϕ; and the last column in the table for the Primary wave contains these numbers, which may be

regarded as proportional to the horizontal displacements in the plane of the ray.

In the case of the Secondary waves the displacement is at right angles to the direction of the ray, and may indeed be in any direction perpendicular thereto. If the displacement be supposed to be along the principal normal to the ray, the associated horizontal displacement is obtained by multiplying by the cosine of ϕ. If, on the other hand, the displacement be regarded as codirectional with the binormal, that is, perpendicular to the plane of the ray, this displacement at the point of emergence will itself be horizontal. Instead of limiting our attention to either of these special directions, we might consider as more satisfactory the average arrangement in which the energy is equally distributed in the two perpendicular directions specified. The squares of the displacements perpendicular to and parallel to the plane of the ray are proportional respectively to $\frac{1}{2}\delta E/\delta A$ and $\frac{1}{2}\delta E/\delta A . \cos^2\phi$; and the square root of the sum of these may be taken as representing the average resultant displacement in the horizontal plane, namely,

$$\sqrt{\{\tfrac{1}{2}\delta E/\delta A(1+\cos^2\phi)\}}.$$

The values are tabulated in the sixth column of the table relating to the Secondary waves.

In these estimates of the displacements the minimum at 65° or 70° is still in evidence, but much less apparent than in the corresponding measures of the energy. It is not surprising, then, that a comparison of the complex records of natural seismic disturbances as given by different types of seismometer at different distances from the epicentre should fail to indicate the presence of this minimum. Moreover, there is a further masking of a possible minimum in virtue of the decay of motion due to viscosity.

The comparison of the horizontal displacements associated with the two types of waves brings out very clearly the tendency for the Primary-wave records, as obtained with the horizontal pendulum, to be smaller at the greater distances than the Secondary-wave records, each being assumed to start with the same energy.

We should expect, therefore, that the advent of the Secondary waves would be more distinctly marked at the greater distances than they seem to be. At moderate arcual distances from the epicentre the Secondary waves frequently show a comparatively large disturbance, and their advent is clearly recognised. Why, then, at arcual distances greater than 110° are the records so uncertain that what one observer calls the

Secondary wave another calls the Primary, while others doubt if either is distinguishable? On the other hand, the Large Surface waves go right round the earth; and in many seismograms certain disturbances which have travelled round the greater arc $(2\pi - 2\alpha)$ can be identified as corresponding in time of start with disturbances which have travelled round the shorter arc 2α. The surface layers of the earth are therefore able to transmit disturbances to distances exceeding the earth's circumference, whereas the Primary and Secondary waves transmitted through the earth seem to be unrecognisable at points distant more than 110° from the epicentre.

In 1906, in a paper published in the *Quarterly Journal of the Geological Society* (vol. lxii), Mr R. D. Oldham put forward the view that the elastic properties of the earth underwent a marked change at a certain depth below the surface. This view was based upon the seismological evidence then available; and he concluded that there was a central nucleus of radius about four-tenths of the earth's radius across which the Secondary waves were transmitted with much smaller velocity than through the parts of the earth outside the nucleus. Certain difficulties in accepting this conclusion were pointed out by Wiechert and Zöppritz;* but although some details of Oldham's speculation may be criticised, there is no doubt that the data now to hand support his main contention that the nucleus of radius 0·4 differs physically from the surrounding shell.

In the calculations given above I have worked from the tables for P and S given by Turner in the British Association Reports; but these are admittedly hypothetical for arcual distances greater than 110°. As may be seen from Tables IV and V, the rays which correspond to this arcual distance reach a depth 3400 km. from the earth's centre, that is, almost exactly half-way down. Long before this depth is reached, however, the velocities of propagation of both the Primary and Secondary waves have ceased to increase with the depth. The remarkably steady increase in both velocities down to these critical depths shows that the moduli $k+4n/3$ and n increase with the depth more rapidly than the density. Then apparently at a depth equal to 0·30 of the earth's radius the ratio n/ρ becomes steady or passes slowly through a maximum, whereas the ratio $(k+4n/3)/\rho$ is still on the increase, and continues increasing till the depth 0·36 is reached. Thereafter it becomes steady or passes slowly through a maximum.

There is therefore a change in the manner in which $(k+4n/3)$ and n

* See their first paper on "Erdbebenwellen," *Göttingen Nachrichten, mathem.-phys. Klasse*, 1907, p. 516.

march with the density, a change which first appears in the rigidity n. Since in round numbers k is about double n, any change occurring in n will produce in $k+4n/3$ a less proportional change. Hence we infer that the elastic changes which appear at the depths mentioned affect first the rigidity n, and thereafter the incompressibility k. In other words, the changing constitution of the earth's material at these critical depths discloses itself most markedly in that elastic characteristic which belongs to solids as distinguished from liquids or non-rigid substances.

Bearing in mind the fact that the data do not carry us further down than about half the earth's radius (corresponding to a ray of arcual range 110°), may we not conclude that at this depth the rigidity becomes small or even zero, so that the distortional wave practically ceases to exist? What becomes of it? It may either be lost in virtue of viscosity in the growingly plastic material, or its energy may pass into the compressional wave form. It is reasonable to regard the incompressibility of this plastic nucleus to be of much the same magnitude as that of the encompassing elastic solid shell. The speed of the compressional wave will therefore be less in the nucleus than in the shell. These suppositions are in accord with the facts that the speed of the Primary wave distinctly diminishes as the depth approaches the position of the inner nucleus, and that the records of the distortional wave as a distinct phenomenon which can be identified are very uncertain at arcual distances greater than 110°. To guard against the possibility of misunderstanding, I should state that the non-existence of the distortional wave beyond this arcual distance does not mean that no disturbances are recorded at times that might be identified with the Secondary waves assumed to emerge there, but that these observed disturbances have not the characteristics of the Secondary-wave disturbances at smaller distances and cannot be distinctly identified as caused by distortional waves passing continuously as such from the original source of the earthquake disturbance.

The material of the earth may then be regarded as essentially an elastic solid down to a depth of half the earth's radius. Throughout about half the thickness of this elastic shell the two elastic constants increase with depth at a greater rate than the density, so that the speeds of propagation of the compressional and distortional waves increase steadily up to values respectively 80 per cent. and 73 per cent. greater than near the surface. Throughout the lower half of the elastic shell the speeds of propagation show a tendency to decrease slowly after reaching their maximum values, and this tendency first declares itself in the distortional wave. There is relatively a fall-off in the rigidity. The suggestion is

that as the nucleus is approached the material of the earth is becoming less of an elastic solid and more of an elastic, highly compressed liquid. The change probably comes on gradually within a comparatively thin shell whose outer and inner radii are, say, 0·5 and 0·4 of the earth's radius. Within this nucleus of radius 0·4 the material of the earth has lost its elastic solid character and can transmit only compressional waves with speed $\sqrt{k/\rho}$, where k is the incompressibility and ρ the density of the material.

It is not possible to make any definite calculations as to the manner in which elastic waves of both types are transformed during transmission across this gradually changing layer. It may be spoken of as a semi-permeable layer, for no distortional wave can pass through it.

We may, however, gain some idea of what occurs by considering the limiting case of an elastic solid passing abruptly into a non-rigid elastic substance of equal compressibility and density. This is one of a general type of problem which I discussed as early as 1888 in a paper read before the Seismological Society of Japan and published in their *Transactions.* The paper was reprinted with additions in the *Philosophical Magazine* (July 1899) under the title "Reflexion and Refraction of Elastic Waves with Seismological Applications."

The details of the present calculation need not be given. It is a simple enough matter to construct the special forms of the equations determining the energies of the refracted and reflected waves in terms of the energy of the incident wave as they apply to the case now imagined. The data are the velocities of the compressional and distortional waves in the elastic shell; and the assumption is that the non-rigid nucleus has the same compressibility as the enclosing shell. If V and U represent the speeds of propagation of the two types of wave in the shell, then

$$V^2 = \frac{k + 4n/3}{\rho} \quad \text{and} \quad U^2 = \frac{n}{\rho}.$$

Hence the square of the speed of propagation of the compressional wave in the nucleus is

$$V'^2 = \frac{k}{\rho} = V^2 - 4U^2/3.$$

Now V and U are given from the seismic calculations, hence the value of V′ is found, and from these the angles of reflection and refraction.

Thus taking from the Tables IV and V the values 12·89 and 6·88 for V and U, we find for V′ the value 10·15. The ratios of these, taken in pairs, give either the refractive index in the usual sense, or what might

be termed the reflective index for the reflection of a type of wave differing from the incident wave. The ratios are

$$V/V' = 1 \cdot 27, \quad V/U = 1 \cdot 87, \quad V'/U = 1 \cdot 47.$$

With the help of these values the angles of reflection and refraction corresponding to any chosen angle of incidence for either type of wave are at once obtained, and the elastic equations referred to above lead to the determination of the relative energies associated with the various waves.

Three cases are to be considered: (I) compressional wave incident in the solid elastic shell giving rise to a compressional refracted wave through the nucleus and two reflected waves, one compressional and one distortional, in the shell; (II) distortional wave incident in the shell giving rise, as in the previous case, to a compressional refracted wave in the nucleus and distortional and compressional reflected waves in the shell; (III) compressional wave incident in the non-rigid nucleus giving rise to two refracted waves in the shell and one reflected wave in the nucleus.

In Table VII on p. 188, arranged in correspondence with the cases just named, the results are given in sufficient detail so as to show the distribution of the energy among the various reflected and refracted rays, and the angles of incidence and refraction corresponding.

The headings a, a_1, a' refer to the energies in the incident reflected and refracted compressional waves, and b, b_1, b' similarly for the distortional waves. The angles θ, θ_1, θ' are the angles of incidence, reflection, and refraction for the compressional ray, and ϕ, ϕ_1, ϕ' the like angles for the distortional ray. Note that the "suffix" always refers to reflected waves in the first medium, and the "dash" to the refracted waves in the second medium.

The results are also shown graphically in fig. 8.

In Table VII the energy of the incident ray for each angle of incidence is taken as unity. But when we consider the cone of rays which emanate from the epicentre and which fall on the surface of the nucleus, we cannot regard these various rays as bringing to that surface equal energies. Along each elemental cone the energy per unit surface falls off as the distance increases, and also falls off in virtue of the obliquity of the angle which the axis of the elemental cone makes with the surface.

To take this into account, suppose the energy to radiate from the epicentre equally all round and to fall on a spherical surface of radius a concentrically situated within the earth of radius b. The energy which crosses the spherical surface of radius $(b-a)$, centre E, passes on to the

Table VII.—Incident, Reflected, and Refracted Rays.

(I) Compressional Wave incident in the elastic Solid.

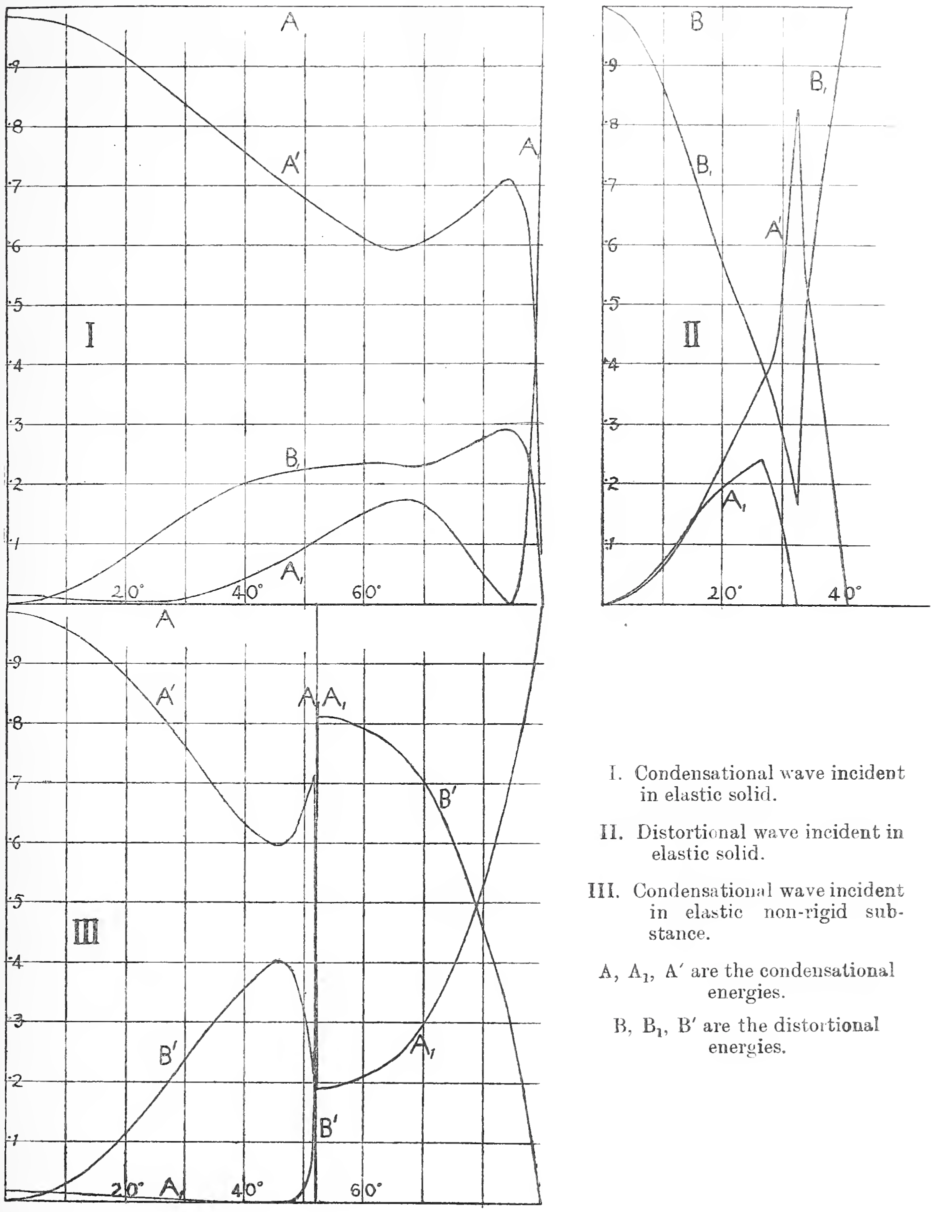
Energies.				Angles of incidence, reflexion, refraction.		
a.	a_1.	b.	a'.	$\theta=\theta_1$.	ϕ.	θ'.
1	0·014	0·00007	0·986	0° 34′	0° 18·5′	0° 27′
1	·013	·007	·980	5 42	3 3	4 29
1	·009	·020	·971	9 28	5 3	7 25
1	·005	·041	·954	14 2	7 28	10 59
1	·002	·125	·873	26 34	13 52	20 36
1	·068	·217	·795	45 0	22 17	33 41
1	·109	·227	·664	51 50	24 45	38 0
1	·150	·234	·616	59 2	27 23	42 24
1	·169	·236	·595	63 26	28 46	44 41
1	·172	·229	·599	68 11	29 53	48 1
1	·068	·270	·662	78 41	31 41	50 29
1	·0006	·290	·7094	84 17	32 14	51 29
1	·016	·286	·698	85 26	32 18	51 35
1	·116	·258	·626	87 8	32 22	51 44
1	·448	·160	·392	88 51	32 25	51 48

(II) Distortional Wave incident in the elastic Solid.

b.	b_1.	a_1.	a'.	$\phi=\phi_1$.	θ_1.	θ'.
1	·953	·026	·021	5° 42′	10° 41′	8° 24′
1	·879	·065	·056	9 28	17 53	14 0
1	·754	·125	·121	14 2	26 55	20 53
1	·612	·183	·205	18 26	36 8	27 38
1	·521	·210	·269	21 48	43 53	33 0
1	·401	·230	·369	26 34	56 35	41 15
1	·176	·000	·824	32 25	90	51 48
1	·454	...	·546	33 41	...	54 24
1	1·000	...	...	43 1	...	90

(III) Compressional Wave incident in the elastic non-rigid Substance.

a.	a_1.	a'.	b'.	$\theta=\theta_1$.	θ'.	ϕ'.
1	·014	·977	·009	5° 42′	7° 16′	3° 54′
1	·013	·960	·027	9 28	12 5	6 26
1	·011	·932	·057	14 2	17 59	9 31
1	·0083	·895	·0967	18 26	23 46	12 29
1	·0067	·860	·1333	21 48	28 16	14 40
1	·004	·806	·190	26 34	34 47	17 46
1	001	·715	·284	33 41	44 55	22 17
1	·00008	·599	·401	45	64 8	28 55
1	·00045	·611	·389	48 1	71 35	30 30
1	·063	·708	·229	51 20	83 45	32 10
1	1	·000	·000	51 50	90	32 25
1	·798	...	·202	51 52	...	32 27
1	·189	...	·811	52 3	...	32 32
1	·203	...	·797	59 2	...	35 47
1	·270	...	·730	68 12	...	39 18
1	·488	...	·512	78 41	...	41 58
1	·692	...	·308	84 17	...	42 45



I. Condensational wave incident in elastic solid.

II. Distortional wave incident in elastic solid.

III. Condensational wave incident in elastic non-rigid substance.

A, A_1, A′ are the condensational energies.

B, B_1, B′ are the distortional energies.

FIG. 8.

corresponding element of the surface of the sphere of radius a, centre C, as indicated by the two close diverging rays from E in fig. 9.

Let β be the angle CEP and θ the angle of incidence of the ray at P, so that the angle PCE has the value $(\theta-\beta)$.

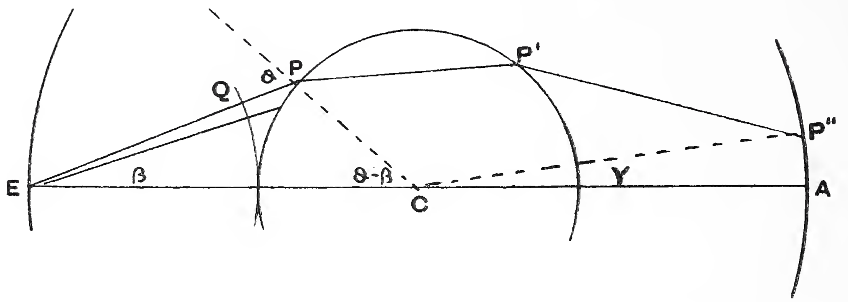


Fig. 9.

The energy which passes across the annulus of area

$$(b-a)d\beta\,.\,2\pi(b-a)\sin\beta$$

at position Q falls on the annulus of area

$$ad(\theta-\beta)\,.\,2\pi a\sin(\theta-\beta)$$

at position P, so that for every unit radiating across the unit area at Q there falls on unit area at P the amount

$$\frac{(b-a)^2 d\beta\sin\beta}{a^2 d(\theta-\beta)\sin(\theta-\beta)}.$$

Now

$$a\sin\theta=b\sin\beta \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

so that

$$ad\theta\cos\theta=bd\beta\cos\beta$$

and

$$\frac{d\beta}{d(\theta-\beta)}=\frac{a\cos\theta}{b\cos\beta-a\cos\theta}$$

and the energy per unit surface of the nucleus is

$$\frac{(b-a)^2 a\cos\theta}{a^2(b\cos\beta-a\cos\theta)}\cdot\frac{\sin\beta}{\sin(\theta-\beta)}=\frac{(b-a)^2\tan\beta}{a(b\cos\beta-a\cos\theta)(\tan\theta-\tan\beta)} \quad . \quad (2)$$

Equation (1) gives for each chosen value of β the corresponding value of θ and its trigonometrical ratios, and by means of equation (2) the energy incident on the nucleus may be calculated. In Table VIII the first column contains values of β increasing by steps of 3° up to 21°, the last being 23° 35′, the position of the tangent line; the second column contains the corresponding values of θ, the angle of incidence on the nucleus;

and the numbers in the third column give the ratios of the energy per unit surface of the nucleus at the position β to the energy at distance $(b-a)$ from the epicentre.

TABLE VIII.—ENERGY DISTRIBUTION.

β.	θ.	Energy per Unit Surface.	Reducing Ratio.	Energy in Nucleus.	Average Values.
0°	0°	1·0000	0·994	0·944	·98
3	7 3′	·9836	·977	·961	·927
6	15 9	·9384	·946	·888	·835
9	23 1	·8638	·893	·771	·730
12	31 19	·7596	·886	·673	·570
15	40 19	·6279	·752	·472	·390
18	50 36	·4702	·672	·316	·240
21	63 38	·2797	·595	·166	·070
23 35′	90	0·0000	0	0	

We have now to consider the energy which passes into the nucleus as each of the rays (β) is refracted at the boundary separating the shell from the nucleus. The data for making this calculation are supplied by Table VII. In this table, however, each incident ray, whatever its inclination, is assumed to have energy unity; but in the present case the energy per unit surface associated with each incident ray other than the ray $\beta=0$ is less than unity, in accordance with the short Table VIII just given. In order to find how much energy passes into the nucleus, we must multiply each number in the third column in this table by the fraction appropriate to the corresponding angle of incidence as given in Table VII.

From the graphs, which are shown on reduced scale in fig. 8, the various energies may be picked off with sufficient accuracy for any required angle of incidence. When this is done for the various angles θ in Table VIII, certain numbers are obtained for the refracted condensational wave. These are tabulated under the heading Reducing Ratio, and form the fourth column in Table VIII. Multiplying these into the corresponding energies per unit surface incident on the nucleus, we obtain in the fifth column the corresponding energies which pass with the refracted condensational wave into the nucleus.

To find the whole energy which passes into the nucleus, we must evaluate the integral

$$\int_{\theta=0}^{\theta=\frac{\pi}{2}} 2\pi a \sin(\theta-\beta)\,.\,e a d(\theta-\beta)$$

where e is the energy per unit surface at position β.

For purposes of approximate quadrature this may be evaluated in the form

$$\sum 2\pi a^2 \bar{e} \int_1^2 \sin(\theta-\beta) d(\theta-\beta) = \sum 2\pi a^2 \bar{e} \{\cos(\theta-\beta)_1 - \cos(\theta-\beta)_2\}$$

where the positions (1) and (2) indicate the beginning and end of a narrow zone on the spherical surface at P, and where $\bar{e}$ is the average value of e over this zone. The average values over the zones bounded by the successive values of $\theta-\beta$ were obtained graphically from the curve showing the relation between $\theta-\beta$ and the energy. They are given in the sixth column of Table VIII.

The final calculation in shown below.

$\theta-\beta$.	Cos $(\theta-\beta)$.	Difference.	Average Energy.	Product.
0° 0′	1			
		·0024	·98	·0024
4 3	·9976			
		·0104	·927	·0096
9 9	·9872			
		·0170	·835	·0142
14 1	·9702			
		·0265	·730	·0193
19 19	·9437			
		·0397	·570	·0226
25 19	·9040			
		·0615	·390	·0240
32 36	·8425			
		·1187	·240	·0285
42 38	·7238			
		·3238	·070	·0227
60 25	·4000			
			Sum	·1433

This fraction multiplied by $2\pi a^2$ represents the energy passing into the nucleus expressed in terms of the energy per unit surface incident on the nucleus along the ray EC.

The whole energy supplied in terms of the same quantity is

$$2\pi(b-a)^2(1-\cos\beta), \qquad \beta = 23° \ 35'.$$

Hence the ratio of the condensational wave energy which passes into the nucleus to the amount which falls on it is

$$\frac{2\pi a^2 \times ·1433}{2\pi(b-a)^2 \times (1-\cos\beta)} = \frac{16 \times ·1433}{36 \times ·0835} = ·762.$$

The condensational energy passing through the nucleus will emerge again into the shell; but in this case the angles of incidence will be included between the values 0° and 51° 50′, which is the angle of total reflection for the condensational wave (see Table VII and diagram III of fig. 8). The proportional loss of energy at this second refraction will be

practically the same as at the first refraction. Hence the condensational waves as they pass through the nucleus and enter the shell again will carry with them ·596($=(\cdot762)^2$) of the original energy, or almost exactly six-tenths. The remainder of the energy will be partly reflected as condensational and distortional waves in the shell at the first surface and as condensational waves in the nucleus at the second surface, and partly transmitted as refracted distortional waves in the shell at the second surface. As may be inferred from diagram III in fig. 8, the refracted distortional wave in the shell (B′) accounts for most of the energy not carried on by the refracted condensational wave (A′). Since ·76 is the fraction of the original energy which penetrates into the nucleus, the fraction of the original energy which proceeds as distortional waves in the shell is not greater than $\cdot76 \times \cdot24 = \cdot18$. This will reach the outer surface considerably later than the condensational wave, and will be simply superposed upon the existing disturbances and quite indistinguishable.

It is interesting to see how the refracted condensational waves will emerge at the outer surface. Let a complete ray be EPP′P″ (fig. 9), meeting the outer surface in P″. What is the angle P″CA in terms of the angle CEP? It is easily shown by the simple laws of geometrical optics that if γ is the angle P″CA and n the refractive index of the nucleus, then

$$\begin{aligned}\tfrac{1}{2}\gamma &= \sin^{-1}\left(\frac{b}{an}\sin\beta\right) - \sin^{-1}\left(\frac{b}{a}\sin\beta\right) + \beta \\ &= \sin^{-1}(1\cdot545\sin\beta) - \sin^{-1}(2\cdot5\sin\beta) + \beta.\end{aligned}$$

The following are the values of γ corresponding to the successive values of β increasing by 3°:—

β	3°	6°	9°	12°	15°	18°	21°	23° 35′
γ	+0° 14′	+0° 18′	−0° 4′	−1° 4′	−3° 28′	−8° 8′	−17° 56′	−56° 30′

The negative signs mean that the corresponding rays EPP′P″ intersect the diameter ECA somewhere between C and A and meet the surface of the earth on the other side of CA. Of the half cone of rays which fall upon the upper hemisphere of the nucleus, as shown in fig. 9, those whose directions lie between 8°·6 and 23°·6 emerge on the lower half of the terrestrial sphere; and of the rays which form the lower half of the cone, the vast majority similarly emerge on the upper part of the plane.

Did no nucleus of the kind supposed exist, the rays emanating from E within the cone of semi-angle 23°·6 would pursue paths not deviating greatly from the chords (see fig. 6), and emerge on the earth's surface over an area bounded by a circle whose radius would subtend at the centre C an angle of 47°·2. The effect of the nucleus is to spread these rays over an area the radius of whose contour subtends an angle of 56°·5. The distribution of energy will not be the same in the two cases; but in the latter case the whole energy has been reduced to six-tenths of the original on account of refraction at the two surfaces. A consideration of the distribution of energy over the various zones, as indicated in Table VIII, also shows that the concentration of rays in the neighbourhood of the antipodal point is of slight importance. This disposes of one of the arguments advanced by Wiechert and Zöppritz against Oldham's hypothesis of a nucleus through which the elastic waves pass with diminished speed.

It is hardly necessary to consider in the same detail the effect of the non-rigid elastic nucleus upon the transmission of the rays originally distortional in the solid shell. A glance at diagram II of fig. 8 shows that the greater part of the incident energy is reflected back as distortional waves in the shell. Moreover, since the speed of the distortional wave in the shell is less than the speed of the condensational wave in the nucleus, the refracted condensational waves diverge as they enter the nucleus, and diverge still more when they pass out into the shell either as condensational or distortional waves. Their divergence is so great and their relative energy values are so small that they will bring to disturbances already existing at the surface where they emerge a quite inappreciable addition.

If instead of the elastic properties changing abruptly from those of a solid elastic shell to those of a non-rigid elastic nucleus of equal compressibility, we have a gradual transition from the solid to the non-rigid with viscosity resisting distortion, the broad results will not differ materially from what has just been established. The distortional waves will be killed out and the energy of the condensational waves largely reduced. What will emerge at the surface of the earth on the further side of the nucleus will be predominantly in the form of condensational waves with no appreciable concentration in the antipodal region. The seismograms obtained by instruments of the horizontal pendulum type will be comparatively inconspicuous and wholly devoid of the well-marked characteristics of seismograms obtained at arcual distances from the epicentre less than 110°.

As the final sections of this paper were being written, my attention was drawn by Dr Harold Jeffreys to his three papers on the "Viscosity of the Earth," published in vols. lxxv, lxxvi, lxxvii of the *Monthly Notices of the Royal Astronomical Society* (1915 to 1917). His main object in these papers is to make the lunar secular acceleration due to tidal friction compatible with the existence of the Eulerian nutation; and in the third paper, in which he introduces a law of viscosity suggested by Sir Joseph Larmor, he refers to its bearing on the transmission of earthquake waves. Maxwell in his second great paper on the dynamical theory of gases (*Phil. Trans.*, 1866) gives a simple mathematical description of the phenomena of viscosity. He considers the strain S and the stress F to be connected by the formula $F = ES$, E being a constant elastic modulus. In a solid body free from viscosity

$$\frac{dF}{dt} = E\frac{dS}{dt}.$$

If there is viscosity, F will tend to disappear at a rate depending on the value of F. If this rate is assumed to be proportional to F, then

$$\frac{dF}{dt} = E\frac{dS}{dt} - \frac{F}{T_1},$$

T_1 being the constant known as the "time of relaxation." Hence

$$ES = F + \frac{1}{T_1}\int F dt$$

is the expression for Maxwell's law of viscosity, and corresponds to the law of "elastico-viscosity" used by Sir G. H. Darwin in his work on tidal friction. Under this law the material yields indefinitely under action of a stress, and when the stress is removed it acquires permanent set. Larmor's suggestion is to make the friction proportional to the rate of straining, so that the equation connecting F and S becomes

$$ES = F - k\frac{dS}{dt} \quad \text{or} \quad E\left(S + T_2\frac{dS}{dt}\right) = F$$

where T_2 is another constant.

Under a constant stress the strain approaches its final value F/E asymptotically. On removal of the stress the strain falls off asymptotically to zero. There is no permanent set. Dr Jeffreys distinguishes this kind by the name "firmo-viscosity." He shows that, in regard to the lunar secular acceleration and the Eulerian nutation, compatibility is secured when T_2 is equal to 371 seconds as an average for the whole earth. He also shows that, with T_2 equal to 1 second, distortional waves of 20 seconds period would not penetrate to more than about 100 km. To

penetrate without appreciable decay to 3000 km. would therefore require a much smaller value of T_2, say one-fiftieth of a second. Consequently, to satisfy the observed phenomena of earthquake waves and also the other conditions studied by Dr Jeffreys, we must assume that T_2 is very small down to a depth of half the earth's radius, that about this depth it begins to become greater, and that at greater depth it becomes so great as to give the average value just mentioned for the whole earth. This high viscosity at the central parts of the earth is to be explained as due presumably to the increased temperature and pressure; but it is not easy to find an explanation in terms of any ordinary theory of the constitution of matter. There is no doubt, however, that the main facts can be co-ordinated in terms of this hypothesis.

The view presented in this paper is that the rigidity of the earth's material breaks down under the influence of the increasing temperature, but that the non-rigid central core retains a measurable compressibility. I have purposely refrained from speaking of this central core as being liquid, since that word connotes properties which may not be possessed by the material at the earth's core. It may, under isotropic stress, remain practically solid as a whole and yet be unable to transmit distortional waves. Such a supposition may not necessarily be incompatible with the other conditions required for the solution of the problems discussed by Dr Jeffreys.

The main conclusions of the present paper may be summarised as follows:—

1. For the first time, by a rigorous mathematical method, the forms of the seismic rays and of the isochronous surfaces have been deduced directly from the data of observation.

2. The seismic rays, both of the condensational and distortional waves, are on the whole concave outwards, indicating that the speeds of propagation increase with depth below the earth's surface until a depth equal to about three-tenths of the earth's radius is reached.

3. At this depth the speeds of propagation tend to or reach a constant value and then fall off slightly for greater depths, certain seismic rays showing a convexity outwards.

4. The data of observation are insufficient to enable us to trace waves which reach a depth lower than six-tenths of the earth's radius.

5. The evidence is that at or near this depth the distortional wave is killed out, so that over arcual distances from the epicentre greater than 120° there is no characteristic appearance of the Secondary wave in the seismograms.

6. The hypothesis suggested by these facts and deductions is that the earth consists of an elastic solid shell down to a depth of about half the earth's radius, that at this depth the rigidity begins to break down, and that finally, at a depth of six-tenths of the earth's radius, the elastic solid shell gives place to a non-rigid nucleus of measurable compressibility. This hypothesis is broadly similar to the views advanced by R. D. Oldham in 1906.

7. H. Jeffreys' suggestion that the phenomena may be co-ordinated in terms of the theory of firmo-viscosity, which he has found serviceable in other lines of research, is worthy of consideration, although here also there are some difficulties to surmount.

Thanks are due to the Carnegie Trust for the Universities of Scotland for financial help in preparing and printing the illustrations and tabular matter.

[Appendix

Appendix.

Table A.—Times of Transit of Primary (P) and Secondary (S) Waves.

Degrees.	P sec.	S sec.	S − P sec.	Degrees.	P sec.	S sec.	S − P sec.	Degrees.	P sec.	S sec.	S − P sec.
1	15	28	13	51	553	991	438	101	855	1565	710
2	31	55	24	52	560	1004	444	102	860	1575	715
3	47	83	36	53	566	1016	450	103	865	1584	719
4	62	110	48	54	573	1029	456	104	870	1593	723
5	77	137	60	55	579	1041	462	105	874	1602	728
6	92	164	72	56	586	1054	468	106	879	1612	733
7	106	190	84	57	592	1066	474	107	884	1621	737
8	121	217	96	58	599	1079	480	108	888	1630	742
9	136	243	107	59	605	1091	486	109	893	1639	746
10	150	269	119	60	612	1103	491	110	897	1648	751
11	164	294	130	61	619	1116	497	111	902	1657	755
12	179	319	140	62	625	1128	503	112	907	1666	759
13	193	344	151	63	632	1141	509	113	911	1674	763
14	206	368	162	64	638	1153	515	114	916	1682	766
15	219	392	173	65	645	1165	520	115	920	1690	770
16	232	415	183	66	651	1177	526	116	925	1698	773
17	245	438	193	67	658	1190	532	117	929	1706	777
18	257	460	203	68	664	1202	538	118	934	1714	780
19	269	482	213	69	671	1214	543	119	938	1722	784
20	281	503	222	70	677	1226	549	120	942	1729	787
21	293	524	231	71	683	1238	555	121	947	1737	790
22	305	545	240	72	690	1250	560	122	952	1744	792
23	317	565	248	73	696	1262	566	123	957	1752	795
24	328	584	256	74	702	1274	572	124	961	1759	798
25	338	603	265	75	709	1286	577	125	966	1766	800
26	348	622	274	76	715	1297	582	126	970	1773	803
27	358	641	283	77	721	1309	588	127	974	1780	806
28	368	659	291	78	727	1320	593	128	978	1787	809
29	378	677	299	79	733	1332	599	129	983	1794	811
30	388	694	306	80	739	1343	604	130	988	1801	813
31	398	711	313	81	745	1355	610	131	992	1807	815
32	407	728	321	82	750	1366	616	132	996	1814	818
33	416	744	328	83	756	1377	621	133	1001	1821	820
34	425	760	335	84	762	1388	626	134	1005	1827	822
35	433	775	342	85	768	1399	631	135	1009	1833	824
36	442	790	348	86	773	1410	637	136	1014	1840	826
37	450	804	354	87	779	1421	642	137	1018	1846	828
38	458	818	360	88	785	1432	647	138	1023	1852	829
39	466	832	366	89	790	1443	653	139	1027	1858	831
40	475	847	372	90	796	1454	658	140	1031	1864	833
41	483	861	378	91	801	1464	663	141	1035	1869	834
42	491	875	384	92	807	1475	668	142	1039	1875	836
43	498	888	390	93	812	1485	673	143	1043	1881	838
44	506	902	396	94	818	1496	678	144	1047	1886	839
45	513	915	402	95	823	1506	683	145	1051	1892	841
46	520	928	408	96	829	1516	687	146	1055	1897	842
47	527	941	414	97	834	1526	692	147	1059	1902	843
48	534	954	420	98	840	1536	696	148	1063	1907	844
49	540	966	426	99	845	1546	701	149	1067	1912	845
50	547	979	432	100	851	1556	705	150	1071	1917	846

Appendix.

Table B.—Tracing of Seismic Rays, with Times of Transit.

Primary Wave.

Ray.	Polar Co-ordinates. Radius. r.	Radian. θ.	Degree. θ.	Cartesian Co-ordinates. x.	y.	$\int \frac{dr}{r} \sqrt{\eta^2 - p^2}$.	$p\theta$.	Time.
P. I	1·00	0·0000	0° 0′	1·000	0·000			
$p=664·8$	·97	·0427	2 27	·969	·0415	14·63	28·4	43
$\phi=48° 41'$	·94	·1205	6 54	·933	·113	22·46	80·1	102·6
	·935	·1695	9 43	·922	·158	22·96	112·7	135·7
	·94	·2184	12 31	·918	·204	...	...	168·8
	·944	·2591	14 51	·923	·245	...	...	200
	·97	·2962	16 58	·928	·283	...	...	228·4
	1·00	·3389	19 25	·943	·332	...	...	271·4
P. II	1·00	0·0000	0 0	1·000	0·000			
$p=598$	·97	·0332	1 54	·97	·032	17·12	9·85	37
$\phi=42° 32'$	·94	·0801	4 35	·937	·075	29·87	47·9	77·8
	·91	·1724	9 53	·896	·157	37·22	103·1	140·3
	·905	·2311	13 14	·881	·207	37·79	138·2	176·0
	·909	·2705	15 30	·876	·243	...	...	200
	·91	·2898	16 36	·872	·260	...	...	211·7
	·94	·3821	21 54	·872	·351	...	...	274·2
	·959	·4118	23 36	·879	·384	...	...	300
	·97	·4290	24 35	·882	·404	...	...	315·0
	1·00	·4622	26 29	·895	·446	...	...	352·0
P. III	1·00	0·0000	0 0	1·000	0·000			
$p=445$	·97	·0197	1 8	·97	·019	21·02	8·76	29·8
$\phi=30° 18'$	·94	·0443	2 32	·939	·042	38·95	19·71	58·7
	·91	·0756	4 20	·907	·069	53·81	33·64	87·5
	·88	·1176	6 44	·874	·104	66·06	52·33	118·4
	·85	·1746	10 3	·837	·148	75·40	77·69	153·1
	·823	·2695	15 26	·793	·219	...	...	200
	·82	·2863	16 24	·785	·232	80·94	127·4	208·3
	·815	·3573	20 28	·764	·285	81·41	159·0	240·4
	·82	·4283	24 32	·746	·341	...	...	272·5
	·836	·4840	27 44	·734	·386	...	...	300
	·85	·5400	30 56	·729	·437	...	...	327·7
	·88	·5970	34 8	·728	·494	...	...	362·4
	·91	·6390	36 37	·730	·543	...	...	393·3
	·915	·6444	36 55	·732	·550	...	...	400
	·94	·6703	38 24	·738	·584	...	...	432·1
	·97	·6949	39 49	·744	·622	...	...	452·0
	1·00	·7146	40 57	·755	·655	...	...	480·8
P. IV	1·00	0·0000	0 0	1·000	0·000			
$p=396·6$	·97	·0173	0 59	·97	·017	22	6·76	28·8
$\phi=26° 39'$	·94	·0375	2 9	·939	·035	41·08	14·7	55·8
	·91	·0625	3 35	·908	·057	57·6	24·4	82·0
	·88	·0936	5 22	·877	·081	71·78	36·6	108·4
	·85	·1333	7 38	·842	·113	83·69	52·1	135·8
	·82	·1868	10 42	·807	·152	93·20	73·0	166·2

APPENDIX.

TABLE B—*continued.*

	r.	θ.	θ.	x.	y.	$\int \frac{dr}{r} \sqrt{\eta^2 - p^2}$.	$p\theta$.	Time.
P. IV—*continued.*	·792	0·2577	14° 46′	·766	·202	...	...	200
	·79	·2673	15 19	·762	·209	100·07	104·5	204·6
	·775	·3419	19 35	·730	·260	102·08	133·7	235·8
	·77	·4183	23 38	·704	·313	102·35	163·6	266
	·775	·4947	28 21	·682	·368	...	...	296·2
	·776	·5046	28 55	·679	·375	...	...	300
	·79	·5693	32 37	·665	·426	...	...	327·4
	·82	·6498	37 14	·652	·496	...	...	365·8
	·85	·7033	40 18	·649	·550	...	...	396·2
	·854	·7088	40 37	·648	·556	...	...	400
	·88	·7430	42 34	·648	·595	...	...	423·6
	·91	·7741	44 21	·651	·636	...	...	450·0
	·94	·7991	45 47	·653	·673	...	...	476·2
	·967	·8169	46 48	·662	·705	...	...	500
	·97	·8193	46 57	·663	·709	...	...	503·2
	1·00	·8366	47 56	·67	·742	...	...	532
P. V	1·00	0·0000	0 0	1·000	0·000			
$p=368·5$	·97	·0153	0 53	·97	·015	22·35	5·64	28·0
$\phi=24° 38'$	·94	·0340	1 57	·94	·032	41·92	12·5	54·4
	·91	·0568	3 15	·908	·052	58·97	20·9	79·9
	·88	·0848	4 51	·876	·074	73·8	31·2	105·0
	·85	·1197	6 51	·844	·101	86·56	44·1	130·7
	·82	·1648	9 26	·809	·134	97·16	60·7	157·9
	·79	·2259	12 57	·770	·177	105·59	83·2	188·8
	·784	·2523	14 27	·757	·195	...	...	200
	·76	·3282	18 48	·720	·245	111·19	120·9	232·1
	·75	·3980	21 39	·697	·277	112·21	146·7	258·9
	·745	·5032	28 50	·653	·359	112·44	185·4	297·8
	·745	·5092	29 10	·650	·363	...	...	300
	·75	·6085	34 52	·615	·429	...	...	336·7
	·76	·6783	38 32	·592	·477	...	...	363·5
	·786	·7645	43 48	·567	·544	...	...	400
	·79	·7805	44 43	·562	·556	...	...	406·8
	·82	·8417	48 13	·546	·612	...	...	437·7
	·85	·8868	50 49	·537	·659	...	...	464·9
	·88	·9217	52 49	·532	·701	...	...	490·6
	·891	·9322	53 25	·531	·716	...	...	500
	·91	·9497	54 25	·530	·740	...	...	515·7
	·94	·9725	55 43	·529	·776	...	...	541·2
	·97	·9911	56 47	·532	·812	...	...	567·6
	1·00	1·0064	57 40	·535	·845	...	...	595·6
P. VI	1·00	0·0000	0 0	1·000	0·000			
$p=364$	·97	·0151	0 52	·97	·015	22·41	5·5	27·9
$\phi=24° 18'$	·94	·0334	1 55	·94	·031	42·08	12·16	54·2
	·91	·0558	3 12	·908	·051	59·29	20·3	79·6
	·88	·0833	4 46	·877	·073	74·17	30·3	104·5
	·85	·1174	6 44	·844	·099	87·05	42·8	129·9
	·82	·1612	9 15	·809	·131	97·84	58·7	156·5
	·79	·2197	12 35	·771	·172	106·54	80·1	186·6
	·777	·2509	14 22	·753	·193	...	...	200
	·76	·3127	17 55	·724	·234	112·59	113·9	226·5

Appendix.

Table B—*continued.*

	r.	θ.	θ.	x.	y.	$\int \frac{dr}{r} \sqrt{\eta^2 - p^2}$.	$p\theta$.	Time.
P. VI—*continued.*	·74	0·4550	26° 5′	·665	·325	114·04	165·7	279·7
	·737	·5103	29 14	·643	·360	...	...	300
	·735	·5719	32 46	·618	·398	114·29	208·3	322·6
	·74	·6887	39 28	·571	·471	...	...	365·5
	·753	·7810	44 45	·535	·530	...	...	400
	·76	·8310	47 37	·512	·562	...	...	418·7
	·79	·9240	52 56	·476	·630	...	...	458·6
	·82	·9826	56 18	·455	·682	...	...	488·7
	·832	1·0012	57 22	·449	·701	...	...	500
	·85	1·0263	58 48	·440	·727	...	...	515·3
	·88	1·0604	60 45	·430	·768	...	...	540·7
	·91	1·0879	62 20	·422	·806	...	...	565·6
	·94	1·1103	63 37	·417	·842	...	...	591·0
	·95	1·1164	63 58	·417	·854	...	...	600
	·97	1·1286	64 40	·415	·877	...	...	617·3
	1·00	1·1437	65 32	·414	·910	...	...	645·2
P. VII	1·00	0·0000	0 0	1·000	0·000			
$p=360·4$	·97	·0149	0 51	·97	·014	22·48	5·37	27·9
$\phi=24° 3′$	·94	·0329	1 53	·937	·031	42·21	11·86	54·1
	·91	·0550	3 9	·908	·050	59·45	19·82	79·3
	·88	·0819	4 42	·877	·072	74·52	29·5	104·0
	·85	·1153	6 36	·844	·098	87·55	41·5	129·1
	·82	·1580	9 3	·810	·129	98·50	56·9	155·4
	·79	·2144	12 17	·772	·168	107·4	77·3	184·7
	·777	·2497	14 18	·753	·192	...	...	200
	·76	·3011	17 15	·726	·226	123·78	108·5	222·3
	·729	·5053	28 57	·638	·353	...	...	300
	·73	·5077	29 5	·638	·355	127·93	183	300·9
	·725	·6445	36 55	·580	·436	128·11	232·3	350·4
	·73	·7813	44 46	·518	·514	...	...	399·9
	·730	·7816	44 47	·518	·514	...	...	400
	·76	·9879	56 36	·418	·635	...	...	478·5
	·777	1·0375	59 27	·395	·669	...	...	500
	·79	1·0746	61 34	·376	·694	...	...	516·1
	·82	1·1310	64 48	·350	·742	...	...	545·4
	·85	1·1737	67 15	·329	·784	...	...	571·7
	·88	1·2071	69 10	·313	·822	...	...	596·8
	·883	1·2106	69 22	·311	·827	...	...	600
	·91	1·2340	70 42	·301	·859	...	...	621·5
	·94	1·2561	71 58	·291	·895	...	...	646·7
	·97	1·2741	73 0	·283	·927	...	...	672·9
	1·00	1·2889	73 51	·278	·961	...	...	700
	1·00	1·2890	73 51	·278	·961	...	...	700·8
P. VIII	1·00	0·0000	0 0	1·000	0·000			
$p=315·9$	·97	·0127	0 44	·97	·012	23·09	4·01	27·1
$\phi=20° 55′$	·94	·0280	1 36	·94	·026	43·56	8·85	52·4
	·91	·0464	2 40	·909	·042	61·68	14·7	76·4
	·88	·0684	3 55	·878	·060	77·83	21·6	99·4
	·85	·0949	5 26	·846	0805	92·19	30·0	122·2
	·82	·1273	7 18	·813	·104	104·80	40·2	145·0
	·79	·1672	9 35	·779	·131	115·84	52·8	168·6

Appendix.

Table B—*continued.*

	r.	θ.	θ.	x.	y.	$\int \frac{dr}{r} \sqrt{\eta^2 - p^2}$.	$p\theta$.	Time.
P. VIII—*continued.*	·76	0·2185	12° 31′	·742	·164	125·21	69·0	194·2
	·749	·2319	13 17	·729	·172	...	...	200
	·73	·2850	16 20	·701	·205	132·86	90·1	223·0
	·70	·3657	20 57	·654	·250	139·77	115·6	255·4
	·67	·4733	27 7	·596	·306	145·46	149·6	295·1
	·668	·4880	27 58	·590	·313	...	...	300
	·64	·6754	38 42	·499	·400	148·89	213·4	362·3
	·634	·7930	45 26	·445	·452	...	...	400
	·635	·7993	45 48	·443	·455	149·31	252·5	401·8
	·64	·9232	52 54	·386	·511	...	...	441·3
	·665	1·0997	63 0	·302	·592	...	...	500
	·67	1·1253	64 28	·289	·604	...	...	508·5
	·70	1·2329	70 38	·232	·660	...	...	548·2
	·73	1·3136	75 16	·185	·706	...	...	580·6
	·746	1·3469	77 10	·166	·727	...	...	600
	·76	1·3801	79 4	·144	·747	...	...	609·4
	·79	1·4314	82 1	·110	·782	...	...	635·0
	·82	1·4713	84 18	·081	·815	...	...	658·5
	·85	1·5037	86 9	·0571	·848	...	...	681·4
	·874	1·5253	87 24	·040	·873	...	...	700
	·88	1·5302	87 40	·035	·879	...	...	704·2
	·91	1·5522	88 56	·017	·910	...	...	727·2
	·94	1·5706	89 59	+ ·0003	·94	...	...	751·2
	·97	1·5859	90 52	− ·015	·97	...	...	776·5
	·997	1·5969	91 30	− ·026	·997	...	...	800
	1·00	1·5986	91 36	− ·028	1·00	...	...	803 6
P. IX	1·00	0·0000	0 0	1·000	0·000			
$p=273·5$	·97	·0108	0 37	·97	·0105	23·59	2·9	26·5
$\phi=18°\ 1'$	·94	·0236	1 21	·94	·022	44·64	6·44	51·1
	·91	·0389	2 14	·91	·036	63·48	10·62	74·1
	·88	·0570	3 16	·878	·050	80·48	15·56	96·0
	·85	·0785	4 30	·847	·067	95·84	21·43	117·3
	·82	·1040	5 58	·816	·085	109·67	28·4	138·1
	·79	·1345	7 42	·783	·106	122·17	36·7	158·9
	·76	·1715	9 50	·750	·130	133·29	46·8	180·1
	·730	·2113	12 6	·714	·153	...	...	200
	·73	·2159	12 22	·713	·156	143·3	59·0	202·3
	·70	·2662	15 15	·677	·184	152·88	72·7	225·6
	·67	·3248	18 37	·635	·214	161·83	88·7	250·5
	·64	·3956	22 40	·592	·247	169·97	108·0	278·0
	·617	·4567	26 10	·554	·272	...	...	300
	·61	·4847	27 46	·540	·284	177·07	133·0	310·1
	·58	·6072	34 47	·477	·331	182·8	166·1	348·9
	·552	·7835	44 53·5	·391	·390	...	...	400
	·55	·8360	47 54	·368	·408	186 21	229·0	415·2
	·545	·9812	56 13	·303	·453	186·71	268·0	454·7
	·55	1·1264	64 32	·236	·496	...	...	494·2
	·551	1·1464	65 41	·227	·502	...	...	500
	·58	1·3552	77 39	·124	·567	...	...	560·5
	·61	1·4777	84 40.	·057	·608	...	...	599·3
	·612	1·4796	84 46	·056	·609	...	...	600
	·64	1·5668	89 46	+ ·0026	·640	...	...	631·4

APPENDIX.

TABLE B—*continued.*

	r.	θ.	θ	x.	y.	$\int \frac{dr}{r}\sqrt{\eta^2-p^2}$.	$p\theta$.	Time.
P. IX—*continued.*	·67	1·6376	93° 50′	− ·045	·669	...	...	658·9
	·70	1·6962	97 11	− ·088	·695	...	...	683·8
	·721	1·7312	99 11	− ·115	·712	...	...	700
	·73	1·7465	100 4	− ·128	·719	...	...	707·1
	·76	1·7909	102 37	− ·166	·741	...	...	729·3
	·79	1·8279	104 44	− ·201	·764	...	...	750·5
	·82	1·8584	106 29	− ·233	·785	...	...	771·3
	·85	1·8839	107 56	− ·262	·808	...	...	792·1
	·861	1·8911	108 21	− ·271	·817	...	...	800
	·88	1·9054	109 10	− ·289	·831	...	...	813·4
	·91	1·9235	110 12	− ·314	·854	...	...	835·3
	·94	1·9388	111 5	− ·338	·877	...	...	858·3
	·97	1·9516	111 49	− ·361	·900	...	...	882·9
	·994	1·9586	112 13	− ·376	·920	...	...	900
	1·00	1·9624	112 26	− ·382	·924	...	...	909·4
P. X	1·00	0·0000	0 0	1·000	0·000			
$p=2·289$	·97	·0089	0 30	·97	·0087	24·02	2·03	25·1
$\phi=15°$	·94	·0193	1 6	·94	·018	45·60	4·43	50·0
	·91	·0317	1 49	·91	·029	65·05	7·26	72·3
	·88	·0462	2 39	·879	·040	82·79	10·57	93·4
	·85	·0632	3 37	·848	·054	99·0	14·46	113·5
	·82	·0831	4 46	·817	·068	113·85	19·02	132·9
	·79	·1064	6 6	·785	·084	127·55	24·69	152·2
	·76	·1337	7 39	·752	·101	140·09	30·61	170·7
	·73	·1655	9 29	·720	·120	151·77	37·87	189·6
	·709	·1843	10 34	·697	·130	...	...	200
	·70	·2007	11 30	·686	·139	163·22	45·93	209·1
	·67	·2402	13 46	·651	·159	174·33	54·98	229·3
	·64	·2854	16 21	·613	·180	185·0	65·3	250·3
	·61	·3378	19 21	·576	·202	195·08	77·3	272·4
	·58	·3992	22 52	·535	·226	204·57	91·37	295·9
	·577	·4110	23 33	·528	·230	...	...	300
	·55	·4730	27 6	·490	·251	213·33	108·2	321·5
	·52	·5639	32 19	·440	·278	221·24	129·0	350·2
	·49	·6861	39 18	·379	·311	227·91	157·0	384·9
	·478	·7468	42 47	·351	·325	...	...	400
	·46	·9093	52 6	·283	·363	232·28	208·1	440·4
	·455	1·0483	60 4	·227	·394	232·87	240·0	472·9
	·458	1·1641	66 42	·181	·421	...	...	500
	·46	1·1873	68 2	·172	·426	...	...	505·4
	·49	1·4106	80 50	·078	·484	...	...	560·9
	·52	1·5327	87 49	·020	·519	...	...	595·6
	·527	1·5466	88 37	+ ·011	·527	...	...	600
	·55	1·6236	93 2	− ·029	·550	...	...	624·3
	·58	1·6974	97 16	− ·073	·577	...	...	649·9
	·61	1·7589	100 47	− ·114	·599	...	...	673·4
	·64	1·8112	103 47	− ·152	·622	...	...	695·5
	·647	1·8209	104 20	− ·159	·627	...	...	700
	·67	1·8564	106 42	− ·189	·643	...	...	716·5
	·70	1·8960	108 38	− ·224	·664	...	...	736·7
	·73	1·9312	110 39	− ·258	·683	...	...	756·2
	·76	1·9630	112 29	− ·291	·702	...	...	775·1

APPENDIX.

TABLE B—*continued.*

	r.	θ.	θ.	x.	y.	$\int \frac{dr}{r} \sqrt{\eta^2 - p^2}$.	$p\theta$.	Time.
P. X—*continued.*	·79	1·9903	114° 2′	− ·322	·721	...	...	793·6
	·799	1·9980	114 19	− ·329	·728	...	...	800
	·82	2·0135	115 22	− ·351	·742	...	...	812·9
	·85	2·0335	116 31	− ·379	·761	...	...	832·3
	·88	2·0505	117 29	− ·406	·780	...	...	852·4
	·91	2·0650	118 19	− ·431	·801	...	...	873·5
	·94	2·0773	119 2	− ·456	·821	...	...	895·8
	·945	2·0791	119 7	− ·461	·825	...	...	900
	·97	2·0878	119 38	− ·479	·843	...	...	919·7
	1·00	2·0967	120 8	− ·505	·863	...	...	945·8
				SECONDARY WAVE.				
S. II	1·00	0·0000	0° 0′	1·000	0·000			
$p=1057{\cdot}4$	·97	·0322	1 51	·97	·031	30·67	34	64·7
$\phi=42°\ 13'$	·94	·0761	4 22	·937	·072	52·87	80·5	133·4
	·91	·1663	9 32	·897	·151	61·72	175·8	237·5
	·905	·2236	12 49	·882	·201	62·78	241·7	304·5
	·91	·2809	16 6	·874	·252	...	...	371·5
	·918	·3056	17 31	·875	·276	...	...	400
	·94	·3711	21 16	·877	·341	...	...	475·6
	·97	·4150	23 47	·888	·391	...	...	534·3
	·995	·4433	25 24	·899	·427	...	...	600
	1·00	·4472	25 37	·902	·432	...	...	609
S. III	1·00	0·0000	0 0	1·000	0·000			
$p=794$	·97	·0195	1 7	·97	·019	37·9	15·5	53·4
$\phi=29°\ 40'$	·94	·0440	2 31	·939	·041	70·04	34·9	104·9
	·91	·0747	4 17	·907	·068	97·3	59·3	156·6
	·88	·1151	6 36	·874	·101	119·55	91·4	211·0
	·85	·1726	9 53	·835	·161	136·33	137·0	273·3
	·82	·2833	16 14	·787	·229	143·0	224·9	367·9
	·817	·3231	18 31	·775	·259	...	...	400
	·815	·3584	20 32	·763	·286	143·8	284·6	428·4
	·82	·4335	24 50	·743	·345	...	...	488·9
	·85	·5442	31 11	·727	·441	...	...	583·5
	·858	·5592	32 3	·727	·455	...	...	600
	·88	·6017	34 28	·727	·498	...	...	645·8
	·91	·6421	36 47	·729	·545	...	...	700·2
	·94	·6728	38 33	·735	·587	...	...	751·9
	·969	·6957	39 52	·744	·621	...	...	800
	·97	·6973	39 57	·744	·623	...	...	803·4
	1·00	·7168	41 4	·754	·657	...	...	856·8
S. IV	1·00	0·0000	0 0	1·000	0·000			
$p=712{\cdot}8$	·97	·0169	0 58	·97	·016	39·41	12·02	51·4
$\phi=26°\ 23'$	·94	·0377	2 10	·94	·036	73·37	26·86	100·2
	·91	·0632	3 37	·908	·057	102·89	45·0	147·9
	·88	·0946	5 25	·877	·083	128·75	67·44	196·2
	·85	·1362	7 48	·842	·116	149·46	97·05	246·5
	·82	·1936	11 6	·803	·157	165·65	138·0	303·7

Appendix.

Table B—*continued.*

	r.	θ.	θ.	x.	y.	$\int \frac{dr}{r} \sqrt{\eta^2 - p^2}$.	pθ.	Time.
S. IV— *continued.*	·79	0·2844	16° 18′	·759	·222	176·81	202·7	379·5
	·785	·3118	17 52	·747	·241	...	...	400
	·775	·3753	21 30	·721	·285	179·84	267·4	447·2
	·77	·4725	27 5	·686	·351	180·20	336·7	516·9
	·775	·5698	32 39	·652	·418	...	...	586·6
	·777	·5879	33 41	·647	·431	...	...	600
	·79	·6607	37 51	·624	·485	...	...	654·3
	·82	·7515	43 3	·599	·568	...	...	730·2
	·85	·8088	46 21	·587	·616	...	...	787·3
	·86	·8193	46 57	·587	·628	...	...	800
	·88	·8504	48 44	·581	·662	...	...	837·6
	·91	·8819	50 32	·579	·703	...	...	885·9
	·94	·9073	51 59	·580	·740	...	...	933·6
	·97	·9282	53 11	·582	·776	...	...	982·4
	·98	·9340	53 31	·583	·788	...	...	1000
	1·00	·9450	54 9	·588	·811	...	...	1033·8
S. VII	1·00	0·0000	0 0	1·000	0·000			
$p=675{\cdot}3$	·97	·0157	0 54	·97	·015	39·98	10·6	50·6
$\phi=24° \ 54'$	·94	·0350	2 0	·939	·033	74·20	23·6	97·8
	·91	·0584	3 21	·908	·053	104·67	39·4	144·1
	·88	·0873	5 0	·876	·077	130·21	58·9	189·1
	·85	·1241	7 7	·843	·110	152·37	83·8	236·2
	·82	·1725	9 53	·808	·143	170·52	116·5	287·0
	·79	·2453	14 3	·766	·192	184·51	165·6	350·1
	·77	·3104	17 47	·733	·235	...	...	400
	·76	·3510	20 7	·714	·261	194·23	236·9	431·1
	·73	·5472	31 21	·623	·380	200·36	369·0	569·4
	·727	·5922	33 56	·603	·406	...	...	600
	·725	·6671	38 13	·570	·449	200·76	450·0	650·8
	·73	·7870	45 6	·515	·517	...	...	732·2
	·742	·8832	50 36	·471	·573	...	...	800
	·76	·7832	56 20	·421	·632	...	...	870·5
	·79	1·0889	62 23	·367	·700	...	...	951·5
	·813	1·1449	65 36	336	·740	...	...	1000
	·82	1·1617	66 34	·328	·752	...	...	1014·6
	·85	1·2101	69 20	·300	·796	...	...	1065·4
	·88	1·2469	71 27	·280	·833	...	...	1112·5
	·91	1·2758	73 6	·264	·870	...	...	1157·5
	·939	1·2937	74 20	·254	·904	...	...	1200
	·94	1·2992	74 26	·252	·907	...	...	1203·8
	·97	1·3185	75 33	·242	·939	...	...	1251·0
	1·00	1·3342	76 27	·234	·972	...	...	1301·6
S. VIII	1·00	0·0000	0 0	1·000	0·000			
$p=592$	·97	·0133	0 46	·97	·013	41·2	7·9	49·1
$\phi=21° \ 39'$	·94	·0295	1 41	·94	·028	77·39	17·5	94·9
	·91	·0488	2 48	·909	·0445	109·61	28·9	138·5
	·88	·0722	4 8	·876	·063	138·1	42·7	180·8
	·85	·1009	5 47	·846	·086	162·98	59·7	227·7
	·82	·1363	7 49	·812	·112	184·61	80·7	265·3
	·79	·1806	10 21	·777	·142	203·1	106·9	310·0
	·76	·2366	13 33	·740	·178	219·08	140·1	359·2

Appendix.

Table B—*continued.*

	r.	θ.	θ.	x.	y.	$\int\frac{dr}{r}\sqrt{\eta^2-p^2}$.	$p\theta$.	Time.
S. VIII—*continued.*	·736	0·2871	16° 27′	·706	·208	...	...	400
	·73	·3031	17 22	·696	·218	233·50	179·4	412·9
	·70	·3844	22 1	·649	·262	246·34	227·6	473·9
	·67	·4931	28 15	·590	·317	256·88	291·9	548·8
	·651	·5748	32 56	·546	·354	...	...	600
	·64	·6882	39 26	·494	·406	263·64	407·4	671·0
	·635	·8074	46 16	·439	·459	264·46	478·0	742·5
	·639	·9033	51 45	·396	·502	...	...	800
	·64	·9266	53 5	·385	·512	...	...	814·0
	·67	1·1217	64 16	·291	·604	...	...	936·2
	·696	1·2142	69 34	·243	·652	...	...	1000
	·70	1·2304	70 30	·234	·660	...	...	1011·1
	·73	1·3117	75 9	·187	·706	...	...	1072·1
	·76	1·3782	78 58	·147	·747	...	...	1125·8
	·79	1·4342	82 10	·107	·783	...	...	1175·0
	·807	1·4590	83 36	·090	·802	...	...	1200
	·82	1·4785	84 43	·075	·817	...	...	1219·7
	·85	1·5139	86 45	·048	·848	...	...	1262·3
	·88	1·5426	88 23	·024	·88	...	...	1304·2
	·91	1·5660	89 44	+ ·004	·910	...	...	1346·5
	·94	1·5853	90 50	– ·014	·94	...	...	1390·1
	·946	1·5888	91 2	– ·017	·946	...	...	1400
	·97	1·6015	91 46	– ·03	·97	...	...	1435·9
	1·00	1·6148	92 31	– 1·044	·999	...	...	1484
S. IX	1·00	0·0000	0 0	1·000	0·000			
$p=509$	·97	·0112	0 39	·97	·0108	42·20	5·7	47·9
$\phi=18°\ 30'$	·94	·0247	1 25	·94	·0232	79·61	12·6	92·2
	·91	·0401	2 18	·91	·0365	113·28	20·4	133·7
	·88	·0591	3 23	·878	·052	143·51	30·1	173·6
	·85	·0818	4 41	·847	·069	170·52	41·6	212·1
	·82	·1090	6 15	·816	·089	194·72	55·5	250·2
	·79	·1416	8 7	·782	·111	216·41	72·1	288·5
	·76	·1802	10 19	·749	·136	236·17	91·7	327·9
	·73	·2239	12 50	·712	·162	255·07	114·0	369·1
	·706	·2594	14 52	·682	·181	...	...	400
	·70	·2737	15 41	·674	·189	273·10	139·3	412·4
	·67	·3316	19 0	·634	·218	290·01	169	459·0
	·64	·4009	22 58	·590	·250	305·46	204	509·0
	·61	·4877	27 57	·539	·286	319·03	248	567·0
	·588	·5471	31 21	·502	·306	...	...	600
	·58	·6055	34 42	·477	·330	330·13	308	638·1
	·55	·8250	47 16	·373	·404	337·20	420	757·2
	·545	·9088	52 4	·335	·430	...	...	800
	·545	·9629	55 10	·311	·447	337·59	490	827·6
	·55	1·1008	63 4	·249	·491	...	...	898
	·574	1·2889	73 51	·160	·551	...	...	1000
	·58	1·3203	75 39	·144	·561	...	...	1017·1
	·61	1·4381	82 24	·0805	·604	...	...	1088·2
	·64	1·5249	87 22	+ ·029	·639	...	...	1146·2
	·67	1·5942	91 20	– ·016	·670	...	...	1196·2
	·672	1·5989	91 37	– ·022	·672	...	...	1200
	·70	1·6521	94 39	– ·057	·698	...	...	1242·8

APPENDIX.

TABLE B—*continued.*

	r.	θ.	θ	x	y	$\int \frac{dr}{r}\sqrt{\eta^2-p^2}$.	$p\theta$.	Time.
S. IX—*continued.*	·73	1·7019	97° 31′	− ·096	·723	...	...	1286·1
	·76	1·7456	100 1	− ·132	·750	...	...	1327·3
	·79	1·7842	102 14	− ·167	·772	...	...	1366·7
	·817	1·8125	103 51	− ·195	·791	...	...	1400
	·82	1·8168	104 6	− ·200	·794	...	...	1405·0
	·85	1·8440	105 39	− ·229	·818	...	...	1443·1
	·88	1·8667	106 57	− ·256	·841	...	...	1481·6
	·91	1·8851	108 3	− ·282	·867	...	...	1521·5
	·94	1·9011	108 55	− ·305	·890	...	...	1563·0
	·966	1·9127	109 35	− ·324	·910	...	...	1600
	·97	1·9146	109 42	− ·327	·913	...	...	1607·3
	1·00	1·9258	110 20	− ·347	·938	...	...	1655·2
S. X	1·00	0·0000	0 0	1·000	0·000			
$p=427·8$	·97	·0092	0 32	·97	·009	43·03	3·95	47
$\phi=15° 28′$	·94	·0203	1 10	·94	·019	81·44	8·67	90·1
	·91	·0332	1 54	·91	·030	116·30	14·2	130·5
	·88	·0484	2 46	·879	·042	147·91	20·71	168·6
	·85	·0664	3 48	·848	·056	176·54	28·41	205·0
	·82	·0876	5 1	·817	·072	202·68	37·46	238·1
	·79	·1124	6 26	·785	·089	226·69	48·07	274·8
	·76	·1409	8 4	·752	·107	249·16	64·57	313·7
	·73	·1726	9 53	·719	·126	271·08	73·82	344·9
	·70	·2078	11 54	·685	·144	292·51	88·9	381·4
	·684	·2273	13 1	·666	·154	...	...	400
	·67	·2473	14 10	·650	·164	313·29	105·8	419·1
	·64	·2923	16 45	·612	·184	333·24	125·0	458·2
	·61	·3444	19 44	·574	·206	352·20	147·3	499·5
	·58	·4055	23 14	·532	·229	370·01	173·5	543·5
	·55	·4838	27 43	·487	·256	386·34	206·9	593·2
	·545	·4954	28 23	·479	·259	...	...	600
	·52	·5772	33 4	·436	·284	400·77	246·9	647·7
	·49	·7083	40 35	·372	·319	411·65	303	714·7
	·463	·8935	51 12	·290	·361	...	...	800
	·46	·9497	54 23	·268	·374	419·17	406·4	825·6
	·455	1·0972	62 52	·207	·405	420·20	469·3	889·5
	·46	1·2453	71 21	·147	·435	...	...	953·4
	·468	1·3465	77 9	·104	·457	...	...	1000
	·49	1·4861	85 9	+ ·0415	·488	...	...	1064·3
	·52	1·6172	92 40	− ·024	·519	...	...	1131·3
	·55	1·7106	98 1	− ·077	·545	...	...	1185·8
	·558	1·7336	99 20	− ·091	·551	...	...	1200
	·58	1·7911	102 30	− ·127	·567	...	...	1235·5
	·61	1·8500	106 0	− ·168	·586	...	...	1279·5
	·64	1·9021	108 59	− ·208	·605	...	...	1320·8
	·67	1·9471	111 34	− ·247	·623	...	...	1359·9
	·70	1·9866	113 50	− ·283	·640	...	...	1397·6
	·706	1·9999	114 35	− ·294	·642	...	...	1400
	·73	2·0218	115 51	− ·318	·657	...	...	1424·1
	·76	2·0535	117 40	− ·353	·673	...	...	1465·3
	·79	2·0821	119 18	− ·386	·689	...	...	1504·3
	·82	2·1068	120 43	− ·420	·707	...	...	1540·9
	·85	2·1280	121 56	− ·450	·722	...	...	1574

Appendix.

Table B—*continued.*

	r.	θ.	θ.	x.	y.	$\int \frac{dr}{r} \sqrt{\eta^2 - p^2}$.	$p\theta$.	Time.
S. X—*continued.*	·87	2·1409	122° 36′	− ·469	·733	...	...	1600
	·88	2·1460	122 58	− ·480	·738	...	...	1610·4
	·91	2·1612	123 50	− ·507	·756	...	...	1648·5
	·94	2·1742	124 34	− ·532	·775	...	...	1688·9
	·97	2·1852	125 12	− ·559	·792	...	...	1774·3
	1·00	2·1944	125 44	− ·584	·812	...	...	1779

Table C.—Isochronic Lines or Wave-Fronts.

Primary Wave.

200 sec.		300 sec.		400 sec.		500 sec.		600 sec.		700 sec.		800 sec.		900 sec.	
x.	y.	x.	y.	x.	y.	x.	y.	x.	y.	x.	y.	x.	y.	x.	y.
·972	·234														
·923	·245	·929	·371												
·876	·243	·879	·384	·852	·523										
·793	·219	·734	·386	·732	·550	·728	·685								
·766	·202	·679	·375	·648	·556	·662	·705								
·757	·195	·650	·363	·567	·544	·531	·716	·527	·850						
·753	·193	·643	·360	·535	·530	·449	·701	·417	·854	·281	·960				
·753	·192	·638	·353	·518	·514	·395	·669	·311	·827	·278	·961	− ·014	·999		
·729	·172	·590	·313	·445	·452	·302	·592	·166	·727	·040	·873	− ·026	·997	− ·368	·930
·714	·153	·554	·272	·391	·390	·227	·502	·056	·609	− ·115	·712	− ·271	·817	− ·376	·920
·697	·130	·528	·230	·351	·325	·181	·421	·011	·474	− ·159	·627	− ·329	·728	− ·461	·825
·681	0	·490	0	·209	0	·094	0								

Secondary Wave.

400 sec.		600 sec.		800 sec.		1000 sec.		1200 sec.		1400 sec.		1600 sec.			
x.	y.	x.	y.	x.	y.	x.	y.	x.	y.	x.	y.	x.	y.		
·960	·281	·912	·410												
·875	·276	·899	·427	·791	·612										
·775	·259	·727	·455	·744	·621	·628	·778								
·747	·241	·647	·431	·587	·628	·583	·788	·377	·926						
·733	·235	·603	·406	·471	·573	·336	·740	·254	·904	·086	·996				
·706	·208	·546	·354	·396	·502	·243	·652	·090	·802	− ·017	·946	− ·248	·969		
·682	·181	·502	·306	·335	·430	·160	·551	− ·022	·672	− ·195	·791	− ·324	·910		
·666	·154	·479	·259	·290	·361	·104	·457	− ·091	·551	− ·294	·642	− ·469	·733		
·627	0	·423	0	·209	0	− ·005	0								

(*Issued separately October* 21, 1919.)

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XV.—An Analysis of an Electron-Transference Hypothesis of Chemical Valency and Combination.

By **John Marshall**, M.A., B.Sc., Scholar of Trinity College, Cambridge; Lecturer in Mathematics, University College, Nottingham. *Communicated by* Professor W. Peddie, D.Sc.

(MS. received March 6, 1919. Read May 5, 1919.)

Introduction.

In this paper it is proposed to analyse in a tentative way the Electron-Transference Hypothesis of Chemical Combination which has been put forward by Lord Kelvin in his paper entitled "Aepinus Atomised" (*Phil. Mag.*, 1902), and by Sir J. J. Thomson (*Phil. Mag.*, 1904).

This hypothesis postulates that on the combination of two or more atoms to form a chemical compound electrons are transferred from atoms playing an electro-positive part to those playing an electro-negative part in the molecule, the number of electrons thus transferred being taken as a measure of the valencies of the respective atoms in the molecule. Such an hypothesis would not lead to conceptions at variance with the facts of electrolytic dissociation.

The methods employed and the assumptions made are similar to those formulated by H. A. Lorentz in his discussion of the Molecular Refractive Index of Mixtures and Compounds (vide *Theory of Electrons*, § 117–130), and by Sir J. J. Thomson in his discussion of the Molecular Refractivity of a collection of atoms (vide *Phil. Mag.*, 1906).

H. A. Lorentz's discussion ignores the contribution given by the positive electrification of the atom to its atomic refractive index except in so far as the positive electrification may be the origin of the force which tends to restore a displaced electron to its position of equilibrium when it is disturbed by the periodic changes in the electro-magnetic forces in the æther when light is incident on the atom.

In Sir J. J. Thomson's paper in *Phil. Mag.*, 1906, the contribution of the positive electrification is not ignored, and the well-known structure of the atom as a sphere of positive electrification in which negative electrons are situated is assumed.

It may be noted at once that the additive law for atomic refractive indices will follow from any structure assumed for the atom, so long as

the actions going on in the separate atoms of a molecule or mixture are to a large extent mutually independent. For if they were not, the refractive index of a compound would be determined principally by the interaction between the atoms, and not, as it is, by their individual properties.

The assumptions made throughout this paper are at best rough approximations to the true state of things; but as they have enabled H. A. Lorentz to deduce the additive law for atomic refractive indices, and Sir J. J. Thomson to give an indication of the discrepancies which might be expected to occur in this law, a further analysis along similar lines may not be considered undesirable.

In Part I of this paper we shall discuss the value of the atomic refractive index in the case of atoms from which electrons have been transferred, ignoring the contribution to this value arising from fields of electrical force due to the vicinity of other atoms or groups of atoms.

In Part II we shall endeavour to obtain a formula for the molecular refractive index of a diatomic molecule, which will allow for the contribution due to electrical action between the atoms of the molecule.

Part I.

(a) *Atomic Refractive Index of a Collection of Atoms each of which is electro-positive to the extent s.*

In this discussion we consider each atom to have lost s electrons by transference to other atoms with which it has become combined.

Each atom will therefore be positively charged to the extent se where e is the charge carried by each negative electron.

We shall neglect the action of the fields of force set up by this chemical combination or by the vicinity of electrical fields due to hydroxyl groups and groups containing multiple bonds.

Let $\rho_r \equiv (\xi_r \eta \zeta_r) \equiv$ Displacement of r^{th} electron from its position of equilibrium at time t, when an electro-magnetic field acts upon it.

$r \equiv (x, y, z) \equiv$ Displacement of centre of sphere of positive electrification.

$F \equiv (X, Y, Z) \equiv$ Force due to incident electro-magnetic field.

$F' \equiv (X', Y', Z') \equiv$ Electric force arising from the polarisation which is due to displacement of electrons.

Hence, assuming that the atom consists of a sphere of positive electrifi-

cation throughout which are distributed negative electrons in positions of equilibrium, we may quote the well-known result:

$$\text{1·01} \qquad X' = \frac{4}{3}\pi N\left(Ex - \sum_{1}^{\mu} e\xi_r\right),$$

with similar equations for Y' and Z'

where N = number of atoms in 1 c.c. of substance.

n = number of electrons in the atom when it is neutral.

$\mu = n - s$.

$E = ne$ = magnitude of positive electrification.

The equations of motion for the positive electrification and the negative electrons we take to be

$$\text{1·02} \qquad M\ddot{x} = (X + X')E - \frac{4}{3}\pi\rho e\sum_{1}^{\mu}(x - \xi_r),$$

$$\text{1·03} \qquad m\sum_{1}^{\mu}\ddot{\xi} = -(X + X')\sum_{1}^{\mu} e + \frac{4}{3}\pi\rho e\sum_{1}^{\mu}(x - \xi_r),$$

where M = mass of positive electrification.

m = mass of a negative electron.

Equations 1·02 and 1·03 implicitly determine the type of action which we are assuming to take place within the atom.

Sir J. J. Thomson and H. A. Lorentz have assumed that the elastic force tending to restore a displaced electron to its position of equilibrium is proportional to its relative displacement.

This constant of proportionality is $\frac{4}{3}\pi\rho e$ in the case of a sphere of uniform positive electrification, and is used in Sir J. J. Thomson's paper in *Phil. Mag.*, 1906, to establish the refractive index of a collection of atoms.

Let

$$t = \frac{4}{3}\pi\rho e$$

and

$$\mathsf{X} = X + X'.$$

If the frequency of the electro-magnetic waves incident on the atoms is p, equation 1·02 and 1·03 become

$$\text{1·04} \qquad -Mp^2x = \mathsf{X}E - t\sum_{1}^{\mu}(x - \xi_r),$$

$$\text{1·05} \qquad -mp^2\sum_{1}^{\mu}\xi_r = -\mathsf{X}\mu e + t\sum_{1}^{\mu}(x - \xi_r).$$

On addition of 1·04 and 1·05 we obtain

1·06 . . . . . $$Mx + m\sum_1^{\mu}\xi_r = -Xse/p^2,$$

which with 1·05 gives

1·07 . . . . . $$x = \frac{m\left[E - \dfrac{tse}{mp^2}\right]X}{t(M+\mu m) - Mmp^2}, \quad \text{and}$$

1·08 . . . $$\sum_1^{\mu}\xi_r = -\left[\frac{M\left(E - \dfrac{tse}{mp^2}\right)}{t(M+\mu m) - Mmp^2} + \frac{se}{mp^2}\right]$$

1·09 . . Let $E_s = E - \dfrac{tse}{mp^2}$, and

1·10 . . . . $$D_s = t(M+mn) - Mmp^2 - tsm$$
$$= D_0 - tsm.$$

1·11 . . . . ∴ $$x = m\,.\,\frac{E_s}{D_s}\,.\,X, \quad \text{and}$$

1·12 . . . $$\sum\xi_r = -\left[M\frac{E_s}{D_s} + \frac{se}{mp^2}\right]X.$$

Substituting these values in equation 1·01, we obtain

1·13 . . . . $$X' = \frac{4}{3}\pi NX\left[\frac{E_s}{D_s}(mE + Me) + \frac{se^2}{mp^2}\right]$$

1·14 . . Let $X' = P_sX$,

$$\text{i.e. } \frac{X'}{P_s} = \frac{X}{1} = \frac{X}{1-P_s},$$

1·15 . . where $$P_s = \frac{4}{3}\pi N\left[\frac{E_s}{D_s}(mE + Me) + \frac{se^2}{mp^2}\right].$$

(*b*) Now, in consequence of the motion of these charges, the electrical current is no longer $\dfrac{K}{4\pi}\cdot\dfrac{dF}{dt}$ where K = specific inductive capacity of the æther, *i.e.* K = 1, but the following vector equation for the current C gives the required modification:

1·16 . . . . . $$C = \frac{1}{4\pi}\cdot\frac{dF}{dt} + N\left(E\dot{r} - \sum_1^{\mu}e\dot{\rho}_r\right)$$
$$= \frac{1}{4\pi}\cdot\frac{d}{dt}(F + 3F')$$
$$= \frac{1}{4\pi}\,\frac{1+2P_s}{1-P_s}\cdot\frac{dF}{dt}, \text{ from 1·14.}$$

Equations of motion in free æther are

1·17 . . . . . . $$4\pi C = \text{curl } H,$$

1·18 . . . . . . $$-\dot{H} = \text{curl } F,$$

where H is the magnetic vector.

Eliminating C and H from equations 1·16, 1·17, 1·18, we have

1·19 . . . . $$\frac{1+2P_s}{1-P_s}\cdot\frac{d^2F}{dt^2}=\text{curl }\dot{H}$$
$$=-\text{curl (curl F)}$$
$$=\Delta^2F-\text{grad (div F)}$$
$$=\Delta^2F$$

since div F = 0 in free æther.

If, however, μ is the refractive index of the medium, the equation of propagation is that of Maxwell, viz.

1·20 . . . . . . $$\mu^2\ddot{F}=\Delta^2F.$$

1·21 . . . . . . $$\mu^2=\frac{1+2P_s}{1-P_s},\text{ giving}$$

1·22 . . . . . . $$P_s=\frac{\mu^2-1}{\mu^2+2}.$$

[The work given in this section (*b*) so far is a vectorial modification of that given in Sir J. J. Thomson's paper in *Phil. Mag.*, 1906.]

1·23 . . $$\text{Specific refractive index}=\frac{\mu^2-1}{\mu^2+2}\cdot\frac{1}{D}=R_s,$$

where D is the density of medium considered.

If N_0 = number of atoms in 1 grm.-molecule of substance,

$\frac{1}{N_0}$ = mass in grms. of 1 molecule.

1·24 . . . $$\therefore\ \frac{N}{N_0}=D=\text{mass of 1 c.c. of substance.}$$

1·25 . . . $$R_s=\frac{4}{3}\pi N_0\left\{\frac{E_s}{D_s}(mE+Me)+\frac{se^2}{mp^2}\right\}$$

A_s = atomic refractive index.

1·26 . . . $$A_s=\frac{4}{3}\pi WN_0\left\{\frac{E_s}{D_s}(mE+Me)+\frac{se^2}{mp^2}\right\}$$
$$=WR_s\text{ where W is atomic weight of substance.}$$

When $s=0$, we obtain the case considered by Sir J. J. Thomson in his paper in *Phil. Mag.*, 1906, viz.

1·27 . . . . $$R_0=\frac{4}{3}\pi N_0\left\{\frac{E(mE+Me)}{\frac{4}{3}\pi\rho(Me+mE)-Mmp^2}\right\}.$$

(*c*) Before we can proceed further, we must consider the magnitude

of the quantities used. We shall adopt electro-magnetic units. We have, therefore,

$$e = 1{\cdot}6 \times 10^{-20}.$$

$$\frac{e}{m} = 1{\cdot}8 \times 10^{-7}.$$

$$\therefore \quad m = 8{\cdot}9 \times 10^{-28}.$$

$$c = 3 \times 10^{10} = \text{velocity of light.}$$

Take

$$\lambda = 6 \times 10^{-5}$$

where λ is the wave-length of incident light.

$$p = \frac{2\pi c}{\lambda} = \frac{2\pi}{\lambda} \text{ in our units,}$$

since c is taken as the unit of velocity.

$$\therefore \quad p = 10^5 \text{ approximately.}$$

The value of $\frac{e}{M}$ in the case of hydrogen is known by electrolytic determination to be

$$9{\cdot}6 \times 10^3,$$

and e being identified with the charge on an electron,

$$M_h = M \text{ for hydrogen} = 1{\cdot}7 \times 10^{-24}.$$

(These values are taken from O. W. Richardson's *Electron Theory of Matter*, chapter i.)

The radius of the hydrogen atom lies between $\cdot 92 \times 10^{-8}$ and $1{\cdot}19 \times 10^{-8}$. (Jeans, *Kinetic Theory of Gases*, p. 347.)

Hence the order of magnitude of

$$t = \frac{4}{3}\pi\rho e = \frac{Ee}{a^3} = n2{\cdot}6 \,.\, 10^{-16},$$

where a, the radius of the atom, is taken as 10^{-8}.

We shall make the assumption, which is in accordance with a large body of experimental evidence, that the mass of an atom is proportional to the number of electrons in it, and hence when neutral to the positive electrification value.

$$\therefore \quad \frac{M}{M_h} = \frac{n}{h},$$

where h = number of electrons in the hydrogen atom.

$$\therefore \quad M = \frac{n}{h} \,.\, 1{\cdot}7 \,.\, 10^{-24}.$$

(d) Adopting the values for the quantities given in (c), we proceed to discuss the relative order of magnitude of the quantities used in equation 1·26.

1·28 . . . . . Let
$$\begin{cases} T = tM = \frac{4}{3}\pi\rho Me \\ a = tmn = \frac{4}{3}\pi pmE \\ b = tm = \frac{4}{3}\pi pme \\ c = Mmp^2 \\ d = \frac{te}{mp^2}. \end{cases}$$

We note at once that

1·29 . . . . . .
$$\begin{cases} cd = eT \text{ and} \\ ea = bE. \end{cases}$$

Also, let

1·30 . . . . . .
$$R = T - c.$$

We have, therefore,

1·31 . . . . .
$$\begin{cases} \frac{a}{T} = \frac{mn}{M} = 5{\cdot}3\,.\,h\,.\,10^{-4} \\ \frac{b}{T} = \frac{m}{M} = 5{\cdot}3\,.\,\frac{h}{n}\,.\,10^{-4} \\ \frac{c}{T} = \frac{mp^2}{t} = \frac{3{\cdot}4}{n}\,.\,10^{-2}. \end{cases}$$

1·32 . . . . Also $d = eT/c = 4{\cdot}7\,.\,n\,.\,10^{-17}$.

(e) The values obtained in sections (c) and (d) for the quantities used are now to be employed to expand our value for A_s.

$$A_s = \frac{4}{3}\pi WN_0\,.\left[\frac{E_s}{D_s}\,.\,(mE + Me) + \frac{se^2}{mp^2}\right]$$

$$= \frac{WN_0}{\rho}\,.\left[\frac{(E - sd)(T + a)}{T + a - c - sb} + sd\right]$$

$$= \frac{WN_0}{\rho}\,.\,\frac{T}{R}\left[(E - sd)\left(1 + \frac{a}{T} - \frac{a}{R} + \frac{sb}{R}\right) + \frac{sdR}{T}\right]$$

to the 1st order in a/T and b/T

$$= \frac{WN_0}{\rho}\,.\,\frac{T}{R}\left[E - sd\left(1 - \frac{R}{T}\right) - (sd - E)\frac{sbT - ac}{RT}\right].$$

1·33 . . .
$$A_s = \frac{WN_0}{\rho}\,.\,\frac{T}{R}\left[E - se - (sd - E)\frac{sbT - ac}{RT}\right]$$

to the 1st order in a/T and b/T.

$$sd > \mathrm{E} \quad \text{if} \quad s > n\,.\,\frac{c}{\mathrm{T}},\ \textit{i.e.}\ \text{if} \quad s > 3{\cdot}4\,.\,10^{-2}$$

and

$$sb\mathrm{T} > ac \quad \text{if} \quad s > n\frac{c}{\mathrm{T}}, \qquad \textit{i.e.} \qquad s > 3{\cdot}4\,.\,10^{-2},$$

which is so since s is a positive integer. Hence the quantity in the square bracket is positive.

Symmetry of the investigation shows that in order to obtain the value for the atomic refractive index when the atoms are electro-negative to the extent s, we need only put $-s$ for s in expression 1·33.

1·34 . $$\therefore \quad \mathrm{A}_{-s} = \frac{\mathrm{WN}_0}{\rho}\,.\,\frac{\mathrm{T}}{\mathrm{R}}\,.\left[\mathrm{E} + se - (sd + \mathrm{E})\frac{sb\mathrm{T} + ac}{\mathrm{RT}}\right].$$

Particular Cases.

(i) $s=0$. Neutral atom which has not lost any electrons by transference.

1·35 . . . . . $$\mathrm{A}_0 = \frac{\mathrm{WN}_0}{\rho}\,.\,\frac{\mathrm{T}}{\mathrm{R}}\mathrm{E}\left(1 - \frac{ac}{\mathrm{RT}}\right).$$

(ii) $s=1$. Monovalent electro-positive atom.

1·36 . . $$\mathrm{A}_{+1} = \frac{\mathrm{WN}_0}{\rho}\,.\,\frac{\mathrm{T}}{\mathrm{R}}\left[\mathrm{E} - e - (d - \mathrm{E})\frac{b\mathrm{T} - ac}{\mathrm{RT}}\right].$$

(iii) $s=-1$. Monovalent electro-negative atom.

1·37 . . . . $$\mathrm{A}_{-1} = \frac{\mathrm{WN}}{\rho}\,.\,\frac{\mathrm{T}}{\mathrm{R}}\left[\mathrm{E} + e - (d + \mathrm{E})\frac{b\mathrm{T} + ac}{\mathrm{RT}}\right].$$

At this stage it might be advisable to compare our formula with experimental values.

$$\mathrm{A}_0 = \frac{\mathrm{WN}_0}{\rho}\,.\,\frac{\mathrm{T}}{\mathrm{R}}\,.\,\mathrm{E} \ \text{approx.}$$

For hydrogen

$$\mathrm{W} = 1$$

$$\mathrm{N}_0 = 6 \times 10^{23}$$

$$\frac{\mathrm{T}}{\mathrm{R}} = 1 \ \text{approx.}$$

$$\therefore \quad \mathrm{A}_0 = 6\,.\,10^{23}\,.\,\frac{4}{3}\pi a^3. \quad a = 10^{-8}$$

$$= 2{\cdot}4,$$

which is of the *same order of magnitude* as the experimental values given in Brühl's paper in *Zeitschrift für physik. Chemie*, 1891, p. 25, and in the usual text-books on physical chemistry, *e.g.* Smiles' book on the *Relation of Chemical Constitution to some Physical Properties.*

(f) The Contribution to the Atomic Refractive Index of the Positive Electrification.

In H. A. Lorentz's work the contribution to the molecular refractive index due to the vibration of the positive electrification is ignored. It is interesting to determine the value of this contribution.

When we ignore the vibration of the positive electrification our equations become

1·38 $$X' = -\frac{4}{3}\pi N \sum_1^{\mu} e\xi_r.$$

1·39 $$m\sum_1^{\mu} \ddot{\xi}_r = -\mathsf{X}\mu e - t\sum^{\mu} \xi_r.$$

1·40 $$(t - mp^2)\sum_1^{\mu} \xi_r = -\mathsf{X}\mu e.$$

1·41 $$\therefore\ X' = \frac{4}{3}\pi N \mathsf{X}\mu e^2/(t - mp^2).$$

1·42 $$A_s^{(l)} = \frac{4}{3}\pi N_0 W e \cdot \frac{E - se}{t - mp^2}$$
$$= \frac{WN_0}{\rho}\frac{T(E - se)}{T - c}$$

where $A_s^{(l)}$ = value of A_s when the vibration of the sphere of positive electrification is neglected.

$$A_s^{(l)} = \frac{WN_0}{\rho R} \cdot T(E - se).$$

1·43 Hence $$\frac{A_s - A_s^{(l)}}{A_s} = \frac{-(sd - E)\dfrac{sbT - ac}{RT}}{E - se - (sd - E)\dfrac{sbT - ac}{RT}}$$

$sbT > ac$, and so approximately.

1·44 $$\frac{A_s - A_s^{(l)}}{A_s} = -\frac{s^2bd}{RE} = -\frac{s^2h}{n} \cdot 10^{-2}$$

if $s \neq 0$,

and $$= -\frac{ac}{RT} \doteqdot -1{\cdot}8 \cdot \frac{h}{n} \cdot 10^{-5}$$

if $s = 0$.

Equation 1·44 shows that

1·45 $$A_s^{(l)} > A_s,$$

and hence the contribution of the vibration of the positive electrification is negative. It is also evident that when $s \neq 0$ this contribution is small, being of order $-\frac{s^2h}{n} \cdot 10^{-2}$, and is of the order $-\frac{1 \cdot 8h}{n} \cdot 10^{-5}$ when $s = 0$.

(*g*) In this section it is proposed to determine if there will be an appreciable difference in the molecular refractivity of a molecule containing two atoms of the same kind according as we calculate the molecular refractivity.

(1) on the assumption that one atom is electro-positive to extent 1, *i.e.* A_{+1} is taken, and the other electro-negative to the extent 1, viz. A_{-1} is taken;

(2) on the assumption that there is no transference of electrons, *i.e.* $2A_0$ will be the value for the molecular refractivity.

We avoid difficulties arising from multiple bonds if we only deal with univalent atoms. It is easy to show from 1·36 and 1·37 that

1·46 $$2A_0 - (A_{+1} + A_{-1}) = 2\frac{N_0 T}{\rho R} \cdot \frac{db}{R}$$
$$= 2\frac{N_0 T^2}{\rho R^2} \cdot \frac{be}{c}, \text{ since } cd = et.$$

1·47 $$\therefore \quad \frac{2A_0 - (A_{+1} + A_{-1})}{2A_0} = \frac{T}{R} \cdot \frac{1}{n} \cdot \frac{b}{c}$$
$$= 1{\cdot}5 \cdot \frac{h}{n} \cdot 10^{-2}.$$

It has been objected to the electron-transference hypothesis that the molecular refractivity of substance *might* differ considerably according as the calculation was based on assumption (1) or on assumption (2) (*vide* Richardson's *Electron Theory of Matter*, p. 575). According to our analysis, no considerable difference is obtainable.

(*h*) *Values of the Atomic Refractivity in the case of the Halogen Elements.*

Different values have been obtained for the atomic refractivities of chlorine and bromine according as (1) gaseous halogen is considered, (2) halogen occurring in organic compounds is considered.

Walden has shown that in certain solvents Br_2 and I_2 can be electrolysed to give liberation of equal quantities of bromine and iodine at each electrode. This fact is not at variance with what would be expected from the electron-transference hypothesis.

In gaseous halogen, therefore, we assume that there is an electro-positive atom and an electro-negative atom, *i.e.* molecular refractivity of gaseous halogen

$$= A_{+1} + A_{-1}.$$

1·48 $$= 2 \cdot \frac{N_0 W}{\rho} \cdot \frac{T}{R} E \text{ approx., from equations 1·36 and 1·37.}$$

On the other hand, we assume that when a halogen atom is combined with a carbon atom in organic compounds of the type $C_nH_{2n+1}Cl_1$ it is electro-negative, and hence its atomic refractivity is given by A_{-1}.

From equations 1·36 and 1·37 it follows that

$$\begin{aligned} 2A_{-1} - (A_{+1} + A_{-1}) &= A_{-1} - A_{+1} \\ &= \frac{N_0W}{\rho} \cdot \frac{T}{R}\left[2e - \frac{2Eb}{R} - \frac{2dea}{TR}\right] \\ &= 2\frac{N_0W}{\rho} \cdot \frac{T}{R}e\left[1 - \frac{2a}{R}\right] \end{aligned}$$

since $dc = eT$ and $nb = a$.

Hence, if we neglect quantities of relative order 10^{-3}, we have

1·49 . . . . . $A_{-1} - A_{+1} = 2N_0W \cdot \frac{T}{\rho R}e$ and

1·50 . . . . . $\frac{A_{-1} - A_{+1}}{A_{-1} + A_{+1}} = \frac{e}{E} = \frac{1}{n}$.

Equations 1·49 and 1·50 show

(*a*) Atomic refractivity of 2Cl or 2Br obtained from organic compounds > atomic refractivity of Cl_2 or Br_2 obtained from gaseous molecule Cl_2 or Br_2.

(*b*) Percentage difference will be of the order $\frac{1}{n} \times 100$ per cent. = 3 per cent. approx. if we take $n = 35$, which is the order of the atomic weight of chlorine. This is in accord with experimental evidence, which indicates that n is proportional to the atomic weight and is usually less than the atomic weight.

The following experimental values for halogen elements are given in Brühl's paper in *Zeitschrift für physik. Chemie*, vol. vii, 1891, p. 25 and p. 179:—

Value of molecular refractive index of gaseous chlorine = 11·54, *i.e.* Cl_2.

Value of molecular refractive index of gaseous bromine = 16·91, *i.e.* Br_2.

Value of twice atomic refractive index of chlorine obtained from organic compounds = 12·028.

Value of twice atomic refractive index of bromine obtained from organic compounds = 17·726.

In each case, therefore, we have a verification of the inequality (*a*).

Also the percentage difference obtainable from equation 1·50 is

$\frac{12·028 - 11·54}{12·028} \times 100 = 4$ per cent. approx. in case of chlorine, and

$\frac{17·73 - 16·91}{17·73} \times 100 = 4·6$ per cent. approx. in case of bromine.

These results are in agreement, so far as order of magnitude is concerned, with the result (b).

These results are derived on the assumption that the additive law, without correction for contributions due to atomic interaction, can be applied. This is so in the case of organic compounds which contain a carbon chain and have no double bindings in their constitution, and also in the case of inorganic compounds which do not display great chemical activity when acted upon by electrical fields of force. Gaseous halogen is quite stable when acted upon by electrical fields of force, and so we have good reason to assume that the additive law for atomic refractivities holds in this case very closely. The values of the atomic refractivity of chlorine and bromine deduced from organic compounds are used as a basis for the determination of the chemical constitution of organic compounds, and the results obtained are very satisfactory.

(i) It is now proposed to discuss the difference in value between the atomic refractivity of an element according as it plays an electro-positive or electro-negative part in a molecule.

When the atom plays an electro-positive part its atomic refractivity is A_{+s}, and when it plays an electro-negative part its atomic refractivity is indicated by A_{-k}.

We have the relation at once that

$$s+k \leq s.$$

From equations 1·36 and 1·37 we have

$$A_{-k}-A_{+s}=\frac{WN_0T}{\rho R}\left[E+ke-(kd+E)\frac{kbT+ac}{RT}-E+se+(sd-E)\frac{sbT-ac}{RT}\right]$$

$$=\frac{WN_0T}{\rho R}(k+s)\left[e-E\frac{b}{R}+(s-k)e\frac{bT}{cR}-\frac{ea}{R}\right]$$

$$=\frac{WN_0T}{\rho R}(k+s)e[1+(s-k).h.10^{-2}] \text{ neglecting } E\frac{b}{R} \text{ and } \frac{ea}{R}.$$

1·51 . . . $\therefore \quad A_{-k}-A_{+s}=\dfrac{WN_0T}{\rho R}(k+s)e$ approx.

1·52 . . . $\dfrac{A_{-k}-A_{+s}}{A_{-k}}=\dfrac{k+s}{n+k}.$

These equations show

(a) $$A_{-k}>A_{+s}.$$

(b) $$\text{Percentage difference}=\frac{k+s}{n+k}\times 100.$$

It is difficult to obtain experimental results sufficiently trustworthy to confute or to support the above results, as the change in the nature of the binding may change the electrical configuration of a molecule.

The simple cases in which multiple bindings do not occur are

$$(1)\ \begin{array}{c} Cl \\ \uparrow \\ Cl \leftarrow C \rightarrow Cl \\ \downarrow \\ Cl \end{array} \qquad (2)\ \begin{array}{c} Cl \\ \uparrow \\ H \rightarrow C \rightarrow Cl \\ \downarrow \\ Cl \end{array} \qquad (3)\ \begin{array}{c} H \\ \downarrow \\ H \rightarrow C \rightarrow Cl \\ \uparrow \\ H \end{array} \qquad (4)\ \begin{array}{c} H \\ \downarrow \\ H \rightarrow C \leftarrow H \\ \uparrow \\ H \end{array}$$

giving values for C denoted by C_{+4}, C_{+2}, C_{-2}, C_{-4} respectively, where C denotes the atomic refractivity of carbon.

The values for chlorine and hydrogen deduced by Brühl in the paper already cited are for the D-line of the spectrum 6·01 and 1·05 respectively, and the values for compounds given above are

$$\begin{array}{rl} CCl_4 = 26{\cdot}59 \text{ giving } & C_{+4} = 2{\cdot}59 \\ CHCl_3 = 21{\cdot}89 \qquad & C_{+2} = 2{\cdot}84 \\ CH_3Cl = 13{\cdot}00 \qquad & C_{-2} = 3{\cdot}85 \\ CH_4 = \ 6{\cdot}62 \qquad & C_{-4} = 2{\cdot}42 \end{array}$$

These values show that with the exception of CH_4 the values of C increase as its electro-negative nature becomes more pronounced, and also taking $n = 12$ the result (*b*) is true—with the exception noted—so far as order of magnitude is concerned, for

$$\frac{C_{-2} - C_{+4}}{C_{-2}} \times 100 = 33 \text{ per cent. by observation and } 42 \text{ per cent. by formula.}$$

$$\frac{C_{-2} - C_{+2}}{C_{-2}} \times 100 = 26 \text{ per cent. by observation and } 28 \text{ per cent. by formula.}$$

The value for oxygen when in the form $\rightrightarrows O$ has been very accurately determined to be 2·29.

From $S \begin{smallmatrix} \nearrow\!\!\nearrow O \\ \searrow\!\!\searrow O \end{smallmatrix}$, S_{+4} is obtainable,

and from $\begin{smallmatrix} H \searrow \\ H \nearrow \end{smallmatrix} S$, S_{-2} can be obtained.

$$\begin{array}{ll} \text{Molecular refractivity of } H_2S = & 9{\cdot}28, \\ \quad ,, \qquad ,, \qquad ,, \quad SO_2 = & 10{\cdot}23, \end{array}$$

and from these we deduce

$$S_{-2} = 7{\cdot}18 \quad \text{and} \quad S_{+4} = 5{\cdot}65,$$

showing (*a*) $S_{-2} > S_{+4}$,

and (*b*) $\dfrac{S_{-2} - S_{+4}}{S_{-2}} \times 100 = \dfrac{153}{7{\cdot}18} = 21$ per cent. from experimental values,

and $= \dfrac{600}{34} = 17$ per cent. by formula,

taking $n = 32$ = atomic weight of sulphur.

This apparent agreement, however, in the case of sulphur must be accepted with caution, as the experimental values obtainable are few.

Part II.—Case of the Diatomic Molecule.

Introduction.

The method of finding the molecular refractivity of a molecule by the addition of the atomic refractivities of the atoms in the molecule gives very accurate results in the case of organic compounds containing carbon, hydrogen, oxygen, and halogen, provided care is taken to use the values of carbon and oxygen corresponding to their chemical bindings in the molecule. This method is used, therefore, as an aid to the determination of the chemical constitution of organic compounds (*vide* chaps. viii and ix of S. Smiles' book on the *Relation of Chemical Constitution to some Physical Properties*). Incidentally we may note that it is this conformity to the additive law in the case of organic compounds which justifies such use of experimental results for purposes of verification as has been made in Part I of this analysis. In the case of organic compounds containing nitrogen, and in that of inorganic salts, the law does not hold to the same degree of approximation.

E.g. Value of molecular refractivity of—

(1) Hydrochloric acid $= M(HCl) = 6{\cdot}70$ by observation, and
$= 6{\cdot}83$ when calculated from $H+Cl$,
giving a difference of $+2$ per cent.

(2) $M(H_2O)$ by observation $= 3{\cdot}82$,
and $M(H_2O)$ calculated from $H_2+O = 4{\cdot}14$.
Difference per cent. $= +8{\cdot}4$.

(3) $M(NO)$ by observation $= 4{\cdot}46$,
$M(NO)$ calculated from $N+O = 4{\cdot}25$,
giving a difference of -5 per cent.

A good many cases of this nature are given by Brühl in the paper already cited, but the above cases will do for purposes of illustration of the value of the discrepancy in the additive law which is encountered in the case of gaseous compounds.

In this part of the paper it is intended to discuss, in a very approximate manner, the divergence from the additive law which will arise when the contribution to the molecular refractivity of a diatomic molecule due to electrical action between the atoms in the molecule is taken into account.

(a) We shall denote the values of the quantities pertaining to the two atoms of the diatomic molecule by the suffixes 1 and 2 respectively. Let atom 1 contain $\mu_1 = n_1 - s$ electrons, and atom 2 contain $\mu_2 = n_2 + s$

electrons; s will therefore measure the electro-positive valency of atom 1 and the electro-negative valency of atom 2.

The total polarisation due to the displacement of the electrons of the two atoms is now equal to the sum of the polarisations due to each separately.

For convenience in our analysis we shall suppose that the electrical force due to the incident electro-magnetic waves has components (X, O, O), and that we are discussing the steady state of the atoms in the field, when the doublets to which they are electrically equivalent have their axes in the direction of the x axis.

The main forced vibration of the negative electrons will therefore be in the x direction, and we shall neglect the contribution of all vibrations not in the direction of the axis of x.

Our equations of motion can now be written—

2·01 . . $$\mathrm{M}_1\ddot{x}_1 = \mathbf{X}\mathrm{E}_1 - \frac{4}{3}\pi\rho_1 e\sum_1^{\mu_1}(x_1 - \xi_r) - \phi_1(\mathrm{D})(x_1 - x_2),$$

2·02 . . $$\mathrm{M}_2\ddot{x}_2 = \mathbf{X}\mathrm{E}_2 - \frac{4}{3}\pi\rho_2 e\sum_1^{\mu_2}(x_2 - \xi'_r) - \phi_2(\mathrm{D})(x_1 - x_2),$$

2·03 . . $$m\sum_1^{\mu_1}\xi_r = -\mathbf{X}\mu_1 e - \frac{4}{3}\pi\rho_1 e\sum_1^{\mu_1}(\xi_r - x_1) + \sum_1^{\mu_1}(\xi_r - x_2)\phi_{21}(\mathrm{D}_r),$$

2·04 . . $$m\sum_1^{\mu_2}\ddot{\xi}'_r = -\mathbf{X}\mu_2 e - \frac{4}{3}\pi\rho_2 e\sum_1^{\mu_2}(\xi'_r - x_2) + \sum_1^{\mu_2}(\xi'_r - x_1)\phi_{12}(\mathrm{D}'_r),$$

2·05 . . $$\mathrm{X}' = \mathrm{N}\left[\mathrm{E}_1 x_1 + \mathrm{E}_2 x_2 - \sum_1^{\mu_1} e\xi_r - \sum_1^{\mu_2} e\xi'_r\right],$$

2·06 . where $\mathbf{X} = \mathrm{X} + \mathrm{X}'$,

X′ being the force due to polarisation of medium arising from displacement of the electrons.

x_1 and x_2 = Displacements in x direction of centres of spheres of positive electrification.

$-\phi_1(\mathrm{D})(x_1 - x_2)$ = Force on positive electrification of 1st atom, due to electrical nature of 2nd atom in its vicinity.

ξ_r and ξ'_r = Displacements in x direction of the r^{th} electrons of the respective atoms.

$\phi_{21}(\mathrm{D}_r)(\xi_r - x_2)$ = Force on r^{th} electron of 1st atom, due to electrical nature of 2nd atom.

D_r = Distance between centre of 2nd atom and r^{th} electron of 1st,

and so on.

We shall determine the nature of the functions ϕ_1, ϕ_2, ϕ_{12} and ϕ_{21} later, but in order to allow of the solution of our equations, we shall make the assumptions implicit in the equations

2·07 . . . $$\sum_1^{\mu_1}(\xi_r - x_2)\phi_{21}(\mathrm{D}_r) = \psi_1\sum_1^{\mu_1}(\xi_r - x$$

2·08 . . $$\sum_1^{\mu_2}(\xi'_r - x_1)\phi_{12}(\mathrm{D}'_r) = \psi_2\sum_1^{\mu_2}(\xi'_r - x_1),$$

where ψ_1 and ψ_2 are functions of the distance between the centres of the two atoms.

Let p be the frequency of the electro-magnetic field of force.

Our equations now become

2·09 . . $$-\mathrm{M}_1p^2x_1 = \mathrm{XE}_1 - t_1\sum_1^{\mu_1}(x_1 - \xi_r) - \phi_1(x_1 - x_2),$$

2·10 . . $$-\mathrm{M}_2p^2x_2 = \mathrm{XE}_2 - t_2\sum_1^{\mu_2}(x_2 - \xi'_r) - \phi_2(x_2 - x_1),$$

2·11 . . $$-mp^2\sum_1^{\mu_1}\xi_r = -\mathrm{X}\mu_1 e - t_1\sum_1^{\mu_1}(\xi_r - x_1) + \psi_1\sum_1^{\mu_1}(\xi_r - x_2)$$

2·12 . . $$-mp^2\sum_1^{\mu_1}\xi'_r = -\mathrm{X}\mu_2 e - t_2\sum_1^{\mu_2}(\xi'_r - x_2) + \psi_2\sum_1^{\mu_2}(\xi'_r - x_1)$$

2·13 . . . where $$\begin{cases} t_1 = \frac{4}{3}\pi\rho_1 e \\ t_2 = \frac{4}{3}\pi\rho_2 e. \end{cases}$$

2·14 . . . . Let $$\begin{cases} y_1 = \frac{4}{3}\pi\rho_1 e\sum_1^{\mu_1}\xi_r \\ y_2 = \frac{4}{3}\pi\rho_2 e\sum_1^{\mu_2}\xi'_r. \end{cases}$$

2·15 . . . . . $$\begin{cases} \frac{y_1}{t_1} = \sum_1^{\mu_1}\xi_r \\ \frac{y_2}{t_2} = \sum_1^{\mu_2}\xi'_r. \end{cases}$$

Equations 2·09, 2·10, 2·11, and 2·12 now become

2·16 . . $$(\phi_1 + t_1\mu_1 - \mathrm{M}_1p^2)x_1 - \phi_1 x_2 - y_1 = \mathrm{XE}_1.$$

2·17 . . $$(\phi_2 + t_2\mu_2 - \mathrm{M}_2p^2)x_2 - \phi_2 x_1 - y_2 = \mathrm{XE}_2.$$

2·18 . . $$\left(1 - \frac{\psi_1}{t_1} - \frac{mp^2}{t_1}\right)y_1 - t_1\mu_1 x_1 + \psi_1\mu_1 x_2 = -\mu_1\mathrm{X}_1 e$$

2·19 . . $\left(1-\frac{\psi_2}{t_2}-\frac{mp^2}{t_2}\right)y_2-t_2\mu_2x_2+\psi_2\mu_2x_1=-X\mu_2e.$

2·20 . . . $X'=\frac{4}{3}\pi N\left\{E_1x_1+E_2x_2-\frac{ey_1}{t_1}-\frac{ey_2}{t_2}\right\}.$

2·21 . . . Let $\begin{cases} f_1=1-\frac{1}{t_1}(mp^2+\psi_1) \\ f_2=1-\frac{1}{t_2}(mp^2+\psi_2), \end{cases}$

and

2·22 . . . . . $\begin{cases} g_1=\phi_1+t_1\mu_1-M_1p^2 \\ g_2=\phi_2+t_2\mu_2-M_2p^2. \end{cases}$

Equations 2·16 to 2·19 now become

2·23 . . . $g_1x_1-\phi_1x_2-y_1=XE_1.$

2·24 . . . $g_2x_2-\phi_2x_1-y_2=XE_2.$

2·25 . . . $f_1y_1-t_1\mu_1x_1+\psi_1\mu_1x_2=-X\mu_1e.$

2·26 . . . $f_2y_2-t_2\mu_2x_2+\psi_2\mu_2x_1=-X\mu_2e.$

Eliminating y_1 and y_2 from these equations, we obtain

2·27 . . $x_1(f_1g_1-\mu_1t_1)+x_2(\psi_1\mu_1-f_1\phi_1)=X(E_1f_1-\mu_1e).$

2·28 . . $x_2(f_2g_2-\mu_2t_2)+x_1(\psi_2\mu_2-f_2\phi_2)=X(E_2f_2-\mu_2e)$

2·29 . . . . Let $\begin{cases} h_1=f_1g_1-\mu_1t_1. \\ h_2=f_2g_2-\mu_2t_2. \end{cases}$

2·30 . . . . . . $\begin{cases} j_1=E_1f_1-\mu_1e. \\ j_2=E_2f_2-\mu_2e. \end{cases}$

2·31 . . . . . . $\begin{cases} \alpha_1=\mu_1\psi_1-f_1\phi_1. \\ \alpha_2=\mu_2\psi_2-f_2\phi_2. \end{cases}$

Equation 2·27 and 2·28 become

2·32 . . . . . $h_1x_1+\alpha_1x_2=Xj_1,$

2·33 . . . . . $h_2x_2+\alpha_2x_1=Xj_2,$

and these equations give

2·34 . . . . . $x_1=X(j_1h_2-j_2\alpha_1)/(h_1h_2-\alpha_1\alpha_2).$

2·35 . . . . . $x_2=X(j_2h_1-j_1\alpha_2)/(h_1h_2-\alpha_1\alpha_2).$

From equations 2·25 and 2·26 we have

2·36 . . . . $y_1=\frac{1}{f_1}(\mu_1t_1x_1-\psi_1\mu_1x_2-X\mu_1e).$

2·37 . . . . $y_2=\frac{1}{f_2}(\mu_2t_2x_2-\psi_2\mu_2x_1-X\mu_2e).$

Substituting these values in 2·20 we obtain

2·38 . . . $$X' = \frac{4}{3}\pi N\left[\left(E_1 - \mu_1 e + e\frac{\psi_2\mu_2}{t_2f_2}\right)x_1 + X\frac{e^2\mu_1}{t_1f_1}\right.$$
$$\left. + \left(E_2 - \mu_2 e + e\frac{\psi_1\mu_1}{t_1f_1}\right)x_2 + X\frac{e^2\mu_2}{t_2f_2}\right]$$
$$= \frac{4}{3}\pi N\left[\left(E_1 - \mu_1 e + e\frac{\psi_2\mu_2}{t_2f_2}\right)\frac{j_1h_2 - j_2a_1}{h_1h_2 - a_1a_2}\right.$$
$$+ \left(E_2 - \mu_2 e + e\frac{\psi_1\mu_1}{t_1f_1}\right)\frac{j_2h_1 - j_1a_2}{h_1h_2 - a_1a_2}$$
$$\left. + \frac{\mu_1e^2}{t_1f_1} + \frac{\mu_2e^2}{t_2f_2}\right]X.$$

2·39 . Let $X' = PX$, then

2·40 . . . $$P = \frac{4}{3}\pi N\left[\left(E_1 - \mu_1 e + e\frac{\mu_2\psi_2}{t_2f_2}\right)\frac{j_1h_2 - j_2a_1}{h_1h_2 - a_1a_2} + \frac{\mu_1e^2}{t_1f_1}\right.$$
$$\left. + \left(E_2 - \mu_2 e + e\frac{\mu_1\psi_1}{t_1f_1}\right)\frac{j_2h_1 - j_1a_2}{h_1h_2 - a_1a_2} + \frac{\mu_2e^2}{t_2f_2}\right].$$

In the same way as in I (*b*) we can show that

$$\left(\frac{\mu^2 - 1}{\mu^2 + 2}\right)_{\text{molecule}} = P_{\text{molecule}}.$$

(*b*) From the form of equation 2·40 it is seen that the additive law does not hold exactly when the contribution due to interaction between the atoms is considered. In order to examine the contribution due to electrical interaction, let

$$P(\text{molecule}) = P_1 + P_2 + P_{12},$$

where P_1 = contribution to P due to atom 1 alone.

P_2 = contribution to P due to atom 2 alone.

P_{12} = contribution to P due to electrical action between atoms 1 and 2.

Then we find

2·41 . . . $$\frac{P_{12}}{\frac{4}{3}\pi N} = \left(E_1 - \frac{e\mu_1}{t_1f_1} + e\frac{\mu_2\psi_2}{t_2f_2}\right)\frac{j_1h_2 - j_2h_1}{h_1h_2 - a_1a_2} + \frac{e^2\mu_1}{t_1f_1}$$
$$+ \left(E_2 - \frac{e\mu_2}{t_2f_2} + e\frac{\mu_1\psi_1}{t_1f_1}\right)\frac{j_2h_1 - j_1h_2}{h_1h_2 - a_1a_2} + \frac{e^2\mu_2}{t_2f_2}$$
$$- \left(E_1 - \frac{e\mu_1}{f'_1}\right)\frac{j'_1}{h'_1} - \frac{e^2\mu_1}{t_1f'_1}$$
$$- \left(E_2 - \frac{e\mu_2}{f'_2}\right)\frac{j'_2}{h'_2} - \frac{e^2\mu_2}{t_2f'_2}$$

where $f'_1 = f_1$, $g'_1 = g_1$, $h'_1 = h_1$, etc., when $\phi_1 = \phi_2 = \psi_1 = \psi_2 = 0$.

(*c*) Sir J. J. Thomson in his paper, *Phil. Mag.*, 1906, assuming the interaction between the atoms to be that which comes into play between the two spheres of positive electricity, concluded that the contribution to

the molecular refractive index "arising from the coupling of the atoms together may easily be comparable with the part due to the corpuscles within the atoms."

Adopting this assumption, we have in our notation

$$\psi_1 = \psi_2 = 0 \quad \text{and} \quad \phi_1 = \phi_2 = \phi,$$

since the interaction is entirely between the positive spheres of electricity.

In expression 2·41 we shall render the term in ϕ explicit by using the following notation:

2·42 $$\begin{cases} a_1 = -f'_1\phi & \text{where } f'_1 = 1 - mp^2/t_1. \\ a_2 = -f'_2\phi & \quad ,, \quad f'_2 = 1 - mp^2/t_2. \end{cases}$$

2·43 $$\begin{cases} g_1 = \phi + g'_1 & \quad ,, \quad g'_1 = t_1\mu_1 - M_1p^2. \\ g_2 = \phi + g'_2 & \quad ,, \quad g'_2 = t_2\mu_2 - M_2p^2. \end{cases}$$

2·44 $$\begin{cases} h_1 = f'_1\phi + h'_1 & \quad ,, \quad h'_1 = f'_1g'_1 - t_1\mu_1. \\ h_2 = f'_2\phi + h'_2 & \quad ,, \quad h'_2 = f'_2g'_2 - t_2\mu_2. \end{cases}$$

Substituting these values in 2·41, we obtain

2·45 $$\frac{P_{12}}{\frac{4}{3}\pi N} = \left(E_1 - \frac{e\mu_1}{f'_1}\right)\left(\frac{h'_2j'_1 + \phi u}{h'_1h'_2 + \phi v} - \frac{j'_1}{h'_1}\right) + \left(E_2 - \frac{e\mu_2}{f'_2}\right)\left(\frac{h'_1j'_2 + \phi u}{h'_1h'_2 + \phi v} - \frac{j'_2}{h'_2}\right)$$

2·46 . . . . . where $u = f_1j'_2 + f_2j'_1$

2·47 . . . . . and $v = f_1h'_2 + f_2h'_1$.

Equation 2·45 can be put into the form

2·48 $$\frac{P_{12}}{\frac{4}{3}\pi N} = \frac{\phi}{h'_1h'_2 + \phi v}\left[u\left(E_1 + E_2 - \frac{e\mu_1}{f'_1} - \frac{e\mu_2}{f'_2}\right) - v\left\{\left(E_1 - \frac{e\mu_1}{f'_1}\right)\frac{j'_1}{h'_1} + \left(E_2 - \frac{e\mu_2}{f'_2}\right)\frac{j'_2}{h'_2}\right\}\right].$$

It is easy to show that the force on the positive electrification of atom 2 due to that of atom 1 is of the order of magnitude of

$$-\frac{2E_1E_2}{D^3}(x_2 - x_1) = -\phi(x_2 - x_1),$$

where D is of the order of the distance between the centres of spheres of positive electrification, *i.e.*

$$D = 0(2 \,.\, 10^{-8}).$$

$$\therefore \quad \phi = \frac{2E_1E_2}{D^3},$$

2·49 . . . . . *i.e.* $\phi = 6{\cdot}4n_1n_2 \,.\, 10^{-17}$

in order of magnitude.

All the equations which follow, and which equate quantities to numerical values, are to be taken to be true only in so far as order of magnitude is concerned.

2·50 . . . $$f'_1 = 1 - \frac{mp^2}{t_1} = 1 - \theta_1,$$

2·51 . . where $$\theta_1 = \frac{c_1}{T_1} = \frac{3 \cdot 4}{n_1} \,.\, 10^{-2}.$$

2·52 . . . $$j'_1 = E_1 f_1 - \mu_1 e = se \quad \text{approx.}$$
$$= 1 \cdot 6 \,.\, s \,.\, 10^{-20}.$$

2·53 . . . $$g'_1 = -M_1 p^2 + t_1 \mu_1 = -M_1 p^2 \left(1 - \frac{a}{c} \frac{\mu_1}{n_1}\right)$$
$$= -M_1 p^2 \quad \text{approx.}$$
$$= -\frac{2n_1}{h} \,.\, 10^{-14}.$$

2·54 . . . . . $$h'_1 = f'_1 g'_1 - t_1 \mu_1$$
$$= -\frac{2n_1}{h} \,.\, 10^{-14}.$$

2·55 . $$u = f'_2 j'_1 + f'_1 j'_2$$
$$= (E_1 + E_2 - \mu_1 e - \mu_2 e) - (\theta_1 + \theta_2)(E_1 + E_2) + e(\theta_2 \mu_1 + \theta_1 \mu_2)$$
$$= -10^{-21}.$$

2·56 . . . . $$v = f'_2 h'_1 + f'_1 h'_2$$
$$= f'_1 f'_2 (g'_1 + g'_2)$$
$$= -\frac{2}{h}(n_1 + n_2) \,.\, 10^{-14}.$$

2·57 . . . . . $$\therefore \quad u \left\{ E_1 + E_2 - \frac{e\mu_1}{f_1} - \frac{e\mu_2}{f_2} \right\}$$
$$= -u(\theta_1 \mu_1 e + \theta_2 \mu_2 e)$$
$$= -6 \cdot 8 \,.\, 10^{-2} \,.\, ue$$
$$= -10^{-42}.$$

2·58 . . . $$v \left\{ \frac{j'_1}{h'_1}\left(E_1 - \frac{\mu_1 e}{f'_1}\right) + \frac{j'_2}{h'_2}\left(E_2 - \frac{\mu_2 e}{f'_2}\right) \right\}$$
$$= v \left\{ \frac{j'^2_1}{h'_1} + \frac{j'^2_2}{h'_2} \right\}$$
$$= vs^2 e^2 \left\{ \frac{1}{h'_1} + \frac{1}{h'_2} \right\}$$
$$= -2 \cdot 6 \,.\, s^2 \,.\, \frac{(n_1 + n_2)^2}{n_1 n_2} \,.\, 10^{-40}.$$

2·59 . . $$\frac{\phi}{h'_1 h'_2 + \phi v} = \frac{6 n_1 n_2 \,.\, 10^{-17}}{\frac{4 n_1 n_2}{h^2} \,.\, 10^{-28} - 1 \cdot 2 n_1 n_2 \,.\, 10^{-30} \frac{(n_1 + n_2)}{h}}$$
$$= 1 \cdot 5 h^2 \,.\, 10^{11} \text{ approx.}$$

2·60 . . $$\frac{P_{12}}{\frac{4}{3}\pi N} = 1{\cdot}5h^2 \,.\, 10^{11} \,.\, 2{\cdot}6 \,.\, s^2 \,.\, \frac{(n_1+n_2)^2}{n_1 n_2} \,.\, 10^{-40}$$
$$= 4h^2 \,.\, \frac{(n_1+n_2)^2}{n_1 n_2} \,.\, 10^{-29}.$$

2·61 . . . . $$\frac{P_{12}}{P_1+P_2} = 4h^2 \,.\, \frac{(n_1+n_2)^2}{n_1 n_2} \,.\, 10^{-5}.$$

This gives percentage contribution due to interaction between the atoms

$$= 4h^2 \,.\, \frac{(n_1+n_2)^2}{n_1 n_2} \,.\, 10^{-3}.$$

This result shows that this assumption for the interaction between the atoms will not account for the observed discrepancies in the additive law for atomic refractivities.

This result is not unexpected, since we have already shown in I (f) that the contribution due to positive electrification is not large compared with that due to the negative electrons.

(d) We shall now endeavour to find the contribution to the molecular refractive index which is due to the interaction of the atoms, on the assumption that a neutral atom behaves like an electrical doublet so far as its action on electrons external to itself is concerned.

In the case, therefore, of an atom which has lost electrons on account of electron transference, we shall assume that the external action of the atom, which is electro-positive, is equivalent to a doublet and a positive charge both situated within the atom.

The previous assumptions that have been made, viz.

$$\sum_1^{\mu_1}(\xi_r - x_2)\phi_{21}(D_r) = \psi_1 \sum_1^{\mu_1}(\xi_r - x_2)$$

and

$$\sum_1^{\mu_2}(\xi'_r - x_1)\phi_{12}(D'_r) = \psi_2 \sum_1^{\mu_2}(\xi'_r - x_1),$$

show that ψ_1 and $\phi_{21}(D_r)$ and ψ_2 and $\phi_{12}(D'_r)$ are of the same order of magnitude.

Consider an electron in the 2nd atom which is acted on by electrical forces due to the distribution of electricity in the 1st atom.

Let ω_1 be the moment of the doublet equivalent to the 1st atom.

Sir J. J. Thomson, in his paper in *Phil. Mag.*, 1914, quotes values for the electrostatic moments of doublets given by Sutherland in *Phil. Mag.*, vol. xxxix, p. 1. These values increase with rise in atomic weight, and we shall take the values of the doublets to be approximately proportional to the atomic weights, and therefore to the number of electrons in the atom.

In the case of hydrogen, the electrostatic moment of the doublet is given as

$$\cdot 75 \times 10^{-18} = \omega_h \text{ in electrostatic units.}$$

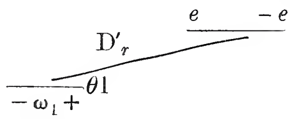
$$\therefore \quad \omega_h = \frac{\cdot 75 \times 10^{-18}}{3 \times 10^{10}} \text{ in electro-magnetic units.}$$

2·62 . . . . . . $\omega_h = 2{\cdot}5 \times 10^{-29}.$

2·63 . . . . . $\therefore \quad \omega_1 = \dfrac{2{\cdot}5 n_1}{h} \times 10^{-29}.$

The r^{th} electron in the 2nd atom, whose displacement is ξ'_r, is displaced an amount $\xi'_r - x_1$ relative to atom 1.

Force in direction of axis of ω_1



where θ is small and is on average zero

$$= \frac{2\omega_1 e}{\left(D_r - \dfrac{\xi'_r - x_1}{2}\right)^3} - \frac{2\omega_1 e}{\left(D_r + \dfrac{\xi'_r - x_1}{2}\right)^3}$$

$$= \frac{6\omega_1 e}{D'^4_r}(\xi'_r - x_1) = \phi_{12}(D'_r)(\xi'_r - x_1).$$

2·64 . . . . . $\therefore \quad \phi_{12}(D'_r) = \dfrac{6\omega_1 e}{D'^4_r}.$

Taking $D'_r = 2 \times 10^{-8}$, we have

2·65 . . . . . $\phi_{12}(D'_r) = \psi_1 = 1{\cdot}5 \,.\, \dfrac{n_1}{h} \,.\, 10^{-17}.$

Force on doublet $-e(\xi'_r - x_1)$ due to charge $+se$ at distance D_r

$$= -\frac{2se^2}{D_r^3}(\xi'_r - x_1) = \phi'_{12}(\xi'_r - x_1).$$

2·66 . . . . . $\therefore \quad \phi'_{12} = -6{\cdot}5 \,.\, s \,.\, 10^{-17}.$

In general

$$6{\cdot}5 s < \frac{1{\cdot}5 n}{h},$$

and we shall therefore assume

2·67 . . . . . $\phi_{12} = \dfrac{1{\cdot}5 n_1}{h} \,.\, 10^{-17} = \psi_1$

and

$$\phi_{21} = \frac{1{\cdot}5 n_2}{h} \,.\, 10^{-17} = \psi_2.$$

We proceed to modify our notation in order to make explicit the contribution to P due to the interaction of the atoms.

2·68 . . $f_1 = f'_1 - \beta_1,$ where $\beta_1 = \dfrac{\psi_1}{t_1} = \dfrac{1{\cdot}5 \times 10^{-1}}{2{\cdot}6 \,.\, h} = \dfrac{1}{17h},$

$$i.e.\ \beta_1 < 1. \quad \therefore \quad \frac{\beta_1}{f'_1} < 1.$$

2·69 . . . $g_1 = g'_1 + \phi_1$ and $\dfrac{\phi_1}{g'_1} = -\cdot 75 \times 10^{-3} n_2.$

2·70 . . . $h_1 = f_1 g_1 - t_1\mu_1$ since $\phi_1 = 6\omega_2 E_1/D_r^4$

$$= 1\cdot 5 n_1 n_2 \,.\, 10^{-17}/h$$

$$= f'_1 g'_1 - t_1\mu_1 + f'_1\phi_1 - \beta_1 g'_1 - \beta_1\phi_1$$

$$= h'_1 + \gamma_1$$

2·71 . . where $\gamma_1 = f'_1\phi_1 - \beta_1 g'_1 - \beta_1\phi_1 = 0(\phi_1)$

$$= \frac{1\cdot 5}{h} n_1 n_2 \,.\, 10^{-17}\,.$$

$$\therefore \quad \frac{\gamma_1}{-h_1} = \cdot 75 \,.\, n_2 \,.\, 10^{-3}. \quad < 1.$$

2·72 . . . $i_1 = E_1 f_1 - \mu_1 e = j'_1 - \beta_1 E_1$

$$\frac{\beta_1 E_1}{j'_1} = \frac{\beta_1 n_1}{s}\,.$$

Equation 2·68 shows that in our work we may take $\beta_1 = \beta_2 = \beta$.

Referring now to the expression 2·41 for P_{12}, we shall work out the values of the different parts of P_{12} in turn.

2·73 . . $E_1 - \dfrac{e\mu_1}{f_1} + \dfrac{e\mu_2\psi_2}{t_2 f_2} = E_1 - \dfrac{e\mu_1}{f'_1} + \dfrac{e\mu_2\psi_2}{t_2 f_2} - \dfrac{e\mu_1\beta}{f'^2_1}$

$$= E_1 - \frac{e\mu_1}{f'_1} + \delta_{12},$$

2·74 . . . where $\delta_{12} = \dfrac{e\mu_2\psi_2}{t_2 f_2} - \dfrac{e\mu_1\beta}{f'^2_1}\,.$

2·75 . . . $j_1 h_2 - j_2 a_1 = (j'_1 - \beta E_1)(h'_2 + \gamma_2) - (j'_2 - \beta_2 E_2) a_1$

$$= j'_1 h'_2 - \epsilon_{12},$$

2·76 . . . where $\epsilon_{12} = \beta_1 E_1 h'_2 + \gamma_2\beta_1 E_1 - \gamma_2 j'_1 + a_1 j'_2 - \beta_2 E_2 a_1.$

2·77 . . . $h_1 h_2 - a_1 a_2 = (h'_1 + \gamma_1)(h'_2 + \gamma_2) - a_1 a_2$

$$= h_1 h'_2 (1 + \kappa),$$

2·78 . . . where $\kappa = \dfrac{\gamma_1}{h'_1} + \dfrac{\gamma_2}{h'_2} + \dfrac{\gamma_1\gamma_2 - a_1 a_2}{h'_1 h'_2}$ and κ is numerically < 1.

2·79 . . . $\therefore \quad \dfrac{j_1 h_2 - j_2 a_1}{h_1 h_2 - a_1 a_2} = \dfrac{j'_1 h'_2 - \epsilon_{12}}{h'_1 h'_2 (1+\kappa)}$

$$= \frac{j''_1}{h'_1} - \frac{\epsilon_{12}}{h'_1 h'_2}(1-\kappa) - \frac{\kappa j''_1}{h'_1}$$

$$= \frac{j''_1}{h'_1} - \Delta_{12}$$

2·80 . . where $\Delta_{12} = \dfrac{\kappa j''_1}{h'_1} + \dfrac{\epsilon_{12}}{h'_1 h'_2}(1-\kappa).$

2·81 . . $$\frac{P_{12}}{\frac{4}{3}\pi N} = \left(E_1 - \frac{e\mu_1}{f'_1} + \delta_{12}\right)\left(\frac{j'_1}{h'_1} - \Delta_{12}\right) + \frac{e^2\mu_1}{t_1 f_1}$$
$$+ \left(E_2 - \frac{e\mu_2}{f'_2} + \delta_{21}\right)\left(\frac{j'_2}{h'_2} - \Delta_{21}\right) + \frac{e^2\mu_2}{t_2 f_2}$$
$$- \left(E_1 - \frac{e\mu_1}{f'_1}\right)\frac{j'_1}{h'_1} - \left(E_2 - \frac{e\mu_2}{f'_2}\right)\frac{j'_2}{h'_2}$$
$$- \frac{e^2\mu_1}{t_1 f'_1} - \frac{e^2\mu_2}{t_2 f'_2}.$$

2·82 . . $$\frac{P_{12}}{\frac{4}{3}\pi N} = \delta_{12}\frac{j'_1}{h'_1} - \Delta_{12}\left(E_1 - \frac{e\mu_1}{f_1} + \delta_{12}\right)$$
$$+ \delta_{21}\frac{j'_2}{h'_2} - \Delta_{21}\left(E_2 - \frac{e\mu_2}{f_2} + \delta_{21}\right)$$
$$+ \frac{e^2\mu_1}{t_1 f''_1}\beta + \frac{e^2\mu_2}{t_2 f''_2}\beta.$$

We proceed to find in an approximate manner the values of δ_{12}, ϵ_{12}, Δ_{12}, etc., and so determine the order of magnitude of P_{12}.

2·83 . . . . . $$\delta_{12} = \frac{e\mu_2\beta}{f_2} - \frac{e\mu_1\beta}{f'^2_1} = 0(se\beta)$$
$$= 1{\cdot}6\beta \,.\, s \,.\, 10^{-20}.$$

2·84 . . $$\epsilon_{12} = \beta_1 h'_2 E_1 + \gamma_2\beta_1 E_1 - \gamma_2 j'_1 + \alpha_1 j'_2 + \beta_2 E_2 \alpha_1$$
$= \beta_1 h'_2 E_1$, which is the greatest term,
$= -\frac{3\beta n_1 n_2}{h} \,.\, 10^{-34}$. (Other terms are of order 10^{-37}.)

2·85 . . . . . $$\kappa = \frac{\gamma_1}{h'_1} + \frac{\gamma_2}{h'_2} + \frac{\gamma_1\gamma_2 - \alpha_1\alpha_2}{h'_1 h'_2}$$
$= -{\cdot}75 \,.\, 10^{-3}(n_1 + n_2)$, *i.e.* κ is numerically less than 1.

2·86 . . . . . $$\Delta_{12} = \kappa\frac{j'_1}{h'_1} + \frac{\epsilon_{12}(1-\kappa)}{h'_1 h'_2}$$
$= \frac{\epsilon_{12}}{h'_1 h'_2}$ on retaining the greatest term,
$= \frac{3\beta h}{4}10^{-6}$, the other terms being of value 10^{-9}.

Now, taking the terms of $P_{12}\Big/\frac{4}{3}\pi N$ separately, we have

2·87 . . . . . $$\delta_{12}\frac{j'_1}{h'_1} = 1{\cdot}3 \,.\, \frac{s^2\beta}{n_1} \,.\, 10^{-26}.$$

2·88 . . . . $$\Delta_{12}\left(E_1 - \frac{e\mu_1}{f_1} + \delta_{12}\right) = 2{\cdot}5\beta h s \,.\, 10^{-26}.$$

2·89 . . . . . $$\frac{e^2\mu_1}{t_1 f'^2_1} \,.\, \beta = 1{\cdot}2\beta \,.\, 10^{-24}.$$

From expressions 2·87, 2·88, and 2·89 we see that the largest terms occurring in 2·82 are $\frac{e^2\mu_1}{t_1 f'^2_1}\beta$ and $\frac{e^2\mu_2}{t_2 f'^2_2}\beta$, the other terms being relative to this one of order 10^{-2} approximately.

Hence we have

2·90 . . . . . $$\frac{P_{12}}{\frac{4}{3}\pi N} = \frac{e^2\mu_1}{t_1 f'^2_1}\beta_1 + \frac{e^2\mu_2}{t_2 f'^2_2}\beta_2,$$

and therefore

2·91 . $$\frac{P_{12}}{P_1 + P_2} = \frac{\beta e^2\left(\frac{\mu_1}{t_1 f'^2_1} + \frac{\mu_2}{t_2 f'^2_2}\right)}{\frac{e^2\mu_1}{t_1 f'^2_1} + \frac{e^2\mu_2}{t_2 f'^2_2} + \left(E_1 - \frac{e\mu_1}{f'_1}\right)\frac{j'_1}{h'_1} + \left(E_2 - \frac{e\mu_2}{f'_2}\right)\frac{j'_2}{h'_2}}.$$

Now

$$\left(E_1 - \frac{e\mu_1}{f'_1}\right)\frac{j'_1}{h'_1} = -0\left(\frac{s^2 h}{2n} \cdot 10^{-26}\right)$$

and

$$\frac{e^2\mu_1}{t_1 f_1} = 0(10^{-24}),$$

and so we may neglect $\left(E_1 - \frac{e\mu_1}{f'_1}\right)\frac{j'_1}{h'_1}$ relatively to $\frac{e\mu_1}{t_1 f_1}$.

2·92 . . . . . $$\therefore \quad \frac{P_{12}}{P_1 + P_2} = \beta \quad \text{approx.},$$

and hence

2·93 . . . $$\left.\begin{array}{c}\text{Percentage contribution to the molecular refractive} \\ \text{index due to interaction of atoms}\end{array}\right\} = 100\beta = \frac{100}{17h}.$$

Now h = number of electrons in the hydrogen atom when neutral,

$$\therefore \quad h \geq 2,$$

since, if $h = 1$ when one electron was transferred from the hydrogen atom, the on'y electrification left would be the positive electrification, and this would contribute a negative quantity to the atomic refractive index, and there is no evidence of the atomic refractivity of hydrogen being negative.

Hence the percentage contribution to the molecular refractivity due to the interaction of the atoms is for $h = 2$ approximately 3 per cent.

This investigation does not presume to give the accurate percentage contribution; it merely gives an indication of the order of magnitude of the percentage, and with this proviso we may say that our result is in agreement with experimental evidence.

(Issued separately January 8, 1920.)

XVI.—A "Duplex" Form of Harmonic Synthetiser and its Mathematical Theory. By J. R. Milne, D.Sc.

(Read July 5, 1915; MS. received July 6, 1919.)

Introduction.

The present paper is really a sequel to one which was published in the Society's *Proceedings* for 1905–06 to describe an improved form of harmonic synthetiser, in which, by the substitution of rotary for reciprocating motion, all sliding parts are eliminated. Each simple harmonic unit of the machine consisted (see fig. 1) of a revolving crank with a pulley at its

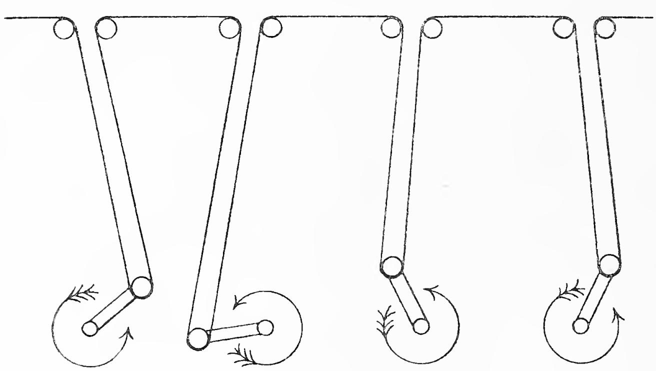


Fig. 1.

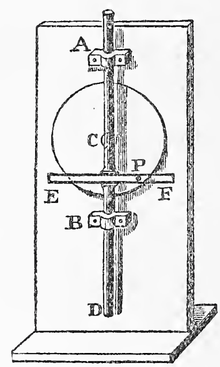


Fig. 2.

outer end down to which a flexible wire was led from fixed pulleys above; and this takes the place of the usual mechanism, shown in fig. 2, in which the slider D, constrained to move in a vertical direction by the guides A and B, has an exact simple harmonic motion imparted to it by means of the pin P fixed in the revolving disc C. Lord Kelvin, in describing his well-known Tide-Predictor, states that he first used the plan of fig. 1, but subsequently altered the machine and introduced that of fig. 2 instead. He gives as the reason, that "This modification, though making the instrument less simple, was rendered in fact necessary by the large range which it was proposed to give for the resultant curve, and which would have required inconveniently great lengths for the straight parts of wire between the upper and lower rows of pulleys to nearly enough annul the geometrical error of the simpler plan." *

* *Proc. Inst. Civil Engineers*, vol. lxv, 1881, p. 16.

In the author's paper of 1906, however, the result of the mathematical discussion is to show that the effect of the "geometrical error" on the accuracy of the instrument is much less than would be supposed.

But in the mathematics a simplifying assumption had to be made (that the diameters of the pulleys employed were negligibly small), and its presence introduces some uncertainty in the conclusion arrived at.

The present paper was commenced solely with the intention of getting rid of this assumption; but in the course of the work *the fortunate discovery was made that a "duplex" disposition of the pulleys of the apparatus—entailing no addition of parts, but rearrangement only—would very nearly annul the error altogether.* This result makes so great a change in the mathematical reasoning that it is better presented afresh, without reference to that given in the former paper.

Definition of "Error."

Any harmonic synthetiser consists of a combination of mechanical units, each giving rise to a linear motion which is exactly, or very approximately, simple harmonic. In the latter case, the measure of the error ϵ is defined as "the displacement of the actual position of the moving part from its theoretical position, *divided by the total range of movement.*"

We shall lead up to the *Duplex Unit* by considering as a simple example the "*Crank and Wire*" *Unit* (fig. 1) employed by Lord Kelvin. In all that follows, it is evident that without loss of generality the crank radius may be taken as unity.

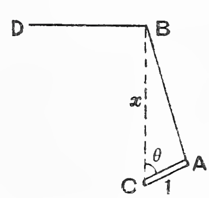


Fig. 3.

In fig. 3 let CA be a crank revolving about its centre C, and AB a cord passing round a pin at B, its end D being supposed to move in the straight line BD. The error of the movement of D, regarded as an approximation to S.H.M., is

$$\epsilon = \tfrac{1}{2}\{\sqrt{(x^2+1-2x\cos\theta)}+\cos\theta - x\} \quad . \quad . \quad . \quad (1)$$

As an example the following table gives some corresponding values of θ and ϵ. The calculation is best made by the more convenient formula

$$\epsilon = \left(\frac{1}{8}\cdot\frac{1-\cos 2\theta}{x-\cos\theta}\right) - \left(\frac{1}{8}\cdot\frac{1-\cos 2\theta}{x-\cos\theta}\right)^2 \frac{1}{x-\cos\theta} \quad . \quad . \quad . \quad (2)$$

the approximation of which is amply sufficient. The value $5\sqrt{2}$ has been chosen for x to enable a comparison to be made later on with a duplex unit of the same dimensions (the case in which $p=5$).

Table I.

θ	0°	15	30	45	60	75	90	105	120	135	150	165	180
	360	345	330	315	300	285	270	255	240	225	210	195	
ϵ	0	·0027	·0101	·0196	·0284	·0341	·0352	·0317	·0247	·0160	·0079	·0021	0

The above results have been graphed in fig. 4, which shows a curve of two almost symmetrical waves. Hence it is apparent that *if two*

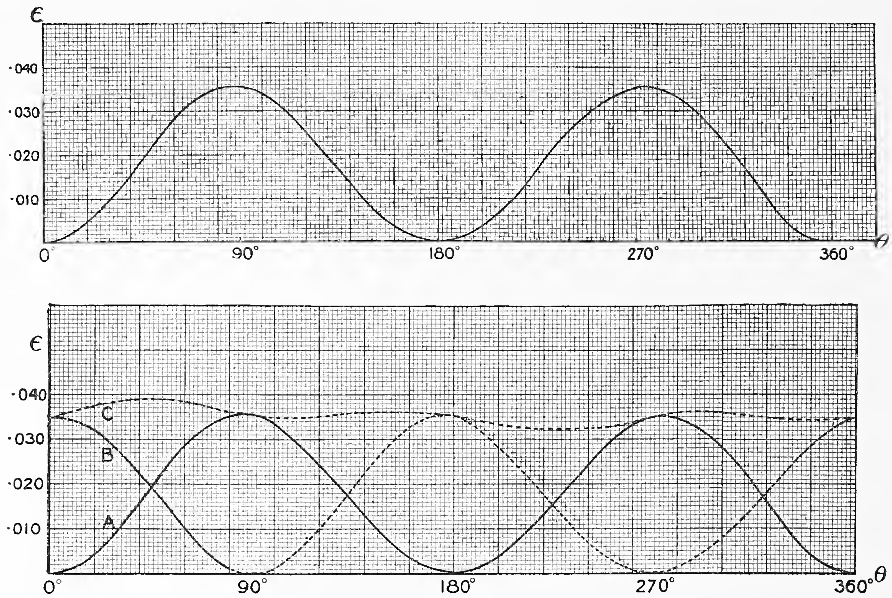


Fig. 4.

A is the original curve.
B is the same curve retarded 90° in phase.
C is the sum of the two.

"crank-and-wire" units could be arranged to work together, one being 90° in phase in advance of the other, the hills and hollows of the two would very nearly cancel each other, and so eliminate the error.

The special design of the apparatus by which this is accomplished will be understood from fig. 5. Imagine a cord fixed at B to be let down under the crank pin P and carried up over the fixed pin A, the end D being constrained to move in the straight line AD. BCP is in itself a complete "crank-and-wire" unit, as also is ACP, and the phase difference between these units can be made 90° by taking BCA equal to 90°.

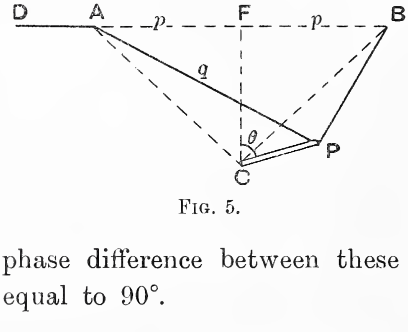


Fig. 5.

Of course such a pair constitutes only one harmonic element of the complete apparatus, and in practice the wire, after passing over A, does not terminate, but is led down to the crank pin of the next pair, and so on.

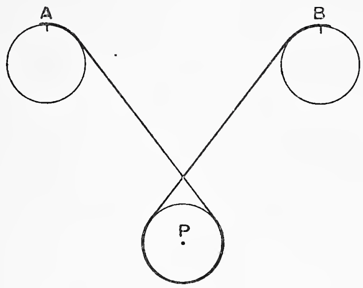


FIG. 6 (a).

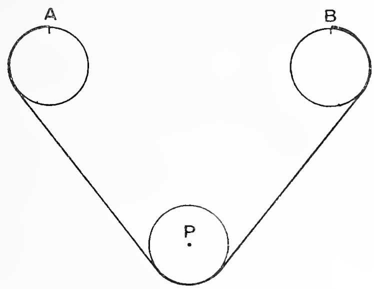


FIG. 6 (b).

In each of the above figures the *centres* of the two top pulleys occupy the positions of the pins A and B in fig. 5; P, the centre of the other pulley, that of the crank pin P in fig. 5.

Also the length of stroke yielded by the duplex unit is not materially inferior to that of a simple unit, as will be shown later.

Device for Eliminating the Diameter of the Pulleys from the Mathematical Analysis.

By merely arranging the wire round the pulleys in the manner shown in figs. 6 (a) or (b) their diameters do not need to be considered, and the mathematical theory which regards pulleys as points is no longer an approximation, but is exact. For it will be seen that in each case the total length of the wire from A to B is equal to its length in fig. 5 from A to B, plus a *constant* length equal to the circumference of a pulley. Probably the arrangement shown in (a) is not so good as that in (b) because it involves a crossed belt.

Expression for the Error.

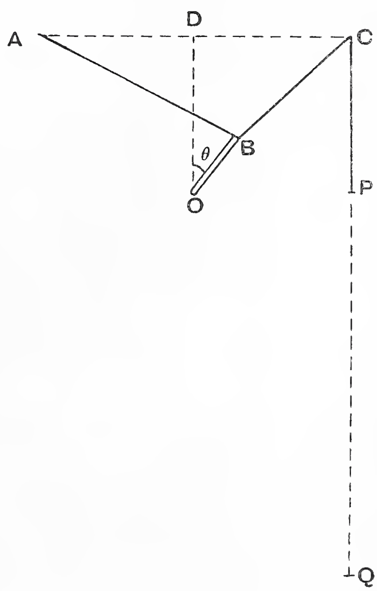


FIG. 7.—DA = DO = DC = p, and OB = 1.

Fig. 7 shows a duplex harmonic unit having a pen P attached to the free end of the wire ABCP, the pen being constrained to move in a vertical line. To investigate its error of motion it is convenient to take our fiducial point Q such that CQ = ABCP, for then QP = AB + BC.

The standard S.H.M. for comparison with the actual motion is one of the same phase and period, having its range in the line CQ and conterminous with that of the pen.* The magnitude of the range is

$$2\sqrt{[2p^2+1+2p]} - 2\sqrt{[2p^2+1-2p]} \quad . \quad . \quad . \qquad (3)$$

The actual position of the pen = QP

$$= \sqrt{[2p^2+1+2p(\sin\theta-\cos\theta)} + \sqrt{[2p^2+1-2p(\sin\theta+\cos\theta)]},$$

and its true position

$$= \{\sqrt{[2p^2+1+2p]} + \sqrt{[2p^2+1-2p]}\} - \cos\theta\{\sqrt{[\ ]} - \sqrt{[\ ]}\}$$

Hence

$$2\epsilon = \left\{\frac{\sqrt{[2p^2+1+2p(\sin\theta-\cos\theta)} + \sqrt{[2p^2+1-2p(\sin\theta+\cos\theta)]} - \{\sqrt{[2p^2+1+2p]} + \sqrt{[2p^2+1-2p]}\}}{\sqrt{[2p^2+1+2p]} - \sqrt{[2p^2+1-2p]}}\right\} + \cos\theta \quad (4)$$

Table of the Error as a Function of the Dimensions of the Duplex Unit and the Phase Angle.

The values of ϵ have been calculated for $p=1$, 4, 5, and 10; and for $\theta=0°$, 15°, 30°, 45°, 60°, 75°, 90°, 105°, 120°, 135°, 165°, and 180°.

As the quantity to be calculated involves the *difference* of the positive and negative parts of the numerator of the fraction, and these are nearly equal in magnitude, it was found necessary to use 7-figure logarithms.

Table II.

$p=1$.														
	θ	0°	15	30	45	60	75	90	105	120	135	150	165	180
	ϵ	0	– ·0154	– ·0575	– ·0873	– ·0682	– ·0296	0	+ ·0165	·0186	·0147	·0079	·0022	0
$p=4$.														
	θ	0°	15	30	45	60	75	90	105	120	135	150	165	180
	ϵ	0	– ·0007	– ·0022	– ·0034	– ·0034	– ·0020	0	+ ·0017	·0025	·0022	·0013	·0004	0
$p=5$														
	θ	0°	15	30	45	60	75	90	105	120	135	150	165	180
	ϵ	0	– ·00041	– ·00134	– ·00210	– ·00211	– ·00128	0	·00112	·00164	·00143	·00087	·00027	0
$p=10$.														
	θ	0°	15	30	45	60	75	90	105	120	135	150	165	180
	ϵ	0	– ·00009	– ·00031	– ·00048	– ·00049	– ·00032	0	·00030	·00044	·00042	·00025	·00008	0

These results are graphed in fig. 8. (The curve for $p=1$ has been drawn to $\frac{1}{30}$ the vertical scale of the other curves in order that it may be

* This is not that particular S.H.M. to which the actual motion is most akin, but it is sufficiently nearly so; and it is far the most convenient both for the user of the instrument and in the calculation of the error. For further information on this point see the previous paper.

shown in its entirety.) The right-hand member of (4) may be expressed as a series—

$$\epsilon = -\frac{1}{8}\cos\theta\,(1-\cos^2\theta)q^2\left\{\left(1+\frac{3}{4}q^2+\frac{5}{8}q^4\right)+\left(\frac{5}{4}+\frac{29}{16}q^2\right)q\,.\cos\theta\right.$$
$$\left.+\left(\frac{7}{8}+\frac{79}{32}q^2\right)q^2\,.\cos^2\theta-\frac{33}{32}q^4\,.\cos^4\theta\right\} \quad . \quad . \quad (5)$$

where

$$q=\frac{2p}{2p^2+1} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (6)$$

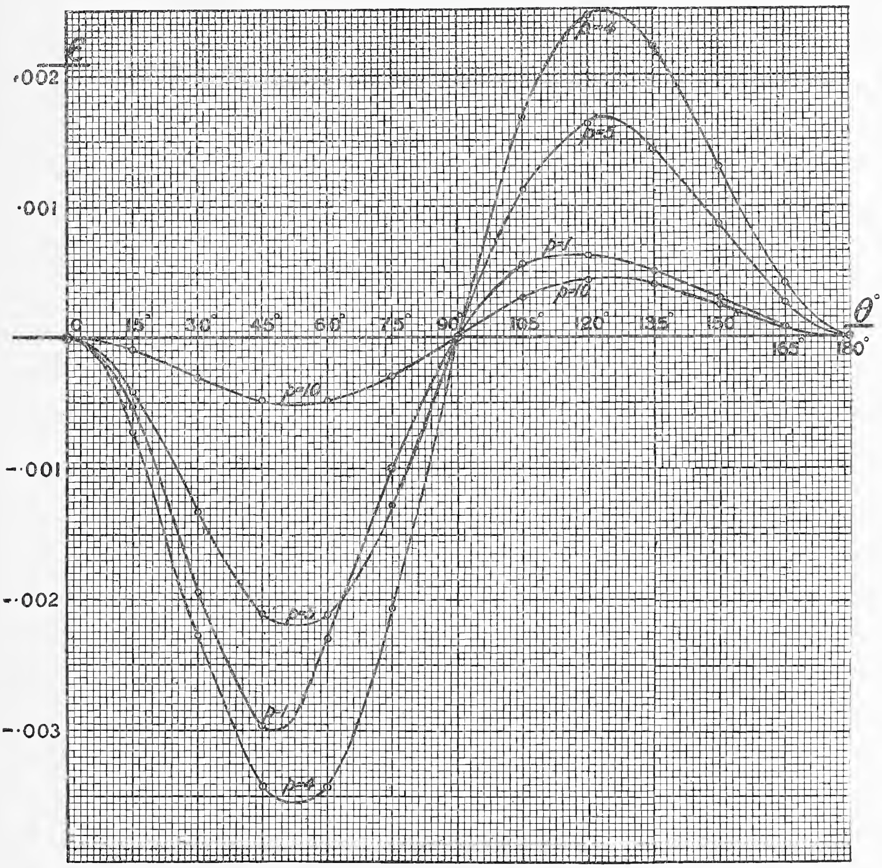


FIG. 8.—Graphs of ϵ for different values of p.

The turning values of ϵ regarded as a function of θ are determined by the equation

$$0=\frac{d}{d\theta}=-\frac{d\epsilon}{d\cos\theta}\sin\theta \quad . \quad . \quad . \quad . \quad . \quad (7)$$

The factor $\sin\theta$ leads to $\theta=0$ or π, values which are evident *a priori* because of our choice of comparison S.H.M. The other factor gives

$$
\begin{aligned}
0 = \frac{d\epsilon}{d\cos\theta} = & (32 + 24q^2 + 20q^4) \\
& + (80q + 116q^3)\cos\theta \\
& - (96 - 12q^2 - 177q^4)\cos^2\theta \\
& - (160q + 232q^3)\cos^3 \\
& - (140q^2 + 560q^4)\cos^4\theta \\
& + 231q^4\cos^6\theta = 0 \quad . \quad . \quad . \quad . \quad . \quad . \quad (8)
\end{aligned}
$$

The solution of this equation is

$$\cos\theta = 0{\cdot}57778 + 0{\cdot}13457q \quad . \quad . \quad . \quad . \quad (9)$$

Greatest Numerical Error.

It appears from Table II that the minimum value of ϵ is in every case numerically greater than its maximum value; hence, confining our attention to the former, we have for the greatest numerical error

$$e = -0{\cdot}04811q^2(1 + {\cdot}7222q + 1{\cdot}21q^2 + 1{\cdot}18q^3). \quad . \quad . \quad . \quad (10)$$

Table III.

p.	q.	$\cos\theta$.	θ.	e.	a.
4	·242424	·6104	52°·38	−·00357	1·4031
5	·196078	·6042	52·83	−·00221	1·4071
7	·141414	·5968	53·36	−·00108	1·4106
10	·099502	·5912	53·76	−·000517	1·4124
16	·062378	·5862	54·11	−·000197	1·4135
∞	0	·5774	54·736	0	1·4142

It will be observed from fig. 8 that as p increases, the positive and negative parts of the curve became more and more alike; but that complete similarity is only attained in the limiting case where $p = \infty$, and the curves have become coincident with the base line.

Amplitude of Harmonic.

The column giving the amplitude, headed a in Table III, is calculated thus:—

From equation 3

$$a = \sqrt{(2p^2 + 1 + 2p)} - \sqrt{(2p^2 + 1 - 2p)}$$

$$\therefore \quad \tfrac{1}{2}a^2 = (2p^2 + 1) - \sqrt{(4p^4 + 1)}\,;$$

and sufficiently nearly

$$\tfrac{1}{2}a^2 = 1 - \frac{1}{(2p)^2} + \frac{1}{(2p)^6} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (11)$$

For all practical purposes the value of a within the range from $p = 4$ to $p = \infty$ may be taken as constant and equal to 1·41.

Equation (10) may be made to furnish a very simple approximation for p as a function of e by the use of (6),

$$p = 0{\cdot}32 + \frac{0\ 22}{\sqrt{(-e)}},$$

and from this the following table of values was obtained:—

Table IV.

$-e$	·005	·004	·003	·002	·001	·005	·0001
p	3·43	3·80	4·34	5·24	7·28	10·16	22·32

Extreme Compactness of Duplex Unit.

We shall proceed to exhibit the use of the foregoing formulæ by taking a numerical example.

Let it be required to find the smallest duplex unit which will describe a simple harmonic motion, having an amplitude of $2\frac{1}{2}$ inches and an obliquity error nowhere exceeding ·01 of an inch.

Here

$$e = \frac{\text{greatest error}}{\text{twice the amplitude}} = \frac{\cdot 01}{5} = \cdot 002\,;$$

hence, from Table IV, $p = 5{\cdot}24$, the unit of measurement being the length of the crank arm.

Now we find from Table III that when $p = 5{\cdot}24$, an arm of 1 inch gives an amplitude of 1·408 inches. Hence, as we require an amplitude of 2·5 inches, the crank must be 2·5/1·408, *i.e.* 1·776 inches long.

Hence, finally, p is $5{\cdot}24 \times 1{\cdot}776 = 9{\cdot}306$ inches in length.

Comparison with Crank and Wire Unit.

The significance of the figures just obtained will best be exhibited by showing how very large the ordinary crank and wire unit of fig. 1 would have to be to produce an equally good result.

We shall first find the angle at which the greatest error of the unit occurs, and thence the value of the error.

Suppose that the crank length in fig. 3 is r, and that the error ϵ and its maximum value e are defined as before.

Then

$$\epsilon = \frac{1}{2r}\left\{\sqrt{x^2 + r^2 - 2xr\cos\theta} - x + r\cos\theta\right\},$$

$$\therefore\quad \frac{d\epsilon}{d\theta} = \frac{1}{2r}\left\{\frac{xr\sin\theta}{\sqrt{x^2 + r^2 - 2xr\cos\theta}} - r\sin\theta\right\}.$$

Hence the greatest error occurs when

$$r^2 - 2xr \cos\theta = 0,$$

i.e. when $\cos\theta = \frac{r}{2x}$,

and therefore $e = r/4x$.

If, then, the value of e be given, this determines x, the necessary size of the unit. In the present example $e = \cdot 002$, and $r = 5/4 = 1\cdot 25$ inches; for it must be remembered that in practice the wire AB in fig. 3 is double, as shown in fig. 1.

Hence $x = 13$ feet, a result which is in very striking contrast with the former for the Duplex Unit.

Related Matters.

For a description of the method employed for varying the amplitude of each harmonic unit during the running of the machine, and for various practical details, reference should be made to the former paper.

Summary.

The present paper is a continuation of one published thirteen years ago describing a new form of harmonic synthetiser. The object is to show that by a slight alteration the simple form of mechanism proposed by Lord Kelvin in 1881, but afterwards rejected by him on acount of "obliquity error," can be made quite satisfactory.

The Author wishes to express his indebtedness to the Trustees of the Moray Fund of Edinburgh University for a grant, and to the Carnegie Trust for the Universities of Scotland for defraying the expense of the illustrations.

(*Issued separately January* 29, 1920.)

OBITUARY NOTICE.

Sir James Alexander Russell, Kt., M.A., M.B., F.R.C.P.E., B.Sc. in Public Health, LL.D., J.P., D.L. By Thomas R. Ronaldson, M.B., F.R.C.P.E.

(MS. received May 19, 1919. Read June 2, 1919.)

About fifty years ago the writer, then beginning his medical studies, paid his first visit to the dissecting-rooms of the Anatomical Department of the University of Edinburgh. The experience was trying, but curiously and greatly relieved by the sight of a tall, alert figure moving from group to group of students, the impression of activity and power being emphasised by a pale face of arresting brightness and intelligence. Such was the writer's introduction to Sir James A. Russell, then Junior Demonstrator of Anatomy under the late Sir William Turner, and the beginning of a valued friendship, ending only with his death.

It is a matter of common observation that the country owes much to its manses, from which, with their plain living and high thinking, so often issue those who do yeoman service in their day and generation. Sir James was one of them. He was the eldest son of the Rev. A. F. Russell, Free Church minister of Kilmodan and South Hall, Argyllshire. He was, however, born on 6th April 1846 in Skye, at Glassellan House, the home of his maternal grandfather, Mr Munro. A few weeks after his birth his mother brought him to Glendaruel, and the old Inn of Tighmor-na-clach became their home until the following year, when the manse was ready for occupation.

There, on the Kyles of Bute,—one of a band of brothers—his boyhood was spent, the strong physique was built up, and the seeds of mental and moral qualities were planted which were so beneficently to bear fruit in after years. Yarn, spun by his nurse from the wool of Highland sheep to the accompaniment of Gaelic folk-song, and woven by the local weaver, was made into kilts by the tailor who came to the family from Skye at stated intervals for the purpose. Education, secular and religious, was carried on at home and in the Stronafian F.C. School, where rumour has it that the excellent and energetic teacher took at times undue advantage of the native garb. Much of boyhood's spare time was spent in boating and sea-fishing. In the later period of his home education he

had, in addition to his father's training, the advantage of the help of Mr Adam Lang, M.A., Aberdeen, and of M. Henri Mouron, a cultured Swiss gentleman, from whom he gained the knowledge of French which was of interest and use to him ever afterwards.

Thus equipped, doubtless with the added responsibility of being the eldest son of the manse with brothers to follow, he proceeded in his sixteenth year to the University of Edinburgh, where he began his studies in 1861 in the Faculty of Arts, and graduated as M.A. Thereafter he entered the Faculty of Medicine, and after a distinguished course graduated in 1868 as M.B., C.M., taking the first place in First Class Honours.

Sir Halliday Croom, in an admirable obituary notice in *The British Medical Journal* of February 1918, says: "Those who remember him recall him as a keen student, fond of all sorts of scientific problems, careful and exact in argument, and ready of speech."

Among the eminent Professors of his time Goodsir influenced him most, and it was to Anatomy that he attached himself after graduation, acting as Demonstrator under the late Sir William Turner, and rising to the post of Senior Demonstrator before demitting office in 1876.

During part of this time he became famous as a "coach" for the professional examinations, few of his students being known to fail.

Recognition of his eminent qualifications as an anatomist and teacher was shown by the offer to him of a professorship of Anatomy in New Zealand in 1874. This, however, he did not see his way to accept.

Although in 1875 he was an applicant for the Chair of Medicine and Anatomy in St Andrews—an unsuccessful one, fortunately, for the future of Edinburgh,—there are clear indications that Anatomy was too exact a science, and presented too limited a field, to satisfy the wide and varied interests of his mind. In a letter dated 14th May 1874 he writes: "I am working two hours a day in the Chemical Laboratory this summer. Turner lets me away for that time. I wish to qualify for getting an appointment of Officer of Public Health should a chance turn up."

To enable him to carry out this wish he graduated in 1875 as B.Sc. in Public Health with First Class Honours—the first B.Sc. in that department.

While Demonstrator of Anatomy he kept in touch with Medicine and Surgery by attending clinical lectures in the Royal Infirmary. In an interesting letter to a medical friend he showed how clearly he grasped the principles of antiseptic surgery and realised the revolution in the treatment of wounds that had begun with Lister.

It is an admirable custom that the inhabitants of Edinburgh should show hospitality to ministers who come from a distance to attend the meetings of the General Assemblies of the Churches. Sir James's father was in this way allocated to Woodville, Canaan Lane, the home of Miss Marianne Wilson, daughter of a well-known naturalist, Mr James Wilson, and niece of the celebrated Christopher North. Thus a friendship began between the families, which finally led to the marriage of Sir James to Miss Wilson in 1876. Having retired from the teaching of Anatomy the same year, and being now in a settled home, where he lived until his death, he was free to follow the natural bent of his mind, which, as has been indicated, was towards Medicine and General Science and their application to public life rather than to the more limited field of Anatomy. The next year was accordingly spent in France and England studying sanitation and the problems of Public Health, the scientific treatment of which was then in its infancy, hoping eventually to get a post as Officer of Public Health.

Returning to Edinburgh in 1877, he inaugurated the class at the Heriot-Watt College on the Theory of Plumbing, especially in relation to sanitary work, and for several years he filled the position of lecturer, and in December of that year gave two lectures, afterwards published, on sanitary houses to builders and plumbers, under the auspices of the Royal Scottish Society of Arts.

How greatly his work in this direction was appreciated is shown in the minute of meeting of the Local Council for Edinburgh and the East of Scotland of the National Registration of Plumbers, held shortly after his death.

Alongside of this he carried on a practice as adviser in sanitary matters, and nothing gave him greater pleasure than to plan and to supervise the carrying out of the sanitation and plumbing of a friend's house.

But the year 1880 was the real beginning of his public life. Elected to the Town Council that year, he became Bailie in 1885, and at the same time Convener of the Public Health Committee, of which he had previously been a member. Finally he was raised to the Civic Chair in 1891, and filled the position of Lord Provost with honour and acceptance for the usual term of three years.

From the date of his election as Town Councillor a new era began in the department of Public Health, which proved to be widespread and far-reaching in its results. It is not difficult to realise the effect of the entrance into a town council, largely composed of commercial men,

of a man of culture and science, trained in Medicine, and who was young, ardent, capable in affairs, single-minded in character and purpose, courteous in manner, and always easy of approach. Sanitary reform was beginning to stir in the minds of public men. The man had come for the hour, and his influence was rapid and decisive.

His association with the Public Health Committee, and the assistance of other public-spirited men, among whom should be specially mentioned the late Sir Henry Littlejohn, led to slums being removed, streets widened, and housing improved. The Fever Hospital, now one of the largest and best of its kind, was initiated and personally watched over in the old Royal Infirmary buildings; and notably the 1891 Act, which is the City's Magna Charta of Public Health, was passed, mainly owing to his foresight, sagacity, and knowledge. In 1881 the death-rate of the city was 18·8 per 1000; it progressively diminished until in 1916 it fell to 14·5.

The electric lighting of the city was installed during his Lord Provostship, and it was due to his scientific knowledge, and to his insistence that it should be kept in the city's hands, that it was the most successful of all town installations. Further, that great improvement, the widening of the North Bridge and of the street between it and the High Street, as well as the coincident enlargement of the North British Railway Station, were accomplished by his courageous shouldering of a great responsibility at a critical juncture.

These are probably the most prominent of his civic successes as Lord Provost. As a by-product of his term of office it has been pointed out by Sir Halliday Croom that during his Lord Provostship, "The profession of Medicine in Edinburgh came to its kingdom, for to every medical institution and to all medical charities he gave, not only his own personal encouragement as Lord Provost, but that of the civic authorities as well, and the members of the profession themselves enjoyed his generous hospitality. He was among the first, if not the very first, medical Lord Provosts of Edinburgh, and it would be a very great advantage to that city, and not to that city only, if men of his calibre, with his scientific and medical knowledge, who had leisure at their disposal, would grace such chairs again." The medical profession has always been held in high esteem by the Town Council, and, with a medical man as Lord Provost, its position reached high-water mark.

But it was not only in civic matters that his services were pre-eminent. His keen mind found outlet in many activities. At an early period he made an effort to have all the children at sea-coast schools, not only at home but in the Colonies, taught signalling by the Morse alphabet,

holding that not only would it prove useful for ships at sea, but that "by exercising a whole class together the children develop that sense of time and rhythm which is essential to all proper co-operation in combined movements, from the pulling of a rope to the marching of a regiment."

As a Volunteer he was a member of No. 4 Company, Q.E.R.V.B., from 1870 to 1877. He became H.M. Inspector of Anatomy in 1881, and in 1890 Assistant Inspector under the Cruelty to Animals Act for Scotland and the North of England, the latter appointment affording full scope for his qualities of tact, patience, and knowledge, and compelling him to the last to keep himself abreast of the developments of Physiology and Pathology.

He was a member of Edinburgh School Board, 1885–1888; Chairman of the Burgh Committee on Secondary Education, 1893–1902; Governor George Heriot's Trust, 1880–1903; ex-officio Chairman of the Board of Management Royal Infirmary during his Lord Provostship; a member of the Board of Management of the Royal Edinburgh Mental Hospital, 1907–1914. He was elected a Fellow and an Examiner of the Royal College of Physicians, Edinburgh.

In 1880 Sir James Russell was elected a Fellow of the Royal Society of Edinburgh, whose meetings he attended with great interest and enjoyment. Although he never contributed a paper to the *Proceedings* or *Transactions*, he often took part in discussions of papers on subjects with which he was acquainted. His mental qualities were strong, not so much in the direction of original investigation and research, as in a marked capacity for understanding and expounding the results of research, and applying them to the good of the community. He contributed a valuable memoir of the late Sir William Turner, which is published in Vol. XXXVI of the *Proceedings*, and which is full of interesting reminiscences of University life. He was also a regular attendant at the meetings of the Royal Society Club, where he delighted his friends with many curious stories of the days when Professor Syme and Sir James Young Simpson added lustre to the medical faculty.

A man of affairs, he was director and chairman of various companies.

As an elder of the Barclay U.F. congregation, he was held in great esteem for faithful and detailed duty to his Church. In that office, as a director of the Edinburgh Medical Missionary Society, and as a member of committee of the French Protestant Church of Edinburgh, he found more outward expression for his simple but deeply religious nature.

Such and other eminent services met with their due meed of public recognition. Not only did he adorn the Civic Chair, but he became Lord Lieutenant of the County of the City of Edinburgh; his Alma Mater conferred on him the degree of LL.D. in 1894, and a few months later Queen Victoria bestowed on him the honour of knighthood.

Born by western seas, reared in a Highland manse, the eldest of a family of sons who—after school life—had to depend largely on their own exertions, early inspired to excel, Sir James was ever the hardest of workers, for whom, until his later years, the usual holiday was scarcely existent. Partly owing to his iron constitution, and partly from the variety of his mental interests, he did not seem to require the relaxations of the ordinary man. But latterly he keenly enjoyed his motor-boat on the Clyde, to the boating and line-fishing of his boyhood being added the mechanical interest of his motor-engine. Towards the end of his life he became subject to bronchitis and to heart weakness, and to these he succumbed on 22nd January 1918, at the age of 72.

To many, the news of his death meant little more than the passing of a useful and distinguished citizen, but to his intimates it meant the loss of a loyal, hospitable, and generous-hearted friend, who would spare neither time nor strength on their behalf, and whose memory will ever be cherished by them.

His first wife died in 1882. In 1897 he married Mary Ruth, daughter of Captain G. B. Prior, R.A., and widow of Captain MacKenzie, Bombay Cavalry, by whom and two daughters he is survived, and by Lady Russell's two daughters by her previous marriage. He was buried in the Dean Cemetery, where so many of Edinburgh's honoured citizens have their last resting-place.

APPENDIX.

CONTENTS.

PROCEEDINGS OF THE STATUTORY GENERAL MEETING

Beginning the 136th Session, 1918–1919.

At the Statutory Meeting of the Royal Society of Edinburgh, held in the Society's Lecture Room, 24 George Street, on Monday, October 28, 1918, at 4.30 p.m.,

Dr A. Crichton Mitchell, Curator of Library, in the Chair,

the Minutes of the last Statutory Meeting of October 22, 1917, were read, approved, and signed.

The Chairman nominated as Scrutineers of the Voting Papers, Surgeon-General Bannerman and Mr C. H. Milne.

Before the ballot was taken, Professor Baily, who had been nominated by the Council for election as the Society's representative on the Heriot Trust, explained that since the Council had made this recommendation inquiry had shown that there were difficulties of a purely legal kind which might prevent him being able to undertake the duties of the Society's representative. While thanking the Council for their expression of confidence, he felt that the simplest course was for him to be allowed to withdraw his name and to nominate instead Dr W. A. Tait as one well able to represent the Society on this important Trust.

The ballot for the election of Office-Bearers, Members of Council, and Representative on the Heriot Trust was then taken.

The Secretary submitted the following Report:—

Mr George Stewart—our Librarian—is still serving in our army as a sergeant in the 4th Royal Scots, which, along with the other regiments of the 52nd Division, returned to France during last year. Shortly after reaching France, Mr Stewart was invalided home, and obtained his first leave since 1915. During his leave he was able to give useful assistance in some rearrangements in the library.

As anticipated last year there has been a considerable falling-off in publication, partly on account of the energies of scientific workers being devoted to war purposes, and partly on account of the necessity for keeping down our expenses. In view of the probable shortage of papers the Council decided to hold meetings once a month only. The number of papers read at our meetings during Session 1917–18 amounted to 25, of which 21 have been, or are being, printed in the *Proceedings* and 3 in the *Transactions*. Of the papers read, 6 were in Mathematics, 4 in Physics, 2 in Meteorology, 1 in Botany, 6 in Chemistry, 1 in Geology, 2 in Zoology, and 3 in Physiology. There were also two addresses given—one in Astronomy and the other in Meteorology. As compared with last year, the most notable difference is in the reduction of *Transactions* papers, which have fallen from 7 to 3. There is practically no change in the number of *Proceedings* papers, but the papers published this Session are on the average much shorter than those published in previous years.

Last March the Society elected 16 new Fellows, and we lost by death and resignation 12 Ordinary Fellows. There are at this moment 634 Ordinary Fellows on our list.

Two prizes were awarded during the year—the Keith Prize to Mr R. C. Mossman, and the Neill Prize to Professor W. H. Lang.

It is my duty to draw attention to the James Scott Prize, which has been founded by the Trustees of the James Scott Bequest. One of the Trustees—Mr Alexander Philip, writer, Brechin—is one of our Fellows. In description of this prize I quote the following excerpt from the Minute of the Trustees as transmitted to the Council:—

> "The Trustees have resolved and hereby resolve that a sum of Two Hundred and Fifty Pounds sterling, representing approximately the nett accumulated income of the said Trust Funds, shall be paid over to the Royal Society of Edinburgh to be held by them in trust, subject to the following conditions:—
>
> "(*a*) The said sum of Two Hundred and Fifty Pounds sterling shall be held by them in trust as a Trust Fund to be known as the James Scott Fund and to be invested by them in name of the Royal Society in trust securities or on deposit receipt with a bank.
>
> "(*b*) The income of the said Trust Fund shall be expended in payment of an honorarium or prize, to be known as the James Scott Prize, for a lecture or essay on the fundamental concepts of natural philosophy, to be awarded triennially or otherwise in accordance with such regulations as the Council of the said Society may from time to time determine.

"(c) In the event of no suitable candidate for the said prize being forthcoming, it shall be competent to the said Society either to add the amount available for the said prize to the principal sum or to carry the same forward to be added to the amount of the prize offered at the next competition; but in no case shall it be competent for the said Society to encroach upon the principal sum, either for the purpose of meeting the expenses of administration or for the purpose of increasing the amount of the said prize.

"ALEXR. PHILIP.
"A. D. TAIT HUTCHISON."

A very serious outlay in all scientific publication is the preparation and printing of plates of illustrations, especially in the biological sciences. The Council is greatly indebted to the help received from the Carnegie Trust for the Universities of Scotland, who have always shown great readiness in giving grants to graduates of the Scottish Universities in aid of the publication of their illustrations. Without this help, which has now been given for a number of years, the Society could not have undertaken the publication of so many valuable papers. It is right that we should express our thanks for the aid thus generously given.

The TREASURER, in submitting his Report for the year, drew special attention to the diminished cost of publication as compared with the previous year. The Council had thus been able to clear off a large part of the outstanding debt. It was also pointed out that the saving effected this year was only provisional, since in due course an increased amount of binding would have to be undertaken if the journals were to be kept in serviceable condition.

Dr PEACH moved the adoption of the Reports, and the reappointment of Messrs LINDSAY, JAMIESON & HALDANE, C.A., as auditors of the accounts for the ensuing Session.
This was unanimously agreed to.

The Scrutineers reported that the Ballot Papers were in order, and that the following had been elected as Office-bearers, Members of Council, and Representative on George Heriot's Trust:—

JOHN HORNE, LL.D., F.R.S., F.G.S., President.
Professor D'ARCY THOMPSON, C.B., B.A., F.R.S.,
Professor JAMES WALKER, D.Sc., Ph.D., LL.D., F.R.S.,
Professor GEORGE A. GIBSON, M.A., LL.D.,
ROBERT KIDSTON, LL.D., F.R.S., F.G.S.,
Professor D. NOËL PATON, M.D., B.Sc., F.R.C.P.E., F.R.S.,
Professor ARTHUR ROBINSON, M.D., M.R.C.S.,
} Vice-Presidents.
CARGILL G. KNOTT, D.Sc., LL.D., General Secretary.
Professor E. T. WHITTAKER, Sc.D., F.R.S.,
J. H. ASHWORTH, D.Sc., F.R.S.,
} Secretaries to Ordinary Meetings.
JAMES CURRIE, M.A., Treasurer.
A. CRICHTON MITCHELL, D.Sc., Hon. D.Sc. (Geneva), Curator of Library and Museum.

ORDINARY MEMBERS OF COUNCIL.

Sir GEORGE A. BERRY, LL.D., M.B., F.R.C.S.E.
JOHN S. FLETT, M.A., D.Sc., LL.D., F.R.S.
Professor MAGNUS MACLEAN, M.A., D.Sc., M.Inst.C.E., M.Inst.E.E.
Professor DAVID WATERSTON, M.A., M.D., F.R.C.S.E.
Professor F. O. BOWER, M.A., D.Sc., F.R.S., F.L.S.
Professor P. T. HERRING, M.D., F.R.C.P.E.
Professor T. J. JEHU, M.A., M.D., F.G.S.
ALEXANDER LAUDER, D.Sc.
The Hon. LORD GUTHRIE, LL.D.
Sir E. SHARPEY SCHAFER, M.D., LL.D., D.Sc. F.R.S.
Professor J. LORRAIN SMITH, M.A., M.D., F.R.S.
W. A. TAIT, D.Sc., M.Inst.C.E.

SOCIETY'S REPRESENTATIVE ON GEORGE HERIOT'S TRUST.

W. A. TAIT, D.Sc., M.Inst.C.E.

The CHAIRMAN, in the name of the Society, thanked the Scrutineers for their services.

PROCEEDINGS OF THE ORDINARY MEETINGS,

Session 1918–1919.

FIRST ORDINARY MEETING.

Monday, November 4, 1918.

Dr John Horne, F.R.S., F.G.S., President, in the Chair.

The President delivered a short Address.

The following Communications were read:—

1. Researches in Optical Activity: the Temperature Rotation Curves for the Tartrates at Low Temperatures. By Dr T. S. PATTERSON and Mr K. L. MOUDGILL. Communicated by Professor A GRAY. (*With Lantern Illustrations.*) *Proc.*, vol. xxxix, pp. 18–34.
2. Amphicheiral Knots. By Miss M. G. HASEMAN. Communicated by the GENERAL SECRETARY. *Trans.*, vol. lii, pp. 597–602.
3. Further Note on the Propagation of Earthquake Waves. By Dr C. G. KNOTT. *Proc.* vol. xxxix, pp. 157–208.

The Rev. J. D. MCCULLOCH signed the Roll, and was duly admitted a Fellow of the Society.

SECOND ORDINARY MEETING.

Monday, December 2, 1918.

Dr John Horne, F.R.S., F.G.S., President, in the Chair.

The following Communications were read:—

1. The Calciferous Glands of Earthworms. By Professor J. STEPHENSON and Dr BAINI PRASHAD. *Trans.*, vol. lii, pp. 455–485.
2. The Prostate Glands of the Earthworms of the Family Megascolecidæ. By Professor J. STEPHENSON and Mr HARU RAM. *Trans.*, vol. lii, pp. 435–453.
3. The Adsorption Isotherm at Low Concentrations. By Dr A. M. WILLIAMS. Communicated by Professor WALKER. *Proc.*, vol. xxxix, pp. 48–55.
4. The Origin of Anticyclones and Depressions. By Lieut. JOHN LOGIE, R.A.F., M.A., B.Sc., F.R.A.S. Communicated by the late Capt. G. W. JONES, R.A.F. *Proc.*, vol. xxxix, pp. 56–77.

THIRD ORDINARY MEETING.

Monday, January 20, 1919.

Dr John Horne, F.R.S., F.G.S., President, in the Chair.

The following Communications were read:—

1. Contributions towards a knowledge of the Anatomy of the Lower Dicotyledons. II: The Anatomy of the Stem of the Berberidaceæ. By Professor HARVEY-GIBSON, C.B.E., and Miss ELSIE HORSMAN. *Trans.*, vol. lii, pp. 501–515.
2. Contributions towards a knowledge of the Anatomy of the Lower Dicotyledons. III: The Anatomy of the Stem of the Calycanthaceæ. By Miss CHRISTINE E. QUINLAN. Communicated by Professor HARVEY-GIBSON, C.B.E. *Trans.*, vol. lii, pp. 517–529.
3. On the Life-History and Bionomics of *Myzus Ribis*, Linn. (Red-Currant Aphis). By Miss MAUD D. HAVILAND. Communicated by Professor BOWER. *Proc.*, vol. xxxix, pp. 78–112.
4. Further Note on Earthquake Waves and the Interior of the Earth. By Dr C. G. KNOTT. *Proc.*, vol. xxxix, pp. 157–208.

FOURTH ORDINARY MEETING.

Monday, February 3, 1919.

Dr John Horne, F.R.S., F.G.S., President, in the Chair.

The following Communications were read :—

1. The Stelar Anatomy of *Platyzoma microphyllum*, R.Br. By Dr J. M'Lean Thompson. (*With Lantern Illustrations.*) *Trans.*, vol. lii, pp. 571–595.
2. The Comparative Anatomy of the Shoulder Girdle and Pectoral Fin of Fishes. By Capt. E. W. Shann, B.Sc. Communicated by Professor W. C. M'Intosh, F.R.S. *Trans.*, vol. lii, pp. 531–570.
3. Note on the Determinant of the Primary Minors of a special set of $(n-1)$-by-n Arrays. By Sir Thomas Muir, F.R.S. *Proc.*, vol. xxxix, pp. 35–40.

Lieut. L. W. G. Malcolm signed the Roll, and was duly admitted a Fellow of the Society.

FIFTH ORDINARY MEETING.

Monday, March 3, 1919.

Dr John Horne, F.R.S., F.G.S., President, in the Chair.

The Annual Election of Fellows took place. The following were elected :—Arthur Robertson Cushny, William John Dundas, Robert Owen Morris, Thomas Stewart Patterson, B. D. Porritt, Alfred Henry Roberts, William Alexander Robertson, Alexander Scott, Alexander Ritchie Scott, William Wright Smith, David Alan Stevenson.

The following Communications were read :—

1. Lantern Demonstration of Colour Blindness; showing what the colour-blind see. By Charles R. Gibson.
2. On the Thermodynamics of Unstable States. By Professor W. Peddie.
3. On Hamilton's Principle and the Modified Function in Analytical Dynamics. By G. H. Livens, M.A. Communicated by Professor Whittaker. *Proc.*, vol. xxxix, pp. 113–119.

SIXTH ORDINARY MEETING.

Monday, May 5, 1919.

Dr John Horne, F.R.S., F.G.S., President, in the Chair.

The Council have awarded :—The Makdougall-Brisbane Prize for the Biennial Period 1916–1918 to Professor A. Anstruther Lawson for his memoirs on the Prothalli of *Tmesipteris Tannensis* and of *Psilotum*, published in the *Transactions* of the Society, together with previous papers on Cytology and on the Gametophytes of various Gymnosperms.
This Prize will be presented at the June Meeting.

The following Communications were read :—

1. Some Conditions influencing the Reaction-velocity of Sodium Nitrite on Blood. By Professor C. R. Marshall. *Proc.*, vol. xxxix, pp. 149–156.
2. On the Mode of Action of Metal Sols. By Professor C. R. Marshall. *Proc.*, vol. xxxix, pp. 143–148.
3. Factors of Circulants. By Professor W. H. Metzler. *Proc.*, vol. xxxix, pp. 41–47.
4. The Cooling of the Soil at Night. By Captain T. Bedford Franklin, B.A. Cantab. Communicated by The General Secretary. *Proc.*, vol. xxxix, pp. 120–136.
5. An Analysis of an Electron Transference Hypothesis of Chemical Valency and Combination. By John Marshall, M.A., B.Sc. Communicated by Professor W. Peddie. *Proc.*, vol. xxxix, pp. 209–233.

Dr William John Dundas, Mr Alfred H. Roberts, Professor Arthur Robertson Cushny, and Dr Thomas Stewart Patterson, signed the Roll, and were duly admitted Fellows of the Society.

SEVENTH ORDINARY MEETING.

Monday, June 2, 1919.

Dr John Horne, F.R.S., F.G.S., President, in the Chair.

The Makdougall-Brisbane Prize for the Biennial Period 1916–1918 was presented to Professor A. Anstruther Lawson (*in absentia*) for his memoirs on the Prothalli of *Tmesipteris Tannensis* and of *Psilotum*, published in the *Transactions* of the Society, together with previous papers on Cytology and on the Gametophytes of various Gymnosperms.

The grounds for the award of the Makdougall-Brisbane Prize to Professor A. Anstruther Lawson are to be found in his researches in three distinct lines. He first found his footing as an investigator in Cytology, and his results found publication in the *Botanical Gazette* and *Annals of Botany*, and later on the Royal Society of Edinburgh published two Memoirs on the behaviour of the Nucleus in division. There may be differences of opinion as to the ultimate verdict on the conclusions therein contained; but the exactitude of his methods, and the beauty of his preparations and his drawings entitle him to a statement of his views; a position which the Society has accepted by giving publicity to these works. A second line of inquiry has been upon the Gametophytes of some of the less common Gymnosperms. In six Memoirs, which are now extensively quoted in special treatises, he has traced the development and structure of the male and female prothallus in as many genera. They thus take their place as substantive contributions to learning. The third line of investigation has been taken up since Professor Lawson was appointed to the Chair in Sydney. It relates to the *Psilotaceæ*, native in New South Wales. He discovered the Gametophytes of both *Psilotum* and *Tmesipteris*. The detailed description of these, with extensive illustration, is now embodied in our *Transactions*. The Society will await with interest further results relating to the embryogeny of both. By such work the last remaining gap in knowledge of the Gametophyte generation in the Pteridophyta has been filled in owing to the activity of Professor Lawson, and other investigators at the Antipodes. The communication of these Memoirs to this Society is a singularly happy event, since they interweave so closely with the work of Dr Kidston and Professor Lang upon the plants of the Lower Devonian Period. Together these contributions are growing into a body of new knowledge of which any scientific society might be justly proud.

The following Communications were read:—

1. Obituary Notice of Sir James Russell. By Dr T. R. Ronaldson. *Proc.*, vol. xxxix, pp. 243–248.
2. On the Presence of Formic Acid in the Stinging Hairs of the Nettle. By Dr Leonard Dobbin. *Proc.*, vol. xxxix, pp. 137–142.
3. X-Ray Optics. Part I. By Dr R. A. Houstoun. *Proc.*, vol. xl.
4. On Pulsations of the Vertical Component of Terrestrial Magnetic Force. By Dr A. Crichton Mitchell. (*With Lantern Illustrations.*)
5. Exhibition of Samples of Encysted Wood, presented by Colonel R. A. Marr, Norfolk, Virginia, U.S.A.

Mr W. W. Smith signed the Roll, and was duly admitted a Fellow of the Society.

PROCEEDINGS OF THE STATUTORY GENERAL MEETING

Ending the 136th Session, 1918–1919.

At the Statutory Meeting of the Royal Society of Edinburgh, held in the Society's Lecture Room, 24 George Street, on Monday, October 27, 1919, at 4.30 p.m.,

Dr John Horne, F.R.S., F.G.S., President, in the Chair,

the Minutes of the last Statutory Meeting of October 28, 1918, were read, approved, and signed.

The Chairman nominated as Scrutineers, the Rev. R. S. Calderwood and Dr A. Morgan.

The Ballot for the Election of Office-Bearers and Members of Council was then taken.

The Secretary submitted the following Report :—

Since the last Report was submitted the war, which influenced in many ways the work of the Society, has come to an end; one important consequence of this is the return of Mr George Stewart, Librarian and Assistant Secretary, to his work in the Society. With the rearrangement of duties Miss Le Harivel, who has acted during the war as temporary Librarian and Assistant Secretary, has been officially appointed Assistant Librarian. Most of our activities are proceeding very much as during the war. The number of papers read at our meetings during Session 1918–1919 was 23, as compared with 25 the preceding year. Of these 13 have been, or are being, published in the *Proceedings* and 6 in the *Transactions*. Of the papers read 4 were in mathematics, 5 in physics, 2 in meteorology, 5 in chemistry, 3 in zoology, and 4 in botany. An address on Colour Blindness, with Lantern Demonstration, was given by Mr C. R. Gibson of Glasgow.

Last year the Society elected 11 new Fellows, and we lost by death 15 Ordinary Fellows and 4 Honorary Fellows.

The Makdougall-Brisbane Prize was awarded to Professor A. A. Lawson.

With the great increase in the cost of publication, and the loss the Society sustained some years ago by fire, it was evident to the Council that, with the ordinary output of papers and publications, there would be a serious deficit during the Session. At the beginning of the Session there was already £200 of debt to clear off incurred by the fire, and a careful estimate showed that the Council could only use £600 for publication purposes, and would be compelled to postpone the binding of serials and journals. These considerations induced the Council to approach the Chancellor of the Exchequer and ask for an increased annual grant. Before presenting their Memorandum the Council had a meeting with the Secretary for Scotland, and encouraged by the reception given by him, the Council prepared the following Memorandum :—

ROYAL SOCIETY OF EDINBURGH.

Application to the Treasury for an Increased Government Grant.

Memorandum.

The Royal Society of Edinburgh was founded by Royal Charter in 1783, in the reign of George III; *ad Statum illius partis Imperii nostri quæ Scotia vocatur accommodata.* According to the Charter the work of the Society was to include Mathematics, Physics, Chemistry, Natural History, Archæology, Philology, and Literature; but, in comparison with the scientific subjects specified, Literature has long occupied a subordinate position. The Society takes rank as the National Scientific Academy in Scotland, in the same manner as the French Academy of Sciences, the Royal Society of London, the Royal Irish Academy, the National Academy of Sciences of Washington, and other like Institutions in their respective countries. It has been, since its foundation, the centre of scientific activity for the whole of Scotland, and has included among its Presidents such distinguished men as Sir James Hall, Sir Walter Scott, the eighth Duke of Argyll, Sir David Brewster, Lord Kelvin, Sir William Turner, and Professor James Geikie.

The chief aim of the Society is the development of original research in Scotland. Its success in this respect has been remarkable. The *Transactions* and *Proceedings* of the Society have always contained papers of high scientific value, some of which have laid the foundations of new branches of science. In recent years there has been a large increase in its publications, especially in those relating to Natural Science.

From 1826 to 1909 the Society occupied rooms in the Royal Institution. These rooms being required for an extension of the Royal Scottish Academy, the Government purchased and equipped the building in George Street at present occupied by the Society, and agreed to allot an annual grant of £600 to assist its scientific work.

The other main source of revenue of the Society is derived from the contributions of its Fellows. These amount to about £890 per annum. The total income is about £2100. The expenditure may be arranged under three heads :—

(1) Publication of scientific researches.

(2) Upkeep of Library, which, with the exception of that belonging to the Royal Society of London, is the most complete library of scientific reference in the United Kingdom.

(3) Salaries and current expenses.

During the twelve years 1902–1914 the annual sum spent on the *Transactions* and *Proceedings* averaged £1050. Before 1906 there was a constant excess of payments over receipts; the balance being met by gifts and bequests from friends of the Society. From 1906 until the beginning of the war the Society was able, with the aid of its grant from the Government, to cope with the papers

presented to it for publication; but since 1914 printing charges have increased 125 per cent., and are not likely to diminish, so that the Society is no longer able to carry on this work efficiently. During the session ending September 30, 1918, the Council has been compelled to decline papers offered to the Society owing to lack of funds available for their publication. This is the more to be regretted since one result of the war has been to show the necessity of encouraging original scientific research in every possible way.

The Council, recognising this necessity, now beg to appeal for an increased grant from the Treasury to enable the Society to meet the enhanced demands made upon its resources. They calculate that an additional sum of £1000 a year will be required in the immediate future, and they therefore ask that the Treasury Grant be increased from £600 to £1600. In this connection it may be pointed out that the Royal Irish Academy, which occupies the same position in Ireland that the Royal Society of Edinburgh occupies in Scotland, receives an annual Government grant of the above amount (£1600). Including this grant, it has a total revenue of nearly £2500, of which about £470 is spent on the publication of scientific papers, and about £400 on literary researches and publications. With a smaller total revenue and a far smaller Government grant, the Royal Society of Edinburgh has expended a much larger amount on the publication of scientific researches. In order to maintain, and still more to improve, its position as a Society for publishing original scientific research in Scotland, an increased grant from the Treasury is essential.

Royal Society of Edinburgh,
22 George Street,
December 30, 1918.

This Memorandum along with a covering letter was sent to all our Ordinary and Honorary Fellows resident in London, also to all the Scottish Members of Parliament. Many cordial replies were received, and generous offers made to help the Society in presenting its case to the Chancellor of the Exchequer. In due course the Council received the following reply from the Chancellor of the Exchequer:—

COPY.

Treasury Chambers,
Whitehall, S.W., 1,
22nd March, 1919.

Dear Sir,—The Chancellor of the Exchequer desires me to express his regret that he has not been in a position to reply at an earlier date to your letter of the 21st January last, asking him to receive a deputation from the Royal Society of Edinburgh to urge the increase of the Government grant made to them from £600 to £1600 per annum.

Mr Chamberlain notes that this request is made on account of the increased cost of printing. Increased grants have not, however, been given on this ground in other similar cases.

As regards the comparison with the Royal Irish Academy, he must point out that the grant made to them is aid, not merely of their publication of scientific researches, but also of their literary activities, such as the publication of Irish manuscripts. It should further be borne in mind that your Society's existing grant is more than proportionate to that assigned to the English Royal Society for similar purposes.

Mr Chamberlain finds that whereas until 1907 the Government grant to the Royal Society of Edinburgh was only £300, and was given to cover the rent paid by them, it has since been increased to £600, although the Society has been housed free at a cost of £20,000 from funds provided by the late Board of Manufactures.

In these circumstances, and having regard to the present condition of national finance, Mr Chamberlain regrets that he cannot propose to Parliament an increase in the grant to the Society at the present time, though he would be ready to reconsider the question along with other similar claims when the financial situation is more favourable; and he fears that no useful purpose would be served by his consenting to receive a deputation on this subject.—Yours faithfully,

R. P. M. Gower.

The General Secretary,
Royal Society of Edinburgh.

The Council replied to this communication in the following terms:—

ROYAL SOCIETY OF EDINBURGH.

COPY.

22 George Street,
April 8, 1919.

Sir,—I am instructed by the Council of the Royal Society of Edinburgh to acknowledge the receipt of your letter dated March 22.

The Council regret that you are unable at the present time to propose an increase of the Government Grant made to the Society, and especially that you decline to receive a deputation on the subject.

I am directed by the Council to point out, in reply to the remarks in your letter about the Royal Irish Academy, that the Council fail to see any difference in principle between the publication of scientific and literary research.

With regard to the other point mentioned, the amount assigned to the Royal Society of London is £1000 for publication and £4000 for direct aid in scientific research. The Royal Society of London have, moreover, much larger invested funds to help in their publication of scientific work. The Council is, therefore, unable to understand how the existing amount granted to the Royal Society of Edinburgh is more than proportionate to that assigned to the English Royal Society for similar purposes. I am further instructed to emphasise the fact that the Society spends all its grant on research—publication being a necessary part of research—and scientific research is acknowledged to be one of the crying needs of the nation. In the meantime all the available funds (for this purpose) are exhausted, and valuable work cannot be published for lack of means.

It should be noted that previously to 1907 the Royal Society of Edinburgh was housed, rent free, in one of the finest buildings of the city, on a commanding site in Princes Street, and was provided with the present house in place of that of which they were then dispossessed.

I have the honour to be, sir, your obedient servant,

C. G. Knott,
General Secretary, R.S.E.

The Rt. Hon. Austen Chamberlain, M.P.,
Treasury Chambers, Whitehall, S.W.

Since it was evident that nothing could in the meantime be effected in the way proposed, one of our Members, resident in London, suggested that the Society should make an appeal to its Fellows to subscribe to a special fund to help the Society over its present difficulties, and at the same time sent £100 as a first contribution. Acting on this suggestion, the Council sent an appeal to all the Fellows of the Society in the following terms:—

ROYAL SOCIETY OF EDINBURGH.

22 George Street,
May 1919.

Urgent.

Dear Sir,—The Council of the Royal Society of Edinburgh made an Appeal last January to the Chancellor of the Exchequer for an increase in the annual grant, in order to enable the Society to publish scientific papers committed to its care. In his reply to our memorandum the Chancellor of the Exchequer referred to several of the important points, and concluded in these words:—"In these circumstances, and having regard to the present condition of national finance, Mr Chamberlain regrets that he cannot propose to Parliament an increase in the grant to the Society at the present time, though he would be ready to reconsider the question along with other similar claims when the financial situation is more favourable."

The great increase in the cost of publication is interfering seriously with the normal work of the Society, and the Council have been considering anxiously how best to meet the situation.

One of our Fellows, resident in London, recently gave a generous donation of £100 so as to help to relieve the financial difficulties in which we find ourselves, and suggested that other Fellows might be in a position to follow his example. The Council have resolved to approach the Fellows in regard to this matter, and have decided to ask for voluntary contributions towards a Special Subscription Fund. They trust that the response to this appeal will be such as to ensure the continued publication of scientific papers during the present session.—Yours very truly,

C. G. Knott.
General Secretary, R.S.E.

Appended is a statement of the amount of money immediately required to place the Society on a satisfactory basis:—

1. Printing and distributing papers still to be published this session	£180	0	0
2. Neill & Co.'s account, 1917—remaining portion of debt	200	0	0
3. Completing the serials got from enemy countries	450	0	0
4. Despatch of *Transactions* and *Proceedings* held up during war	60	0	0
	£890	0	0

It should be further noted that the Binding of Serials is very much in arrears, to meet which a large additional sum will be required.

By September 1919 the whole amount subscribed to the "Special Subscription Fund" was £775, 16s. 6d., and after a second appeal this was increased to £1072, 17s. 6d.

Owing to the generosity of many of its Fellows, the Society has now been able to clear off the debts of former years, and to meet our present expenses for publication. Of the whole sum subscribed to the Special Subscription Fund there remains in hand on October 27, 1919, the sum of £771, which will be carried over to ease the financial stress in meeting the expenses specially referred to in the Second Appeal, viz.:—Completing the Serials from Foreign Countries; the despatch of *Transactions* and *Proceedings* held up during the war, and the printing and distribution of the remaining papers which belong to the last session. It should be noted, however, that the general situation remains as before, viz. that in view of the continuing high prices the present income of the Society is not sufficient for it to continue to publish to nearly the same extent as heretofore the results of scientific research.

During the year the Society appointed two delegates, Sir E. Sharpey Schafer and Dr C. G. Knott, to the Conference of the International Association. The first meeting was held in Paris towards the end of November 1918, and the arrangements made at that time have resulted in the formation of two important International Unions, viz. that of Astronomy and Geophysics. Professor George Forbes and Mr M'Ewan were appointed delegates to the Union of Astronomy, and Dr Knott and Dr Crichton Mitchell to the Union of Geophysics.

Professor Lapworth and Professor Hudson Beare were chosen as representatives of the Society to the Watt Centenary Celebration held in Birmingham in September last.

The Treasurer in submitting his Report for the year compared the Income and Expenditure with that of the previous year, and called attention to the fact that the deficit of £301, 16s. 11d. on the year's working had been met by transferring that sum from the "Special Subscription Fund" to the "General Fund."

Dr E. M. Wedderburn moved the adoption of the Reports, and the reappointment of Messrs Lindsay, Jamieson & Haldane, C.A., as auditors of the accounts for the ensuing Session.

This was unanimously agreed to.

The Scrutineers reported that the Ballot Papers were in order, and that the following had been elected as Office-Bearers and Members of Council:—

Professor Frederick O. Bower, M.A., D.Sc., LL.D., F.R.S., F.L.S., President.

Professor G. A. Gibson, M.A., LL.D.,
Robert Kidston, LL.D., F.R.S., F.G.S.,
Professor D. Noël Paton, M.D., B.Sc., LL.D., F.R.C.P.E., F.R.S.,
Professor Arthur Robinson, M.D., M.R.C.S.,
Sir George A. Berry, M.B., C.M., LL.D., F.R.C.S.E.,
Professor William Peddie, D.Sc.,
— Vice-Presidents.

Cargill G. Knott, D.Sc., LL.D., General Secretary.

Professor E. T. Whittaker, Sc.D., F.R.S.,
J. H. Ashworth, D.Sc., F.R.S.,
— Secretaries to Ordinary Meetings.

James Currie, M.A., LL.D., Treasurer.

A. Crichton Mitchell, D.Sc., Hon. D.Sc. (Geneva), Curator of Library and Museum.

ORDINARY MEMBERS OF COUNCIL.

Professor P. T. Herring, M.D., F.R.C.P.E.
Professor T. J. Jehu, M.A., M.D., F.G.S.
Alexander Lauder, D.Sc., F.I.C.
The Hon. Lord Guthrie, LL.D.
Professor R. A. Sampson, M.A., D.Sc., F.R.S.
Professor J. Lorrain Smith, M.A., M.D., F.R.S
W. A. Tait, D Sc., M.Inst.C.E.
Surgeon-General W. B. Bannerman, C.S.I., I.M.S., M.D., D.Sc.
Henry Moubray Cadell, of Grange, B.Sc.
Professor Arthur Robertson Cushny, M.A., M.D., LL.D., F.R.S.
Principal Sir James Alfred Ewing, K.C.B., M.A., B.Sc., LL.D., M.Inst.C.E., F.R.S.
George James Lidstone, F.F.A., F.I.A.

SOCIETY'S REPRESENTATIVE ON GEORGE HERIOT'S TRUST.

W. A. Tait, D.Sc., M.Inst.C.E.

The Chairman, in the name of the Society, thanked the Scrutineers for their services.

THE KEITH, MAKDOUGALL-BRISBANE, NEILL, GUNNING VICTORIA JUBILEE, AND JAMES SCOTT PRIZES.

The above Prizes will be awarded by the Council in the following manner:—

I. KEITH PRIZE.

The Keith Prize, consisting of a Gold Medal and from £40 to £50 in Money, will be awarded in the Session 1921–1922 for the "best communication on a scientific subject, communicated,* in the first instance, to the Royal Society of Edinburgh during the Sessions 1919–1920 and 1920–1921." Preference will be given to a paper containing a discovery.

II. MAKDOUGALL-BRISBANE PRIZE.

This Prize is to be awarded biennially by the Council of the Royal Society of Edinburgh to such person, for such purposes, for such objects, and in such manner as shall appear to them the most conducive to the promotion of the interests of science; with the *proviso* that the Council shall not be compelled to award the Prize unless there shall be some individual engaged in scientific pursuit, or some paper written on a scientific subject, or some discovery in science made during the biennial period, of sufficient merit or importance in the opinion of the Council to be entitled to the Prize.

1. The Prize, consisting of a Gold Medal and a sum of Money, will be awarded before the close of the Session 1920–1921, for an Essay or Paper having reference to any branch of scientific inquiry, whether Material or Mental.

2. Competing Essays to be addressed to the Secretary of the Society, and transmitted not later than 8th July 1920.

3. The Competition is open to all men of science.

4. The Essays may be either anonymous or otherwise. In the former case, they must be distinguished by mottoes, with corresponding sealed billets, superscribed with the same motto, and containing the name of the Author.

5. The Council impose no restriction as to the length of the Essays, which may be, at the discretion of the Council, read at the Ordinary Meetings of the Society. They wish also to leave the property and free disposal of the manuscripts to the Authors; a copy, however, being deposited in the Archives of the Society, unless the paper shall be published in the Transactions.

* For the purposes of this award the word "communicated" shall be understood to mean the date on which the manuscript of a paper is received in its final form for printing, as recorded by the General Secretary or other responsible official.

6. In awarding the Prize, the Council will also take into consideration any scientific papers presented * to the Society during the Sessions 1916–17, 1917–18, whether they may have been given in with a view to the prize or not.

III. NEILL PRIZE.

The Council of the Royal Society of Edinburgh having received the bequest of the late Dr Patrick Neill of the sum of £500, for the purpose of "the interest thereof being applied in furnishing a Medal or other reward every second or third year to any distinguished Scottish Naturalist, according as such Medal or reward shall be voted by the Council of the said Society," hereby intimate:

1. The Neill Prize, consisting of a Gold Medal and a sum of Money, will be awarded during the Session 1921–1922.

2. The Prize will be given for a Paper of distinguished merit, on a subject of Natural History, by a Scottish Naturalist, which shall have been presented * to the Society during the two years preceding the fourth Monday in October 1921,—or failing presentation of a paper sufficiently meritorious, it will be awarded for a work or publication by some distinguished Scottish Naturalist, on some branch of Natural History, bearing date within five years of the time of award.

IV. GUNNING VICTORIA JUBILEE PRIZE.

This Prize, founded in the year 1887 by Dr R. H. Gunning, is to be awarded quadrennially by the Council of the Royal Society of Edinburgh, in recognition of original work in Physics, Chemistry, or Pure or Applied Mathematics.

Evidence of such work may be afforded either by a Paper presented to the Society, or by a Paper on one of the above subjects, or some discovery in them elsewhere communicated or made, which the Council may consider to be deserving of the Prize.

The Prize consists of a sum of money, and is open to men of science resident in or connected with Scotland. The first award was made in the year 1887. The next award will be made in 1920.

In accordance with the wish of the Donor, the Council of the Society may on fit occasions award the Prize for work of a definite kind to be undertaken during the three succeeding years by a scientific man of recognised ability.

V. JAMES SCOTT PRIZE.

This Prize, founded in the year 1918 by the Trustees of the James Scott Bequest, is to be awarded triennially, or at such intervals as the Council of the Royal Society of Edinburgh may decide, "for a lecture or essay on the fundamental concepts of Natural Philosophy."

The first award will be in the year 1921.

* For the purposes of this award the word "presented" shall be understood to mean the date on which the manuscript of a paper is received in its final form for printing, as recorded by the General Secretary or other responsible official.

RESOLUTIONS OF COUNCIL IN REGARD TO THE MODE OF AWARDING PRIZES.

(*See Minutes of Meeting of January* 18, 1915.)

I. With regard to the Keith and Makdougall-Brisbane Prizes, which are open to all Sciences, the mode of award will be as follows :—

1. Papers or essays to be considered shall be arranged in two groups, A and B, —Group A to include Astronomy, Chemistry, Mathematics, Metallurgy, Meteorology and Physics; Group B to include Anatomy, Anthropology, Botany, Geology, Pathology, Physiology, and Zoology.
2. These two Prizes shall be awarded to each group in alternate biennial periods, provided papers worthy of recommendation have been communicated to the Society.
3. Prior to the adjudication the Council shall appoint, in the first instance, a Committee composed of representatives of the group of Sciences which did not receive the award in the immediately preceding period. The Committee shall consider the Papers which come within their group of Sciences, and report in due course to the Council.
4. In the event of the aforesaid Committee reporting that within their group of subjects there is, in their opinion, no paper worthy of being recommended for the award, the Council, on accepting this report, shall appoint a Committee representative of the alternate group to consider papers coming within their group and to report accordingly.
5. Papers to be considered by the Committees shall fall within the period dating from the last award in groups A and B respectively.

II. With regard to the Neill Prize, the term "Naturalist" shall be understood to include any student in the Sciences composing group B, namely, Anatomy, Anthropology, Botany, Geology, Pathology, Physiology, Zoology.

AWARDS OF THE KEITH, MAKDOUGALL-BRISBANE, NEILL, AND GUNNING PRIZES.

I. KEITH PRIZE.

1ST BIENNIAL PERIOD, 1827–29.—Dr BREWSTER, for his papers "on his Discovery of Two New Immiscible Fluids in the Cavities of certain Minerals," published in the Transactions of the Society.

2ND BIENNIAL PERIOD, 1829–31.—Dr BREWSTER, for his paper "on a New Analysis of Solar Light," published in the Transactions of the Society.

3RD BIENNIAL PERIOD, 1831–33.—THOMAS GRAHAM, Esq., for his paper "on the Law of the Diffusion of Gases," published in the Transactions of the Society.

4TH BIENNIAL PERIOD, 1833–35.—Professor J. D. FORBES, for his paper "on the Refraction and Polarization of Heat," published in the Transactions of the Society.

5TH BIENNIAL PERIOD, 1835–37.—JOHN SCOTT RUSSELL, Esq., for his researches "on Hydrodynamics," published in the Transactions of the Society.

6TH BIENNIAL PERIOD, 1837–39.—Mr JOHN SHAW, for his experiments "on the Development and Growth of the Salmon," published in the Transactions of the Society.

7TH BIENNIAL PERIOD, 1839–41.—Not awarded.

8TH BIENNIAL PERIOD, 1841–43.—Professor JAMES DAVID FORBES, for his papers "on Glaciers," published in the Proceedings of the Society.

9TH BIENNIAL PERIOD, 1843–45.—Not awarded.

10TH BIENNIAL PERIOD, 1845–47.—General Sir THOMAS BRISBANE, Bart., for the Makerstoun Observations on Magnetic Phenomena, made at his expense, and published in the Transactions of the Society.

11TH BIENNIAL PERIOD, 1847–49.—Not awarded.

12TH BIENNIAL PERIOD, 1849–51.—Professor KELLAND, for his papers "on General Differentiation, including his more recent Communication on a process of the Differential Calculus, and its application to the solution of certain Differential Equations," published in the Transactions of the Society.

13TH BIENNIAL PERIOD, 1851–53.—W. J. MACQUORN RANKINE, Esq., for his series of papers "on the Mechanical Action of Heat," published in the Transactions of the Society.

14TH BIENNIAL PERIOD, 1853–55.—Dr THOMAS ANDERSON, for his papers "on the Crystalline Constituents of Opium, and on the Products of the Destructive Distillation of Animal Substances," published in the Transactions of the Society.

15TH BIENNIAL PERIOD, 1855–57.—Professor BOOLE, for his Memoir "on the Application of the Theory of Probabilities to Questions of the Combination of Testimonies and Judgments," published in the Transactions of the Society.

16TH BIENNIAL PERIOD, 1857–59.—Not awarded.

17TH BIENNIAL PERIOD, 1859–61.—JOHN ALLAN BROUN, Esq., F.R.S., Director of the Trevandrum Observatory, for his papers "on the Horizontal Force of the Earth's Magnetism, on the Correction of the Bifilar Magnetometer, and on Terrestrial Magnetism generally," published in the Transactions of the Society.

18TH BIENNIAL PERIOD, 1861–63.—Professor WILLIAM THOMSON, of the University of Glasgow, for his Communication "on some Kinematical and Dynamical Theorems."

19TH BIENNIAL PERIOD, 1863–65.—Principal FORBES, St Andrews, for his "Experimental Inquiry into the Laws of Conduction of Heat in Iron Bars," published in the Transactions of the Society.

20TH BIENNIAL PERIOD, 1865–67.—Professor C. PIAZZI SMYTH, for his paper "on Recent Measures at the Great Pyramid," published in the Transactions of the Society.

21ST BIENNIAL PERIOD, 1867–69.—Professor P. G. TAIT, for his paper "on the Rotation of a Rigid Body about a Fixed Point," published in the Transactions of the Society.

22ND BIENNIAL PERIOD, 1869–71.—Professor CLERK MAXWELL, for his paper "on Figures, Frames, and Diagrams of Forces," published in the Transactions of the Society.

23RD BIENNIAL PERIOD, 1871-73.—Professor P. G. TAIT, for his paper entitled "First Approximation to a Thermo-electric Diagram," published in the Transactions of the Society.

24TH BIENNIAL PERIOD, 1873-1875.—Professor CRUM BROWN, for his Researches "on the Sense of Rotation, and on the Anatomical Relations of the Semicircular Canals of the Internal Ear."

25TH BIENNIAL PERIOD, 1875-77.—Professor M. FORSTER HEDDLE, for his papers "on the Rhombohedral Carbonates," and "on the Felspars of Scotland," published in the Transactions of the Society.

26TH BIENNIAL PERIOD, 1877-79.—Professor H. C. FLEEMING JENKIN, for his paper "on the Application of Graphic Methods to the Determination of the Efficiency of Machinery," published in the Transactions of the Society; Part II having appeared in the volume for 1877-78.

27TH BIENNIAL PERIOD, 1879-81.—Professor GEORGE CHRYSTAL, for his paper "on the Differential Telephone," published in the Transactions of the Society.

28TH BIENNIAL PERIOD, 1881-83.—THOMAS MUIR, Esq., LL.D., for his "Researches into the Theory of Determinants and Continued Fractions," published in the Proceedings of the Society.

29TH BIENNIAL PERIOD, 1883-85.—JOHN AITKEN, Esq., for his paper "on the Formation of Small Clear Spaces in Dusty Air," and for previous papers on Atmospheric Phenomena, published in the Transactions of the Society.

30TH BIENNIAL PERIOD, 1885-87.—JOHN YOUNG BUCHANAN, Esq., for a series of communications, extending over several years, on subjects connected with Ocean Circulation, Compressibility of Glass, etc.; two of which, viz., "On Ice and Brines," and "On the Distribution of Temperature in the Antarctic Ocean," have been published in the Proceedings of the Society.

31ST BIENNIAL PERIOD, 1887-89.—Professor E. A. LETTS, for his papers on the Organic Compounds of Phosphorus, published in the Transactions of the Society.

32ND BIENNIAL PERIOD, 1889-91.—R. T. OMOND, Esq., for his contributions to Meteorological Science, many of which are contained in vol. xxxiv of the Society's Transactions.

33RD BIENNIAL PERIOD, 1891-93.—Professor THOMAS R. FRASER, F.R.S., for his papers on *Strophanthus hispidus*, Strophanthin, and Strophanthidin, read to the Society in February and June 1889 and in December 1891, and printed in vols. xxxv, xxxvi, and xxxvii of the Society's Transactions.

34TH BIENNIAL PERIOD, 1893-95.—Dr CARGILL G. KNOTT, for his papers on the Strains produced by Magnetism in Iron and in Nickel, which have appeared in the Transactions and Proceedings of the Society.

35TH BIENNIAL PERIOD, 1895-97.—Dr THOMAS MUIR, for his continued communications on Determinants and Allied Questions.

36TH BIENNIAL PERIOD, 1897-99.—Dr JAMES BURGESS, for his paper "on the Definite Integral $\frac{2}{\sqrt{\pi}}\int_0^t \epsilon^{-t^2}dt$, with extended Tables of Values," printed in vol. xxxix of the Transactions of the Society.

37TH BIENNIAL PERIOD, 1899-1901.—Dr HUGH MARSHALL, for his discovery of the Persulphates, and for his Communications on the Properties and Reactions of these Salts, published in the Proceedings of the Society.

38TH BIENNIAL PERIOD, 1901-03.—Sir WILLIAM TURNER, K.C.B., LL.D., F.R.S., etc., for his memoirs entitled "A Contribution to the Craniology of the People of Scotland," published in the Transactions of the Society, and for his "Contributions to the Craniology of the People of the Empire of India," Parts I, II, likewise published in the Transactions of the Society.

39TH BIENNIAL PERIOD, 1903-05.—THOMAS H. BRYCE, M.A., M.D., for his two papers on "The Histology of the Blood of the Larva of *Lepidosiren paradoxa*," published in the Transactions of the Society within the period.

40TH BIENNIAL PERIOD, 1905-07.—ALEXANDER BRUCE, M.A., M.D., F.R.C.P.E., for his paper entitled "Distribution of the Cells in the Intermedio-Lateral Tract of the Spinal Cord," published in the Transactions of the Society within the period.

41ST BIENNIAL PERIOD, 1907-09.—WHEELTON HIND, M.D., B.S., F.R.C.S., F.G.S., for a paper published in the Transactions of the Society, "On the Lamellibranch and Gasteropod Fauna found in the Millstone Grit of Scotland."

42ND BIENNIAL PERIOD, 1909-11.—Professor ALEXANDER SMITH, B.Sc., Ph.D., of New York, for his researches upon "Sulphur" and upon "Vapour Pressure," appearing in the Proceedings of the Society.

43RD BIENNIAL PERIOD, 1911–1913.—JAMES RUSSELL, Esq., for his series of investigations relating to magnetic phenomena in metals and the molecular theory of magnetism, the results of which have been published in the Proceedings and Transactions of the Society, the last paper having been issued within the period.

44TH BIENNIAL PERIOD, 1913–15.—JAMES HARTLEY ASHWORTH, D.Sc., for his papers on "Larvæ of Lingula and Pelagodiscus," and on "Sclerocheilus," published in the Transactions of the Society, and for other papers on the Morphology and Histology of Polychæta.

45TH BIENNIAL PERIOD, 1915–17.—ROBERT C. MOSSMAN, for his work on the Meteorology of the Antarctic Regions, which originated with the important series of observations made by him during the voyage of the "Scotia" (1902–1904), and includes his paper "On a Sea-Saw of Barometric Pressure, Temperature, and Wind Velocity between the Weddell Sea and the Ross Sea," published in the Proceedings of the Society.

II. MAKDOUGALL-BRISBANE PRIZE.

1ST BIENNIAL PERIOD, 1859.—Sir RODERICK IMPEY MURCHISON, on account of his Contributions to the Geology of Scotland.

2ND BIENNIAL PERIOD, 1860–62.—WILLIAM SELLER, M.D., F.R.C.P.E., for his "Memoir of the Life and Writings of Dr Robert Whytt," published in the Transactions of the Society.

3RD BIENNIAL PERIOD, 1862–64.—JOHN DENIS MACDONALD, Esq., R.N., F.R.S., Surgeon of H.M.S. "Icarus," for his paper "on the Representative Relationships of the Fixed and Free Tunicata, regarded as Two Sub-classes of equivalent value; with some General Remarks on their Morphology," published in the Transactions of the Society.

4TH BIENNIAL PERIOD, 1864–66.—Not awarded.

5TH BIENNIAL PERIOD, 1866–68.—Dr ALEXANDER CRUM BROWN and Dr THOMAS RICHARD FRASER, for their conjoint paper "on the Connection between Chemical Constitution and Physiological Action," published in the Transactions of the Society.

6TH BIENNIAL PERIOD, 1868–70.—Not awarded.

7TH BIENNIAL PERIOD, 1870–72.—GEORGE JAMES ALLMAN, M.D., F.R.S., Emeritus Professor of Natural History, for his paper "on the Homological Relations of the Cœlenterata," published in the Transactions, which forms a leading chapter of his Monograph of Gymnoblastic or Tubularian Hydroids—since published.

8TH BIENNIAL PERIOD, 1872–74.—Professor LISTER, for his paper "on the Germ Theory of Putrefaction and the Fermentive Changes," communicated to the Society, 7th April 1873.

9TH BIENNIAL PERIOD, 1874–76.—ALEXANDER BUCHAN, A.M., for his paper "on the Diurnal Oscillation of the Barometer," published in the Transactions of the Society.

10TH BIENNIAL PERIOD, 1876–78.—Professor ARCHIBALD GEIKIE, for his paper "on the Old Red Sandstone of Western Europe," published in the Transactions of the Society.

11TH BIENNIAL PERIOD, 1878–80.—Professor PIAZZI SMYTH, Astronomer-Royal for Scotland, for his paper "on the Solar Spectrum in 1877–78, with some Practical Idea of its probable Temperature of Origination," published in the Transactions of the Society.

12TH BIENNIAL PERIOD, 1880–82.—Professor JAMES GEIKIE, for his "Contributions to the Geology of the North-West of Europe," including his paper "on the Geology of the Faroes," published in the Transactions of the Society.

13TH BIENNIAL PERIOD, 1882–84.—EDWARD SANG, Esq., LL.D., for his paper "on the Need of Decimal Subdivisions in Astronomy and Navigation, and on Tables requisite therefor," and generally for his Recalculations of Logarithms both of Numbers and Trigonometrical Ratios,—the former communication being published in the Proceedings of the Society.

14TH BIENNIAL PERIOD, 1884–86.—JOHN MURRAY, Esq., LL.D., for his papers "On the Drainage Areas of Continents, and Ocean Deposits," "The Rainfall of the Globe, and Discharge of Rivers," "The Height of the Land and Depth of the Ocean," and "The Distribution of Temperature in the Scottish Lochs as affected by the Wind."

15TH BIENNIAL PERIOD, 1886–88.—ARCHIBALD GEIKIE, Esq., LL.D., for numerous Communications, especially that entitled "History of Volcanic Action during the Tertiary Period in the British Isles," published in the Transactions of the Society.

16TH BIENNIAL PERIOD, 1889–90.—Dr LUDWIG BECKER, for his paper on "The Solar Spectrum at Medium and Low Altitudes," printed in vol. xxxvi, Part I, of the Society's Transactions.

17TH BIENNIAL PERIOD, 1890–92.—HUGH ROBERT MILL, Esq., D.Sc., for his papers on "The Physical Conditions of the Clyde Sea Area," Part I being already published in vol. xxxvi of the Society's Transactions.

18TH BIENNIAL PERIOD, 1892–94.—Professor JAMES WALKER, D.Sc., Ph.D., for his work on Physical Chemistry, part of which has been published in the Proceedings of the Society, vol. xx, pp. 255–263. In making this award, the Council took into consideration the work done by Professor Walker along with Professor Crum Brown on the Electrolytic Synthesis of Dibasic Acids, published in the Transactions of the Society.

19TH BIENNIAL PERIOD, 1894–96.—Professor JOHN G. M'KENDRICK, for numerous Physiological papers, especially in connection with Sound, many of which have appeared in the Society's publications.

20TH BIENNIAL PERIOD, 1896–98.—Dr WILLIAM PEDDIE, for his papers on the Torsional Rigidity of Wires.

21ST BIENNIAL PERIOD, 1898–1900.—Dr RAMSAY H. TRAQUAIR, for his paper entitled "Report on Fossil Fishes collected by the Geological Survey in the Upper Silurian Rocks of Scotland," printed in vol. xxxix of the Transactions of the Society.

22ND BIENNIAL PERIOD, 1900–02.—Dr ARTHUR T. MASTERMAN, for his paper entitled "The Early Development of *Cribrella oculata* (Forbes), with remarks on Echinoderm Development," printed in vol. xl of the Transactions of the Society.

23RD BIENNIAL PERIOD, 1902–04.—Mr JOHN DOUGALL, M.A., for his paper on "An Analytical Theory of the Equilibrium of an Isotropic Elastic Plate," published in vol. xli of the Transactions of the Society.

24TH BIENNIAL PERIOD, 1904–06.—JACOB E. HALM, Ph.D., for his two papers entitled "Spectroscopic Observations of the Rotation of the Sun," and "Some Further Results obtained with the Spectroheliometer," and for other astronomical and mathematical papers published in the Transactions and Proceedings of the Society within the period.

25TH BIENNIAL PERIOD, 1906–08.—D. T. GWYNNE-VAUGHAN, M.A., F.L.S., for his papers, 1st, "On the Fossil Osmundaceæ," and 2nd, "On the Origin of the Adaxially-curved Leaf-trace in the Filicales," communicated by him conjointly with Dr R. Kidston.

26TH BIENNIAL PERIOD, 1908–10.—ERNEST MACLAGAN WEDDERBURN, M.A., LL.B., for his series of papers bearing upon "The Temperature Distribution in Fresh-water Lochs," and especially upon "The Temperature Seiche."

27TH BIENNIAL PERIOD, 1910–12.—JOHN BROWNLEE, M.A., M.D., D.Sc., for his contributions to the Theory of Mendelian Distributions and cognate subjects, published in the Proceedings of the Society within and prior to the prescribed period.

28TH BIENNIAL PERIOD, 1912–14.—Professor C. R. MARSHALL, M.D., M.A., for his studies "On the Pharmacological Action of Tetra-alkyl-ammonium Compounds."

29TH BIENNIAL PERIOD, 1914–16.—ROBERT ALEXANDER HOUSTOUN, Ph.D., D.Sc., for his series of papers on "The Absorption of Light by Inorganic Salts," published in the Proceedings of the Society.

30TH BIENNIAL PERIOD, 1916–18.—Professor A. ANSTRUTHER LAWSON for his Memoirs on "The Prothalli of *Tmesipteris Tannensis* and of *Psilotum*," published in the *Transactions* of the Society, together with previous papers on Cytology and on The Gametophytes of various Gymnosperms.

III. THE NEILL PRIZE.

1ST TRIENNIAL PERIOD, 1856–59.—Dr W. LAUDER LINDSAY, for his paper "on the Spermogones and Pycnides of Filamentous, Fruticulose, and Foliaceous Lichens," published in the Transactions of the Society.

2ND TRIENNIAL PERIOD, 1859–61.—ROBERT KAYE GREVILLE, LL.D., for his contributions to Scottish Natural History, more especially in the department of Cryptogamic Botany, including his recent papers on Diatomaceæ.

3RD TRIENNIAL PERIOD, 1862–65.—ANDREW CROMBIE RAMSAY, F.R.S., Professor of Geology in the Government School of Mines, and Local Director of the Geological Survey of Great Britain, for his various works and memoirs published during the last five years, in which he has applied the large experience acquired by him in the Direction of the arduous work of the Geological Survey of Great Britain to the elucidation of important questions bearing on Geological Science.

4TH TRIENNIAL PERIOD, 1865–68.—Dr WILLIAM CARMICHAEL M'INTOSH, for his paper "on the Structure of the British Nemerteans, and on some New British Annelids," published in the Transactions of the Society.

5TH TRIENNIAL PERIOD, 1868–71.—Professor WILLIAM TURNER, for his papers "on the Great Finner Whale; and on the Gravid Uterus, and the Arrangement of the Fœtal Membranes in the Cetacea," published in the Transactions of the Society.

6TH TRIENNIAL PERIOD, 1871–74.—CHARLES WILLIAM PEACH, Esq., for his Contributions to Scottish Zoology and Geology, and for his recent contributions to Fossil Botany.

7TH TRIENNIAL PERIOD, 1874–77.—Dr RAMSAY H. TRAQUAIR, for his paper "on the Structure and Affinities of *Tristichopterus alatus* (Egerton)," published in the Transactions of the Society, and also for his contributions to the Knowledge of the Structure of Recent and Fossil Fishes.

8TH TRIENNIAL PERIOD, 1877–80.—JOHN MURRAY, Esq., for his paper "on the Structure and Origin of Coral Reefs and Islands," published (in abstract) in the Proceedings of the Society.

9TH TRIENNIAL PERIOD, 1880–83.—Professor HERDMAN, for his papers "on the Tunicata," published in the Proceedings and Transactions of the Society.

10TH TRIENNIAL PERIOD, 1883–86.—B. N. PEACH, Esq., for his Contributions to the Geology and Palæontology of Scotland, published in the Transactions of the Society.

11TH TRIENNIAL PERIOD, 1886–89.—ROBERT KIDSTON, Esq., for his Researches in Fossil Botany, published in the Transactions of the Society.

12TH TRIENNIAL PERIOD, 1889–92.—JOHN HORNE, Esq., F.G.S., for his Investigations into the Geological Structure and Petrology of the North-West Highlands.

13TH TRIENNIAL PERIOD, 1892–95.—ROBERT IRVINE, Esq., for his papers on the Action of Organisms in the Secretion of Carbonate of Lime and Silica, and on the solution of these substances in Organic Juices. These are printed in the Society's Transactions and Proceedings.

14TH TRIENNIAL PERIOD, 1895–98.—Professor COSSAR EWART, for his recent Investigations connected with Telegony.

15TH TRIENNIAL PERIOD, 1898–1901.—Dr JOHN S. FLETT, for his papers entitled "The Old Red Sandstone of the Orkneys" and "The Trap Dykes of the Orkneys," printed in vol. xxxix of the Transactions of the Society.

16TH TRIENNIAL PERIOD, 1901–04.—Professor J. GRAHAM KERR, M.A., for his Researches on *Lepidosiren paradoxa*, published in the Philosophical Transactions of the Royal Society, London.

17TH TRIENNIAL PERIOD, 1904–07.—FRANK J. COLE, B.Sc., for his paper entitled "A Monograph on the General Morphology of the Myxinoid Fishes, based on a Study of Myxine," published in the Transactions of the Society, regard being also paid to Mr Cole's other valuable contributions to the Anatomy and Morphology of Fishes.

1ST BIENNIAL PERIOD, 1907–09.—FRANCIS J. LEWIS, M.Sc., F.L.S., for his papers in the Society's Transactions "On the Plant Remains of the Scottish Peat Mosses."

2ND BIENNIAL PERIOD, 1909–11.—JAMES MURRAY, Esq., for his paper on "Scottish Rotifers collected by the Lake Survey (Supplement)," and other papers on the "Rotifera" and "Tardigrada," which appeared in the Transactions of the Society—(this Prize was awarded after consideration of the papers received within the five years prior to the time of award: see Neill Prize Regulations).

3RD BIENNIAL PERIOD, 1911–13.—Dr W. S. BRUCE, in recognition of the scientific results of his Arctic and Antarctic explorations.

4TH BIENNIAL PERIOD, 1913–15.—ROBERT CAMPBELL, D.Sc., for his paper on "The Upper Cambrian Rocks at Craigeven Bay, Stonehaven," and "Downtonian and Old Red Sandstone Rocks of Kincardineshire," published in the Transactions of the Society.

5TH BIENNIAL PERIOD, 1915–17.—W. H. LANG, F.R.S., M.B., D.Sc., for his paper in conjunction with Dr R. KIDSTON, F.R.S., on *Rhynia Gwynne-Vaughani*, Kidston and Lang, published in the Transactions of the Society, and for his previous investigations on Pteridophytes and Cycads.

IV. GUNNING VICTORIA JUBILEE PRIZE.

1ST TRIENNIAL PERIOD, 1884–87.—Sir WILLIAM THOMSON, Pres. R.S.E., F.R.S., for a remarkable series of papers "on Hydrokinetics," especially on Waves and Vortices, which have been communicated to the Society.

2ND TRIENNIAL PERIOD, 1887–90.—Professor P. G. TAIT, Sec. R.S.E., for his work in connection with the "Challenger" Expedition, and his other Researches in Physical Science.

3RD TRIENNIAL PERIOD, 1890–93.—ALEXANDER BUCHAN, Esq., LL.D., for his varied, extensive, and extremely important Contributions to Meteorology, many of which have appeared in the Society's publications.

4th Triennial Period, 1893-96.—John Aitken, Esq., for his brilliant Investigations in Physics, especially in connection with the Formation and Condensation of Aqueous Vapour.

1st Quadrennial Period, 1896-1900.—Dr T. D. Anderson, for his discoveries of New and Variable Stars.

2nd Quadrennial Period, 1900-04.—Sir James Dewar, LL.D., D.C.L., F.R.S., etc., for his researches on the Liquefaction of Gases, extending over the last quarter of a century, and on the Chemical and Physical Properties of Substances at Low Temperatures: his earliest papers being published in the Transactions and Proceedings of the Society.

3rd Quadrennial Period, 1904-08.—Professor George Chrystal, M.A., LL.D., for a series of papers on "Seiches," including "The Hydrodynamical Theory and Experimental Investigations of the Seiche Phenomena of Certain Scottish Lakes."

4th Quadrennial Period, 1908-12.—Professor J. Norman Collie, Ph.D., F.R.S., for his distinguished contributions to Chemistry, Organic and Inorganic, during twenty-seven years, including his work upon Neon and other rare gases. Professor Collie's early papers were contributed to the Transactions of the Society.

5th Quadrennial Period, 1912-16.—Sir Thos. Muir, C.M.G., LL.D., F.R.S., for his series of Memoirs upon "The Theory and History of Determinants and Allied Forms," published in the Transactions and Proceedings of the Society between the years 1872 and 1915.

ABSTRACT

OF

THE ACCOUNTS OF JAMES CURRIE, ESQ., LL.D.

As Treasurer of the Royal Society of Edinburgh.

SESSION 1918–1919.

I. ACCOUNT OF THE GENERAL FUND.

CHARGE.

1. Arrears of Contributions at 30th September 1918		£96 12 0
2. Contributions for present Session :—		
1. 194 Fellows at £2, 2s. each	£407 8 0	
86 Fellows at £3, 3s. each	270 18 0	
	£678 6 0	
2. Fees of Admission and Contributions of twelve new Fellows at £4, 4s. each	50 8 0	
3. Commutation Fee in lieu of future Contributions of one Fellow	25 4 0	
		753 18 0
3. Contribution for 1919–1920 paid in advance		2 2 0
4. Interest received—		
Interest on £7830 five per cent. War Loan, 1929–47, Untaxed	£391 10 0	
Annuity from Edinburgh and District Water Trust, less Tax, £15, 15s.	36 15 0	
Interest on Deposit Receipts	28 4 11	
		456 9 11
5. Transactions and Proceedings		58 5 11
6. Annual Grant from Government		600 0 0
7. Income Tax repaid for year to 5th April 1919		15 15 0
8. Receipts from Sale of Napier Tercentenary Memorial Volume		26 12 9
Amount of the Charge		**£2009 15 7**

DISCHARGE.

1. TAXES, INSURANCE, COAL AND LIGHTING :—		
Inhabited House Duty	£0 6 3	
Insurance	24 16 2	
Coal, etc., to 4th September 1919	34 9 8	
Gas to 12th May 1919	3 1 0	
Electric Light to 19th September 1919	5 6 11	
Water, 1918–19	4 4 0	
		£72 4 0
2. SALARIES :—		
General Secretary, 1918–19	£100 0 0	
Librarian and Assistant Secretary	145 0 0	
Assistant Librarian	104 0 0	
Office Keeper	94 10 0	
Treasurer's Clerk	35 0 0	
		478 10 0
Carry forward		£550 14 0

Brought forward		.	£550 14 0
3. Expenses of Transactions :—			
Neill & Co., Ltd., Printers		£370 10 2	
Hislop & Day, Ltd., Engravers		39 7 11	
Orrock & Son, Bookbinders		39 13 9	
M'Farlane & Erskine, Lithographers		29 17 6	
Alex. S. Huth, Lithographer		19 16 0	
		£499 5 4	
Less—			
Carnegie Trustees — Grants towards Messrs Fisher's, Thompson's, Robinson's, and Waterston's Papers	£139 4 0		
Royal Society, London—Grants towards Messrs Fisher's and Thompson's Papers	25 0 0		
Private Contribution towards Mr R. A. Fisher's Paper	30 0 0		
		194 4 0	
			305 1 4
4. Expenses of Proceedings :—			
Neill & Co., Ltd., Printers		£578 0 9	
Hislop & Day, Ltd., Engravers		14 9 0	
			592 9 9
5. Books, Periodicals, Newspapers, etc. :—			
James Thin, Bookseller		£184 9 5	
R. Grant & Son, Booksellers		7 3 2	
W. Green & Son, Ltd., Booksellers		1 4 3	
International Catalogue of Scientific Literature		17 0 0	
Robertson & Scott, News Agents		7 10 5	
Ray Society, Subscription		1 1 0	
Palæontographical Society, Do.		1 1 0	
Board of Scientific Societies, London, Donation		5 0 0	
Williams & Norgate		1 8 0	
Arthur F. Bird		2 5 6	
			228 2 9
6. Other Payments :—			
Neill & Co., Ltd., Printers		£99 18 7	
E. Sawers, Purveyor		19 18 0	
S. Duncan, Tailor (uniforms)		10 13 0	
S. Heddle—Bonus		10 0 0	
Andrew H. Baird		3 10 0	
Lindsay, Jamieson & Haldane, C.A., Auditors		10 10 0	
Post Office Telephone Rent		12 0 0	
A. Cowan & Sons, Ltd.		5 15 7	
Orrock & Son, Bookbinders		2 2 6	
Gillies & Wright, Joiners		14 14 8	
R. Graham, Slater		7 1 0	
Edward & Co., Engineers		4 13 9	
G. Waterston & Sons, Ltd., Stationers		3 15 3	
A. Black & Co., Brushmakers		3 15 2	
M'Farlane & Erskine, Lithographers		2 15 0	
Travelling Expenses of two Delegates to the Inter-Allied Conference of Scientific Academies in Paris		30 0 0	
Petty Expenses, Postages, Carriage, etc.		99 15 6	
			340 18 0
7. Arrears of Contributions outstanding at 30th September 1919 :—			
Present Session		£46 4 0	
Previous Sessions		45 3 0	
			91 7 0
Amount of the Discharge			**£2108 12 10**

Amount of the Charge	**£2009 15 7**
Amount of the Discharge	**2108 12 10**
Excess of Payments over Receipts for 1918–1919	**£98 17 3**
FLOATING BALANCE DUE BY THE SOCIETY at 30th September 1918	202 19 8
Balance due by the Society at 30th September 1919	**£301 16 11**
Less—Transferred from Special Subscription Fund	301 16 11

SPECIAL SUBSCRIPTION FUND.

Subscriptions received			£1046 15 6
Amount transferred to General Fund to meet Deficiency on Accounts to 30th September 1919		£301 16 11	
Balance at 30th September 1919 :—			
Due by Union Bank of Scotland, Ltd., on Account Current	£731 10 0		
Due by Treasurer	13 8 7		
		744 18 7	
			1046 15 6

Note.—The above balance is referred to in Secretary's Report for the year.

II. ACCOUNT OF THE KEITH FUND

To 30th September 1919.

CHARGE.

1. BALANCE due by Union Bank of Scotland, Ltd., on Deposit Receipt at 30th September 1918		£57 7 3
2. INTEREST RECEIVED :—		
On £650 five per cent. War Loan, 1929–47, Untaxed	£32 10 0	
On Deposit Receipt, Union Bank of Scotland, Ltd.	2 1 10	
		34 11 10
		£91 19 1

DISCHARGE.

1. Alex. Kirkwood & Son, Engravers, for Gold Medal awarded to Mr R. C. Mossman for 1915–17		£18 5 2
2. BALANCE due by Union Bank of Scotland, Ltd., at 30th September 1919 :—		
On Deposit Receipt	£59 9 1	
On Account Current	14 4 10	
		73 13 11
		£91 19 1

III. ACCOUNT OF THE NEILL FUND

To 30th September 1919.

CHARGE.

1. BALANCE due by Union Bank of Scotland, Ltd., on Deposit Receipt at 30th September 1918		£32 12 8
2. INTEREST RECEIVED :—		
On £300 five per cent. War Loan, 1929–47, Untaxed	£15 0 0	
On Deposit Receipt, Union Bank of Scotland, Ltd.	1 3 2	
		16 3 2
		£48 15 10

DISCHARGE.

1. Alex. Kirkwood & Son, Engravers, for Gold Medal awarded to Professor W. H. Lang for 1915–17		£17 7 6
2. BALANCE due by Union Bank of Scotland, Ltd., at 30th September 1919 :—		
On Deposit Receipt	£30 2 10	
On Account Current	1 5 6	
		31 8 4
		£48 15 10

IV. ACCOUNT OF THE MAKDOUGALL-BRISBANE FUND

To 30th September 1919.

CHARGE.

1. Balance due by Union Bank of Scotland, Ltd., at 30th September 1918 :—		
On Deposit Receipt		£64 3 8
On Account Current		10 0 0
		£74 3 8
2. Interest Received :—		
On £400 five per cent. War Loan, 1929–47, Untaxed	£20 0 0	
On Deposit Receipts, Union Bank of Scotland, Ltd.	2 17 1	
		22 17 1
		£97 0 9

DISCHARGE.

1. Prof. A. A. Lawson—Money Portion of Prize, 1916–18	£20 3 1
2. Alex. Kirkwood & Son, Engravers, for two Gold Medals awarded to Dr R. A. Houstoun for 1914–16 and Prof. A. A. Lawson for 1916–18	41 2 2
3. Balance due by Union Bank of Scotland, Ltd., on Deposit Receipt at 30th September 1919	35 15 6
	£97 0 9

V. ACCOUNT OF THE MAKERSTOUN MAGNETIC METEOROLOGICAL OBSERVATION FUND

To 30th September 1919.

CHARGE.

1. Balance due by Union Bank of Scotland, Ltd., on Account Current at 30th September 1918	£16 2 11
2. Interest Received :— On £250 five per cent. War Loan, 1929–47, Untaxed	12 10 0
	£28 12 11

DISCHARGE.

1. W. C. M'C. Lewis—In aid of publication of the Annual Table of Constants, etc.	£5 0 0
2. Balance due by Union Bank of Scotland, Ltd., on Account Current at 30th September 1919	23 12 11
	£28 12 11

VI. ACCOUNT OF THE GUNNING VICTORIA JUBILEE PRIZE FUND

To 30th September 1919.

(Instituted by Dr R. H. Gunning of Edinburgh and Rio de Janeiro.)

CHARGE.

1. Balance due by Union Bank of Scotland, Ltd., at 30th September 1918 :—		
On Deposit Receipt		£55 1 10
On Account Current		14 5 0
		£69 6 10
2. Interest Received :—		
On £570 five per cent. War Loan, 1929–47, Untaxed	£28 10 0	
On Deposit Receipt, Union Bank of Scotland, Ltd.	2 12 4	
		31 2 4
		£100 9 2

DISCHARGE.

1. Balance due by Union Bank of Scotland, Ltd., at 30th September 1919 :—				
On Deposit Receipt		£57	14	2
On Account Current		42	15	0
		£100	9	2

VII. ACCOUNT OF THE JAMES SCOTT PRIZE FUND

To 30th September 1919.

CHARGE.

1. Balance due by Union Bank of Scotland, Ltd., Brechin, on Deposit Receipt at 30th September 1918	£250	0	0
2. Interest Received :—			
On Deposit Receipt, Union Bank of Scotland, Ltd.	14	4	0
	£264	4	0

DISCHARGE.

1. Balance due by Union Bank of Scotland, Ltd., on Deposit Receipt at 30th September 1919	£264	4	0

STATE OF THE FUNDS BELONGING TO THE ROYAL SOCIETY OF EDINBURGH

As at 30th *September* 1919.

1. GENERAL FUND—

1. £7830 five per cent. War Loan, 1929–47, at $94\frac{3}{8}$ per cent.	£7389	11	3
2. £52, 10s. Annuity of the Edinburgh and District Water Trust, equivalent to £875 at $112\frac{1}{2}$ per cent.	984	7	6
3. Deposit Receipt Union Bank of Scotland, Ltd., being balance of Legacy received, during 1917–18, from the Trustees of the late Mr Robert Mackay Smith, £500 *less* legacy duty £50	450	0	0
4. Arrears of Contributions, as per preceding Abstract of Accounts	91	7	0
5. Balance of Special Subscription Fund	744	18	7
Amount	£9660	4	4

Exclusive of Library, Museum, Pictures, etc., and Furniture in the Society's Rooms at George Street, Edinburgh.

2. KEITH FUND—

1. £650 five per cent. War Loan, 1929–47, at $94\frac{3}{8}$ per cent.				£613	8	9
2. Balance due by Union Bank of Scotland, Ltd. :—						
On Deposit Receipt	£59	9	1			
On Account Current	14	4	10			
				73	13	11
Amount				£687	2	8

3. NEILL FUND—

1. £300 five per cent. War Loan, 1929–47, at $94\frac{3}{8}$ per cent.				£283	2	6
2. Balance due by Union Bank of Scotland, Ltd. :—						
On Deposit Receipt	£30	2	10			
On Account Current	1	5	6			
				31	8	4
Amount				£314	10	10

4. MAKDOUGALL-BRISBANE FUND—

1. £400 five per cent. War Loan, 1929–47, at $94\frac{3}{8}$ per cent.	£377	10	0
2. Balance due by Union Bank of Scotland, Ltd., on Deposit Receipt	35	15	6
Amount	£413	5	6

5. **MAKERSTOUN MAGNETIC METEOROLOGICAL OBSERVATION FUND—**

1. £250 five per cent. War Loan, 1929–47, at 94$\frac{3}{8}$ per cent.		£235 18 9
2. Balance due by Union Bank of Scotland, Ltd., on Account Current		23 12 11
AMOUNT		£259 11 8

6. **GUNNING VICTORIA JUBILEE PRIZE FUND**—Instituted by Dr Gunning of Edinburgh and Rio de Janeiro—

1. £570 five per cent. War Loan, 1929–47, at 94$\frac{3}{8}$ per cent.		£537 18 9
2. Balance due by Union Bank of Scotland, Ltd. :—		
On Deposit Receipt	£57 14 2	
On Account Current	42 15 0	
		100 9 2
AMOUNT		£638 7 11

7. **JAMES SCOTT PRIZE FUND—**

Balance due by Union Bank of Scotland, Ltd., on Deposit Receipt	£264 4 0

8. **TAIT MEMORIAL FUND—**

This Fund consists mainly of War Loan, and is to mature for a period of about ten years from 1918, when it is expected to yield about £75 per annum.

EDINBURGH, 17*th October* 1919.—We have examined the preceding Accounts of the Treasurer of the Royal Society of Edinburgh for the Session 1918–1919, and have found them to be correct. The securities of the various Investments at 30th September 1919, as noted in the above Statement of Funds (with the exception of No. 8), have been exhibited to us.

LINDSAY, JAMIESON & HALDANE, C.A.,
Auditors.

THE COUNCIL OF THE SOCIETY.

October 1919.

PRESIDENT.

PROFESSOR FREDERICK O. BOWER, M.A., D.Sc., LL.D., F.R.S., F.L.S.

VICE-PRESIDENTS.

PROFESSOR GEORGE A. GIBSON, M.A., LL.D., Professor of Mathematics, University, Glasgow.
ROBERT KIDSTON, LL.D., F.R.S., F.G.S.
PROFESSOR D. NOËL PATON, M.D., B.Sc., LL.D., F.R.C.P.E., F.R.S., Professor of Physiology, University, Glasgow.
PROFESSOR A. ROBINSON, M.D., M.R.C.S., Professor of Anatomy, University, Edinburgh.
SIR GEORGE A. BERRY, M.B., C.M., LL.D., F.R.C.S.E.
PROFESSOR WILLIAM PEDDIE, D.Sc., Professor of Natural Philosophy in University College, Dundee.

GENERAL SECRETARY.

CARGILL G. KNOTT, D.Sc., LL.D.

SECRETARIES TO ORDINARY MEETINGS.

PROFESSOR E. T. WHITTAKER, Sc.D., F.R.S., Professor of Mathematics, University, Edinburgh.
J. H. ASHWORTH, D.Sc., F.R.S., Lecturer on Invertebrate Zoology, University, Edinburgh.

TREASURER.

JAMES CURRIE, M.A., LL.D.

CURATOR OF LIBRARY AND MUSEUM.

A. CRICHTON MITCHELL, D.Sc., Hon. D.Sc. (Geneva).

COUNCILLORS.

PROFESSOR P. T. HERRING, M.D., F.R.C.P.E.
PROFESSOR T. J. JEHU, M.A., M.D., F.G.S.
ALEXANDER LAUDER, D.Sc.
THE HON. LORD GUTHRIE, LL.D.
PROFESSOR R. A. SAMPSON, M.A., D.Sc., F.R.S.
PROFESSOR J. LORRAIN SMITH, M.A., M.D., F.R.S.
W. A. TAIT, D.Sc., M.Inst.C.E.
SURGEON-GENERAL W. B. BANNERMAN, C.S.I., I.M.S., M.D., D.Sc.
HENRY MOUBRAY CADELL, of Grange, B.Sc.
PROFESSOR ARTHUR ROBERTSON CUSHNY, M.A., M.D., LL.D., F.R.S.
PRINCIPAL SIR JAMES ALFRED EWING, K.C.B., M.A., B.Sc., LL.D., M.Inst.C.E., F.R.S.
GEORGE JAMES LIDSTONE, F.F.A., F.I.A.

ALPHABETICAL LIST OF THE ORDINARY FELLOWS OF THE SOCIETY,

Corrected to January 15, 1920.

N.B.—*Those marked* * *are Annual Contributors.*

B. prefixed to a name indicates that the Fellow has received a Makdougall-Brisbane Medal.
K. ,, ,, ,, Keith Medal.
N. ,, ,, ,, Neill Medal.
V. J. ,, ,, ,, the Gunning Victoria Jubilee Prize.
C. ,, ,, ,, contributed one or more Communications to the Society's TRANSACTIONS or PROCEEDINGS.

Date of Election.			Service on Council, etc.
1898	C.	* Abercromby, the Rt. Hon. Lord, LL.D., 62 Palmerston Place, Edinburgh	
1898		Adami, Prof. J. G., M.A., M.D. (Camb., M'Gill, and Belfast), LL.D., F.R.S., Professor of Pathology in M'Gill University, Montreal	
1896		* Affleck, Sir Jas. Ormiston, M.D., LL.D., F.R.C.P.E., 38 Heriot Row, Edinburgh	
1895		Alford, Robert Gervase, M.Inst.C.E., Three Gables, Woodburn Park Road, Tunbridge Wells, Kent	
1889		Alison, John, M.A., Head Master, George Watson's College, Edinburgh 5	
1894		Allan, Francis John, M.D., C.M. Edin., M.O.H. City of Westminster, Westminster City Hall, Charing Cross Road, London	
1888	C.	Allardice, R. E., M.A., Professor of Mathematics in Stanford University, Palo Alto, Santa Clara Co., California	
1906		Anderson, Daniel E., M.D., B.A., B.Sc., Green Bank, Merton Lane, Highgate, London, N.	
1883		Anderson, Sir Robert Rowand, LL.D., 16 Rutland Square, Edinburgh	
1905		* Anderson, William, M.A., Head Science Master, George Watson's College, Edinburgh, 6 Lockharton Crescent, Edinburgh 10	
1903		Anderson-Berry, David, M.D., LL.D., F.R.S.L., M.R.A.S., F.S.A. (Scot.), Versailles, Highgate, London, N.	
1905		* Andrew, George, M.A., B.A., H.M.I.S., Balgillo Cottage, Seafield Road, Broughty Ferry	
1881	C.	Anglin, A. H., M.A., LL.D., M.R.I.A., Professor of Mathematics, Queen's College, Cork	
1915		Anthony, Charles, M.Inst.C.E., M. Am. Soc. C.E., F.R.San.I., F.R.A.S., F.R.Met.S., F.R.M.S., F.C.S., General Manager, Water Works Company, Vieytes Esq. Gorriti, Bahia Blanca, Argentina	
1906		Appleton, Colonel Arthur Frederick, F.R.C.V.S., Nylstroom, Smoke Lane, Reigate 15	
1899		Appleyard, James R., Royal Technical Institute, Salford, Manchester	
1910	C.	Archibald, E. H., B.Sc., Professor of Chemistry, University of British Columbia, Vancouver, Canada	
1907		* Archibald, James, M.A., 31 Leamington Terrace, Edinburgh	
1911	C. K.	* Ashworth, James Hartley, D.Sc., F.R.S., Professor of Zoology, University of Edinburgh (SECRETARY), 69 Braid Avenue, Edinburgh	1912–14, 1915–18. Sec. 1918–
1907		* Badre, Muhammad, Ph.D., Almuneerah, Cairo, Egypt 20	
1896	C.	* Baily, Francis Gibson, M.A., M.Inst.E.E., Professor of Electrical Engineering, Heriot-Watt College, Edinburgh, Newbury, Colinton, Midlothian	1909–12.

Date of Election			Service on Council, etc.
1877	C.	Balfour, I. Bayley, M.A., Sc.D., M.D., LL.D., F.R.S., F.L.S., King's Botanist in Scotland, Professor of Botany in the University of Edinburgh and Keeper of the Royal Botanic Garden, Inverleith House, Edinburgh	1888–91.
1905	C.	Balfour-Browne, William Alexander Francis, M.A., Barrister-at-Law, Oaklands, Fenstanton, near St Ives, Hunts	
1892	C.	Ballantyne, J. W., M.D., F.R.C.P.E., 19 Rothesay Terrace, Edinburgh	
1918		* Balsillie, David, B.Sc., F.G.S., Assistant in the Chemistry Department, University, Edinburgh, 14 Greyfriars Garden, St Andrews 25	
1902	C.	Bannerman, W. B., C.S.I., I.M.S., M.D., D.Sc., Surgeon General, Indian Medical Service, 11 Strathearn Place, Edinburgh	1919–
1889		Barbour, A. H. F., M.A., M.D., LL.D., F.R.C.P.E., 4 Charlotte Square, Edinburgh	
1886		Barclay, A. J. Gunion, M.A., 3 Chandos Avenue, Oakleigh Park, London, N.	
1883	C.	Barclay, G. W. W., M.A., Raeden House, Aberdeen	
1903		Bardswell, Noël Dean, M.D., M.R.C.P. Ed. and Lond., King Edward VII Sanatorium, Midhurst 30	
1914	C.	* Barkla, Charles Glover, D.Sc., F.R.S., Professor of Natural Philosophy in the University of Edinburgh, 20 Hermitage Drive, Edinburgh	1915–18.
1882	C.	Barnes, Henry, O.B.E., M.D., LL.D., 6 Portland Square, Carlisle	
1904		Barr, Sir James, M.D., LL.D., F.R.C.P. Lond., 72 Rodney Street, Liverpool	
1874		Barrett, Sir William F., F.R.S., M.R.I.A., formerly Professor of Physics, Royal College of Science, Dublin, 31 Devonshire Place, London, W. 1	
1887		Bartholomew, J. G., LL.D., F.R.G.S., The Geographical Institute, Duncan Street, Edinburgh 35	1909–12.
1895	C.	Barton, Edwin H., D.Sc., F.R.S., A.M.Inst.E.E., F.P.S.L., Professor of Experimental Physics, University College, Nottingham	
1904		* Baxter, William Muirhead, Glenalmond, Sciennes Gardens, Edinburgh	
1913		Beard, Joseph, F.R.C.S. (Edin.), M.R.C.S. (Eng.), L.R.C.P. (Lond.), D.P.H. (Camb.), Medical Officer of Health and School Medical Officer, City of Carlisle, 8 Carlton Gardens, Carlisle	
1888		Beare, Thomas Hudson, B.Sc., M.Inst.C.E., J.P., Professor of Engineering in the University of Edinburgh	1907–09. V-P 1909–15.
1897	C.	* Beattie, John Carruthers, D.Sc., Vice-Chancellor and Principal, The University, Cape Town 40	
1892		Beck, Sir J. H. Meiring, Kt., M.D., M.R.C.P.E., Drostdy, Tulbagh, Cape Province, South Africa	
1893	C. B.	Becker, Ludwig, Ph.D., Regius Professor of Astronomy in the University of Glasgow, The Observatory, Glasgow	
1882	C.	Beddard, Frank E., M.A. Oxon., F.R.S., Prosector to the Zoological Society of London, Zoological Society's Gardens, Regent's Park, London	
1887		Begg, Ferdinand Faithfull, 46 Saint Aubyns, Hove, Sussex	
1906		Bell, John Patrick Fair, F.Z.S., Fulforth, Witton Gilbert, Durham 45	
1916		* Bell, Robert John Tainsh, M.A., D.Sc., Lecturer in Mathematics in the University of Glasgow, 146 Hyndland Road, Glasgow	
1915		Bell, Walter Leonard, M.D.Edin., F.S.A.Scot., 123 London Road, North Lowestoft, Suffolk	
1893	C.	Berry, Sir George A., M.B., C.M., LL.D., F.R.C.S.E. (VICE-PRESIDENT), 31 Drumsheugh Gardens, Edinburgh	1916–19. V-P 1919–
1897	C.	Berry, Richard J. A., M.D., F.R.C.S.E., Professor of Anatomy in the University of Melbourne, Victoria, Australia	
1904		* Beveridge, Erskine, LL.D., St Leonards Hill, Dunfermline 50	
1880	C.	Birch, De Burgh, C.B., M.D., Emeritus Professor of Physiology in the University of Leeds	
1907		* Black, Frederick Alexander, Solicitor, 59 Academy Street, Inverness	
1884	C.	Black, John S., M.A., LL.D., 125 St James' Court, London, S.W. 1	1891–94, 1916–18. Cur. 1906–16.
1897	C.	* Blaikie, Walter Biggar, LL.D., The Loan, Colinton	1914–17.
1904	C.	* Bles, Edward J., M.A., D.Sc., Elterholm, Cambridge 55	
1918		* Blight, Francis James, Chairman and Managing Director of Charles Griffin & Co., Ltd., Publishers, Tregenna, Wembley, Middlesex	
1894		Bolton, Herbert, M.Sc., F.G.S., F.Z.S., Director of the Bristol Museum and Art Gallery, Bristol, 58 Coldharbour Road, Redland, Bristol	

Date of Election.			Service on Council, etc.
1915		* Boon, Alfred Archibald, D.Sc., F.I.C., B.A., Professor of Chemistry, Heriot-Watt College, Edinburgh	
1872	C.	Bottomley, J. Thompson, M.A., D.Sc., LL.D., F.R.S., F.C.S., 13 University Gardens, Glasgow	
1886	C.	Bower, Frederick O., M.A., D.Sc., LL.D., F.R.S., F.L.S. (President), Regius Professor of Botany in the University of Glasgow, 1 St John's Terrace, Hillhead, Glasgow 60	1887–90, 1893–96, 1907–09, 1917–19 V-P 1910–16. P 1919–
1884	C.	Bowman, Frederick Hungerford, D.Sc., F.C.S. (Lond. and Berl.), F.I.C., A.Inst.C.E., A.Inst.M.E., M.Inst.E.E., etc., 77 Acomb Street, Whitworth Park, Manchester	
1901		Bradbury, J. B., M.D., Downing Professor of Medicine, University of Cambridge	
1916		Bradley, Francis Ernest, M.A., M.Com., LL.D., Barrister-at-Law, Examiner to the Council of Legal Education, Bank of England Chambers, Tib Lane, Manchester	
1903	C.	* Bradley, O. Charnock, M.D., D.Sc., Principal, Royal Dick Veterinary College, Edinburgh	1907–10, 1915–17.
1886		Bramwell, Byrom, M.D., F.R.C.P.E., LL.D., 23 Drumsheugh Gardens, Edinburgh 65	1890–93.
1907		* Bramwell, Edwin, M.D., F.R.C.P.E., F.R.C.P. Lond., 23 Drumsheugh Gardens, Edinburgh	
1918		* Bremner, Alexander, M.A., D.Sc., Headmaster, Demonstration School, Training Centre, Aberdeen, 13 Belgrave Terrace, Aberdeen	
1912		Bridger, Adolphus Edward, M.D. (Edin.), F.R.C.P. (Edin.), B.Sc. (Paris), B.L. (Paris), Foley Lodge, Langham Street, London, W.	
1916	C.	* Briggs, Henry, D.Sc., A.R.S.M., Professor of Mining, Heriot-Watt College, Allermuir, Liberton, Midlothian	
1895		Bright, Sir Charles, M.Inst.C.E., M.Inst.E.E., F.R.Aë.S., F.Inst.Radio.E., F.R.A.S., F.R.G.S., Leigh Grange, Kent, and Athenæum Club, Pall Mall, London, S.W. 70	
1893		Brock, G. Sandison, M.D., 6 Corso d'Italia, Rome, Italy	
1901	C.	* Brodie, W. Brodie, M.B., Thaxted, Dunmow, Essex	
1907		Brown, Alexander, M.A., B.Sc., Professor of Applied Mathematics, The University, Cape Town	
1864	C. K. B.	Brown, Alex. Crum, M.A., M.D., D.Sc., F.R.C.P.E., LL.D., F.R.S., Emeritus Professor of Chemistry in the University of Edinburgh, 8 Belgrave Crescent, Edinburgh	1865–68, 1869–72, 1873–75, 1876–78, 1911–13. Sec. 1879–1905. V-P 1905–11.
1898		* Brown, David, F.C.S., F.I.C., J.P., Willowbrae House, Willowbrae Road, Edinburgh 75	
1911		* Brown, David Rainy, Chemical Manufacturer (J. F. Macfarlan & Co.), 93 Abbeyhill, Edinburgh	
1883	C.	Brown, J. J. Graham, M.D., F.R.C.P.E., 3 Chester Street, Edinburgh	
1885	C.	Brown, J. Macdonald, M.D., F.R.C.S., 64 Upper Berkeley Street, Portman Square, London, W.	
1909	B. C.	* Brownlee, John, M.A., M.D., D.Sc., the National Institute for Medical Research, Mount Vernon, Hampstead, N.W. 3	
1912		* Bruce, Alexander Ninian, D.Sc., M.D., 8 Ainslie Place, Edinburgh 80	
1906	C. N.	* Bruce, William Speirs, LL.D., Director of the Scottish Oceanographical Laboratory, Edinburgh, Antarctica, Joppa, Midlothian	1909–12.
1898	C. K.	* Bryce, T. H., M.A., M.D. (Edin.), Professor of Anatomy in the University of Glasgow, 2 The University, Glasgow	1911–14.
1870	C. K.	Buchanan, John Young, M.A., F.R.S., 26 Norfolk Street, Park Lane, London, W.	1878–81, 1884–86.
1905		Bunting, Thomas Lowe, M.D., 27 Denton Road, Scotswood, Newcastle-on-Tyne	
1902		* Burgess, A. G., M.A., Rector of The Academy, Rothesay, Blythswood, Rothesay 85	

Date of Election.			Service on Council, etc.
1887		Burnet, Sir John James, LL.D., R.S.A., Architect, 239 St Vincent Street, Glasgow	
1888		Burns, Rev. T., D.D., F.S.A. Scot., Minister of Lady Glenorchy's Parish Church, Croston Lodge, Chalmers Crescent, Edinburgh	
1917		* Burnside, George Barnhill, Admiralty Experimental Station, Shandon, Dumbartonshire	
1915		* Butchart, Raymond Keiler, B.Sc., University College, Dundee, 5 Briarwood Terrace, West Park Road, Dundee	
1896		* Butters, J. W., M.A., B.Sc., Rector of Ardrossan Academy 90	
1887	C.	Cadell, Henry Moubray, of Grange, B.Sc., Linlithgow	1919–
1910		* Calderwood, Rev. Robert Sibbald, Minister of Cambuslang, The Manse, Cambuslang, Lanarkshire	
1893	C.	Calderwood, W. L., Inspector of Salmon Fisheries of Scotland, South Bank, Canaan Lane, Edinburgh	
1894		Cameron, James Angus, M.D., Medical Officer of Health, Firhall, Nairn	
1905	C.	Cameron, John, M.D., D.Sc., M.R.C.S. Eng., Dalhousie University, Halifax, Nova Scotia 95	
1904		* Campbell, Charles Duff, Scottish Liberal Club, Princes Street, Edinburgh	
1918		* Campbell, John Menzies, L.D.S. (Glas.), D.D.S. (Toronto), L.D.S. (Ontario), 14 Buckingham Terrace, Glasgow, W.	
1915	C. N.	* Campbell, Robert, D.Sc., Lecturer in Petrology, University of Edinburgh, 7 Muirend Avenue, Juniper Green, Midlothian	
1899	C.	* Carlier, Edmund W. W., M.D., M.Sc., F.E.S., Professor of Physiology, University, Birmingham	
1910		Carnegie, David, M.Inst.C.E., M.Inst.Mech.E., M.I.S.Inst., "Woodlands," Beckenham Hill, Kent 100	
1905	C.	* Carse, George Alexander, M.A., D.Sc., Lecturer on Natural Philosophy, University of Edinburgh, 3 Middleby Street, Edinburgh	
1901		Carslaw, H. S., M.A., D.Sc., Professor of Mathematics in the University of Sydney, New South Wales	
1905		Carter, Joseph Henry, F.R.C.V.S., Stone House, Church Street, Burnley, Lancashire	
1898		* Carter, Wm. Allan, O.B.E., M.Inst. C.E., Stamford Hall, Gullane	1911–14
1898		Carus-Wilson, Cecil, F.R.G.S., F.G.S., Waldegrave Park, Strawberry Hill, Middlesex, and Sandacres Lodge, Parkstone-on-Sea, Dorset 105	
1908		Cavanagh, Thomas Francis, M.D., The Hospital, Bella Coola, B.C., Canada	
1882		Cay, W. Dyce, M.Inst.C.E., Junior Carlton Club, Pall Mall, London, S.W. 1	
1899		Chatham, James, Actuary, c/o Robert Murrie, Esq., 28 St Andrew Square, Edinburgh	
1912		Chaudhuri, Banawari Lal, B.A. (Cal.), B.Sc. (Edin.), Assistant Superintendent, Natural History Section, Indian Museum, 120 Lower Circular Road, Calcutta, India	
1874		Chiene, John, C.B., M.D., LL.D., F.R.C.S.E., Emeritus Professor of Surgery in the University of Edinburgh, Barnton Avenue, Davidson's Mains 110	1884–86, 1904–06.
1891		Clark, John B., M.A., Head Master of Heriot's Hospital School, Lauriston, Garleffin, 146 Craiglea Drive, Edinburgh	
1911		* Clark, William Inglis, D.Sc., 29 Lauder Road, Edinburgh	
1903		* Clarke, William Eagle, LL.D., F.L.S., Keeper of the Natural History Collections in the Royal Scottish Museum, Edinburgh, 35 Braid Road, Edinburgh	
1909		Clayton, Thomas Morrison, M.D., D.Hy., B.Sc., D.P.H., Medical Officer of Health, Gateshead, 13 The Crescent, Gateshead-on-Tyne	
1913		* Cleghorn, Alexander, M.Inst.C.E., Marine Engineer, 14 Hatfield Drive, Kelvinside, Glasgow 115	
1904	C.	Coker, Ernest George, M.A., D.Sc., Hon. D.Sc. (Sydney), F.R.S., M.Inst.C.E., M.Inst.E.E., Professor of Civil and Mechanical Engineering, University of London, University College, Gower Street, London, W.C.	
1904		Coles, Alfred Charles, M.D., D.Sc., York House, Poole Road, Bournemouth, W.	
1888	V. J. C.	Collie, John Norman, Ph.D., D.Sc., LL.D., F.R.S., F.C.S., F.I.C., F.R.G.S., Professor of Organic Chemistry in the University College, Gower Street, London	
1904		* Colquhoun, Walter, M.A., M.B., 18 Walmer Crescent, Ibrox, Glasgow	
1909	C.	* Comrie, Peter, M.A., B.Sc., Head Mathematical Master, Boroughmuir Junior Student Centre, 19 Craighouse Terrace, Edinburgh 120	
1886		Connan, Daniel M., M.A.	
1905		* Corrie, David, F.C.S., Nobel's Explosives Company, Polmont, Stirlingshire	

Date of Election.			Service on Council, etc.
1914		* Coutts, William Barron, M.A., B.Sc., 33 Dalhousie Terrace, Edinburgh, Royal Garrison Artillery, Ordnance College, Woolwich, London. S.E.	
1911		* Cowan, Alexander C., Papermaker, Valleyfield House, Penicuik, Midlothian	
1916	C.	Craig, E. H. Cunningham, B.A. (Cambridge), Geologist and Mining Engineer, The Dutch House, Beaconsfield 125	
1908		Craig, James Ireland, M.A., B.A., Woolwich House, The Drive, Sydenham, London, S.E. 26	
1875		Craig, William, M.D., F.R.C.S.E., Lecturer on Materia Medica to the College of Surgeons, 71 Bruntsfield Place, Edinburgh	
1903		Crawford, Lawrence, M.A., D.Sc., Professor of Pure Mathematics, The University, Cape Town	
1870		Crichton-Browne, Sir Jas., M.D., LL.D., D.Sc., F.R.S., Lord Chancellor's Visitor and Vice-President and Treasurer of the Royal Institution of Great Britain, 45 Hans Place, S.W., and Royal Courts of Justice, Strand, London	
1916		* Crombie, James Edward, M.A., LL.D., Millowner, Parkhill House, Dyce, Aberdeenshire 130	
1886		Croom, Sir John Halliday, M.D., F.R.C.P.E., Professor of Midwifery in the University of Edinburgh, late President, Royal College of Surgeons, Edinburgh, 25 Charlotte Square, Edinburgh	
1914		* Cumming, Alexander Charles, D.Sc., O.B.E., Lecturer in Chemistry, University, Edinburgh, 2 Relugas Road, Edinburgh	
1917		* Cunningham, Brysson, D.Sc., B.E., M.Inst.C.E., Civil Engineer, 16 Beechwood Road, Sanderstead, Surrey	
1898		* Currie, James, M.A. Cantab., LL.D. (Treasurer), Larkfield, Goldenacre, Edinburgh	Treas. 1906–
1919		* Cushny, Arthur Robertson, M.A., M.D., LL.D., F.R.S., Professor of Materia Medica and Pharmacology, University, Edinburgh 135	
1904		* Cuthbertson, John, Secretary, West of Scotland Agricultural College, 6 Charles Street, Kilmarnock	
1885		Daniell, Alfred, M.A., LL.B., D.Sc., Advocate, The Athenæum Club, Pall Mall, London	
1884		Davy, R., F.R.C.S. Eng., Consulting Surgeon to Westminster Hospital, Burstone Manor, Bow, North Devon	
1917		* Day, T. Cuthbert, Partner of the firm of Hislop & Day, 36 Hillside Crescent, Edinburgh	
1894		Denny, Sir Archibald, Bart., LL.D., Cardross Park, Cardross, Dumbartonshire 140	
1869	C. V. J.	Dewar, Sir James, Kt., M.A., LL.D., D.C.L., D.Sc., F.R.S., V.P.C.S., Jacksonian Professor of Natural and Experimental Philosophy in the University of Cambridge, and Fullerian Professor of Chemistry at the Royal Institution of Great Britain, London	1872–74.
1905		* Dewar, James Campbell, C.A., 27 Douglas Crescent, Edinburgh	
1906		* Dewar, Thomas William, M.D., F.R.C.P., Kincairn, Dunblane	
1884		Dickson, the Right Hon. Charles Scott, Lord Justice-Clerk, K.C., LL.D., 22 Moray Place, Edinburgh	
1888	C.	Dickson, Henry Newton, M.A., D.Sc., C.B.E., 160 Castle Hill, Reading 145	
1876	C.	Dickson, J. D. Hamilton, M.A., Senior Fellow and formerly Tutor, St Peter's College, Cambridge	
1885	C.	Dixon, James Main, M.A., Litt. Hum. Doctor, Professor of English, University of Southern California, University Avenue, Los Angeles, California, U.S.A.	
1897		* Dobbie, James Bell, F.Z.S., 12 South Inverleith Avenue, Edinburgh	
1904		* Dobbie, Sir James Johnston, Kt., M.A., D.Sc., LL.D., F.R.S., Principal of the Government Laboratories, London, 4 Vicarage Gate, Kensington, London, W.	1905–08.
1881	C.	Dobbin, Leonard, Ph.D., Lecturer on Chemistry in the University of Edinburgh, 6 Wilton Road, Edinburgh 150	1904–07, 1913–16.
1918		* Dodd, Alexander Scott, B.Sc., F.I.C., F.C.S., City Analyst for Edinburgh, 20 Stafford Street, Edinburgh	
1905		* Donaldson, Rev. Wm. Galloway, J.P., F.R.G.S., F.E.I.S., The Manse, Forfar	
1882	C.	Dott, David B., F.I.C., Memb. Pharm. Soc., Ravenslea, Musselburgh	
1918		* Douglas, Carstairs Cumming, M.D., D.Sc., Professor of Medical Jurisprudence and Hygiene, Anderson's College, Glasgow, 2 Royal Crescent, Glasgow	
1910		* Douglas, Loudon MacQueen, Author and Lecturer, 29 W. Saville Terrace, Newington, Edinburgh 155	

Date of Election.			Service on Council, etc.
1908	C.	Drinkwater, Harry, M.D., M.R.C.S. (Eng.), F.L.S., Lister House, Wrexham, North Wales	
1901		* Drinkwater, Thomas W., L.R.C.P.E., L.R.C.S.E., Chemical Laboratory, Surgeons' Hall, Edinburgh	
1917		* Dron, Robert W., A.M.Inst.C.E., 65 Renfield Street, Glasgow	
1919		Dundas, William John, W.S., LL.D., Crown Agent for Scotland, 11 Drumsheugh Gardens, Edinburgh	
1904		* Dunlop, William Brown, M.A., 4A St Andrew Square, Edinburgh 160	
1903		Dunstan, John, M.R.C.V.S., Inversnaid, Liskeard, Cornwall	
1892	C.	Dunstan, M. J. R., M.A., F.I.C., F.C.S., Principal, South-Eastern Agricultural College, Wye, Kent	
1906	C.	Dyson, Sir Frank Watson, Kt., M.A., LL.D., F.R.S., Astronomer Royal, Royal Observatory, Greenwich	1907–10.
1893		Edington, Alexander, M.D., Howick, Natal	
1904		* Edwards, John, LL.D., 4 Great Western Terrace, Kelvinside, Glasgow 165	
1904		* Elder, William, M.D., F.R.C.P.E., 4 John's Place, Leith	
1875		Elliot, Daniel G., American Museum of Natural History, Central Park West, New York, N.Y., U.S.A.	
1906	C.	* Ellis, David, D.Sc., Ph.D., Lecturer in Botany and Bacteriology, Glasgow and West of Scotland Technical College, Glasgow	
1897	C.	* Erskine-Murray, James Robert, D.Sc., 16 Elmfield Road, Bromley, Kent	
1884		Evans, William, F.F.A., 38 Morningside Park, Edinburgh 170	
1879	C. N.	Ewart, James Cossar, M.D., F.R.C.S.E., F.R.S., F.Z.S., Regius Professor of Natural History, University of Edinburgh, Craigybield, Penicuik, Midlothian	1882–85, 1904–07. V-P 1907–12.
1902		* Ewen, John Taylor, B.Sc., M.I.Mech.E., H.M. Inspector of Schools, 104 King's Gate, Aberdeen	
1878	C.	Ewing, Sir James Alfred, K.C.B., M.A., B.Sc., LL.D., M.Inst.C.E., F.R.S., J.P., Principal of the University of Edinburgh, formerly Director of Naval Education, Admiralty, 16 Moray Place, Edinburgh	1888–91, 1919–
1900	C.	Eyre, John W. H., M.D., M.S. (Dunelm), D.P.H. (Camb.), Professor of Bacteriology, Guy's Hospital, London	
1910		* Fairgrieve, Mungo M'Callum, M.A. (Glasg.), M.A. (Cambridge), Master at the Edinburgh Academy, 37 Queen's Crescent, Edinburgh 175	
1907	C.	Falconer, John Downie, M.A., D.Sc., F.G.S., Lecturer on Geography, The University, Glasgow	
1888	C.	Fawsitt, Charles A., Coney Park, Bridge of Allan	
1883	C.	Felkin, Robert W., M.D., F.R.G.S., Whare Ra, Havelock North, Hawk's Bay, New Zealand	
1899		* Fergus, Andrew Freeland, M.D., 22 Blythswood Square, Glasgow	
1907		* Fergus, Edward Oswald, 12 Clairmont Gardens, Glasgow 180	
1904		* Ferguson, James Haig, M.D., F.R.C.P.E., F.R.C.S.E., 7 Coates Crescent, Edinburgh	
1898		* Findlay, Sir John R., M.A. Oxon., K.B.E., 3 Rothesay Terrace, Edinburgh	
1899		* Finlay, David W., B.A., M.D., LL.D., F.R.C.P., D.P.H., Emeritus Professor of Medicine in the University of Aberdeen, Honorary Physician to His Majesty in Scotland, Balgownie, Helensburgh	
1911		Fleming, John Arnold, F.C.S., etc., Pottery Manufacturer, 136 Glebe Street, St Rollox, Glasgow, Locksley, Helensburgh	
1906		* Fleming, Robert Alexander, M.A., M.D., F.R.C.P.E., Assistant Physician, Royal Infirmary, 10 Chester Street, Edinburgh 185	
1900	C. N.	* Flett, John S., M.A., D.Sc., LL.D., F.R.S., O.B.E., Director of the Geological Survey of Scotland, 33 George Square, Edinburgh	1916–19.
1872		Forbes, Professor George, M.A., M.Inst.C.E., M.Inst.E.E., F.R.S., F.R.A.S., 11 Little College Street, Westminster, S.W.	
1892		Ford, John Simpson, F.C.S., 7 Corrennie Drive, Edinburgh	
1910		* Fraser, Alexander, Actuary, 17 Eildon Street, Edinburgh	
1896		* Fraser, John, M.B., F.R.C.P.E., formerly one of H.M. Commissioners in Lunacy for Scotland, 54 Great King Street, Edinburgh 190	
1915		* Fraser, Rev. Joseph Robert, U.F. Manse, Kinneff, Scotland	
1914		* Fraser, William, Managing Director, Neill & Co., Ltd., Printers, 17 Eildon Street, Edinburgh	
1891		Fullarton, J. H., M.A., D.Sc., 23 Porchester Gardens, London, W.	
1891		Fulton, T. Wemyss, M.D., Scientific Superintendent Scottish Fishery Board, 41 Queen's Road, Aberdeen	

Date of Election.			Service on Council, etc.
1907		* Galbraith, Alexander, "Ravenswood," Dalmuir, Dumbartonshire 195	
1918		* Galloway, T. Lindsay, M.A., F.G.S., Assoc. M.Inst.C.E., M.Inst.M.E., Coalmaster, Kilchrist, Campbeltown, Argyllshire	
1888	C.	Galt, Alexander, D.Sc., Keeper of the Technological Department, Royal Scottish Museum, Edinburgh, St Margaret's, Craiglockhart, Midlothian	
1901		Ganguli, Sanjiban, M.A., Principal, Maharaja's College, and Director of Public Instruction, Jaipur State, Jaipur, India	
1899		Gatehouse, T. E., A.M.Inst.C.E., M.Inst.M.E., M.Inst.E.E., Fairfield, 128 Tulse Hill, London, S.W.	
1909	C.	* Geddes, Rt. Hon. Sir Auckland C., K.C.B., M.D., President of Board of Trade, 7 Whitehall Gardens, London, S.W. 1 200	
1880	C.	Geddes, Patrick, Professor of Botany in University College, Dundee, and Lecturer on Zoology, Ramsay Garden, University Hall, Edinburgh	
1861	C. B.	Geikie, Sir Archibald, O.M., K.C.B., D.C.L. Oxf., D.Sc., LL.D., Ph.D., Late Pres. R.S., Foreign Member of the Reale Accad. Lincei, Rome, of the National Acad. of the United States, of the Academies of Stockholm, Christiania, Göttingen, Corresponding Member of the Institute of France and of the Academies of Berlin, Vienna, Munich, Turin, Belgium, Philadelphia, New York, etc., Shepherd's Down, Haslemere, Surrey	1869–72, 1874–76, 1879–82.
1914		Gemmell, John Edward, M.B., C.M., Hon. Surgeon Hospital for Women and Maternity Hospital; Hon. Gynæcologist, Victoria Central Hospital, Liscard, 28 Rodney Street, Liverpool	
1909		* Gentle, William, B.Sc., 12 Mayfield Road, Edinburgh	
1914		* Gibb, Sir Alexander, K.B.E., C.B., R.M., Director-General of Civil Engineering, Ministry of Transport, 6 Whitehall Gardens, London, S.W. 1 205	
1916		* Gibb, A. W., D.Sc., Lecturer in Geology, The University, Aberdeen, 1 Belvidere Street, Aberdeen	
1910	C.	* Gibb, David, M.A., B.Sc., Lecturer in Mathematics, Edinburgh University, 15 South Lauder Road, Edinburgh	
1917	C.	* Gibson, Alexander, M.B., Ch.B., F.R.C.S. (Eng.), Professor of Anatomy in the Medical College, Winnipeg, Canada	
1912	C.	* Gibson, Arnold Hartley, D.Sc., Professor of Engineering, University College, Dundee	
1910		* Gibson, Charles Robert, Lynton, Mansewood, by Pollokshaws 210	
1890		Gibson, George A., M.A., LL.D. (Vice-President), Professor of Mathematics in the University of Glasgow, 10 The University, Glasgow	1905–08, 1912–13. V-P 1917–
1911		Gidney, Henry A. J., L.M. and S. Socts. Ap. (Lond.), F.R.C.S. (Edin.), D.P.H. (Camb.), D.O. (Oxford), Army Specialist Public Health, c/o Thomas Cook & Sons, Ludgate Circus, London	
1900		Gilchrist, Douglas A., B.Sc., Professor of Agriculture and Rural Economy, Armstrong College, Newcastle-upon-Tyne	
1880		Gilruth, George Ritchie, Surgeon, Springbank, Bridge of Allan	
1907		Gilruth, John Anderson, M.R.C.V.S., D.V.Sc. (Melb.), Administrator, Government House, Darwin Northern Territory, Australia 215	
1909		* Gladstone, Hugh Steuart, M.A., M.B.O.U., F.Z.S., 40 Lennox Gardens, London, S.W.	
1911		Gladstone, Reginald John, M.D., F.R.C.S. (Eng.), Lecturer and Senior Demonstrator of Anatomy, King's College, University of London, 22 Regent's Park Terrace, London, N.W.	
1898		* Glaister, John, M.D., F.R.F.P.S. Glasgow, D.P.H. Camb., Professor of Forensic Medicine in the University of Glasgow, 3 Newton Place, Glasgow	
1910		Goodall, Joseph Strickland, M.B. (Lond.), M.S.A. (Eng.), Lecturer on Physiology, Middlesex Hospital, London, Annandale Lodge, Vanbrugh Park, Blackheath, London, S.E.	
1901		Goodwillie, James, M.A., B.Sc., Liberton, Edinburgh 220	
1913	C.	* Gordon, William Thomas, M.A., D.Sc. (Edin.), B.A. (Cantab.), Lecturer in Geology, University of London, King's College, Strand, W.C.	
1897		Gordon-Munn, John Gordon, M.D., Heigham Hall, Norwich	
1891		Graham, Richard D., 12 Strathearn Road, Edinburgh	
1898	C.	* Gray, Albert, A., M.D., 4 Clairmont Gardens, Glasgow	
1883	C.	Gray, Andrew, M.A., LL.D., F.R.S., Professor of Natural Philosophy in the University of Glasgow 225	1903–06. V-P 1906–09.
1910		Gray, Bruce M'Gregor, C.E., A.M.Inst.C.E., Westbourne Grove, Selby, Yorkshire	

Date of Election.			Service on Council, etc.
1909	C.	* Gray, James Gordon, D.Sc., Lecturer in Physics in the University of Glasgow, 11 The University, Glasgow	1913–15.
1918		* Gray, Wm. Forbes, F.S.A. (Scot.), Editor and Author, 8 Mansionhouse Road, Edinburgh	
1897		Greenlees, Thomas Duncan, M.D. Edin., Yèresco, Fordingbridge, Hants	
1905	C.	* Gregory, John Walter, D.Sc., F.R.S., Professor of Geology in the University of Glasgow, 4 Park Quadrant, Glasgow 230	1908–11.
1906		Greig, Edward David Wilson, C.I.E., M.D., D.Sc., Major, H.M. Indian Medical Service, United Service Club, Calcutta, India	
1905		Greig, Sir Robert Blyth, LL.D., F.Z.S., Board of Agriculture for Scotland, 29 St Andrew Square, Edinburgh	
1910		* Grimshaw, Percy Hall, Assistant Keeper, Natural History Department, The Royal Scottish Museum, 49 Comiston Drive, Edinburgh	
1899		* Guest, Edward Graham, M.A., B.Sc., 5 Newbattle Terrace, Edinburgh	
1907	C.	* Gulliver, Gilbert Henry, D.Sc., A.M.I.Mech.E., 99 Southwark Street, London, S.E. 235	
1911	C.	* Gunn, James Andrew, M.A., M.D., D.Sc., Department of Pharmacology, University Museum, Oxford	
1888	C.	Guppy, Henry Brougham, M.B., Rosario, Salcombe, Devon	
1916		* Guthrie, The Hon. Lord, LL.D., Judge of the Court of Session, 13 Royal Circus, Edinburgh	1918–
1911		* Guy, William, F.R.C.S., L.R.C.P., L.D.S.Ed., Consulting Dental Surgeon, Edinburgh Royal Infirmary; Dean, Edinburgh Dental Hospital and School; Lecturer on Human and Comparative Dental Anatomy and Physiology, 11 Wemyss Place, Edinburgh	
1911		Hall-Edwards, John Francis, L.R.C.P. (Edin.), Hon. F.R.P.S., Senior Medical Officer in charge of X-ray Department, General Hospital, Birmingham, 141A and 141B Great Charles Street (Newhall Street), Birmingham 240	
1918		* Hardie, P. S., M.A., B.Sc., Lecturer in Physics, Sultania Training College, Cairo, Egypt	
1896	C.	* Harris, David Fraser, B.Sc. (Lond.), D.Sc. (Birm.), M.D., F.S.A. Scot., Professor of Physiology in the Dalhousie University, Halifax, Nova Scotia	
1914		Harrison, Edward Philip, Ph.D., Professor of Physics, Presidency College, University of Calcutta, The Observatory, Alipore, Calcutta	
1917		* Harrison, John, O.B.E., J.P., LL.D., Convener of the Heriot-Watt College Committee, Chairman of the Edinburgh Public Library, Rockville, Napier Road, Edinburgh	
1888	C.	Hart, D. Berry, M.D., F.R.C.P.E., 13 Northumberland Street, Edinburgh 245	
1914	C.	Harvey-Gibson, Robert John, C.B.E., M.A., D.L. and J.P. for the County Palatine of Lancaster, Mem. Roy. Dub. Soc., Professor of Botany, University of Liverpool, 18 Gambier Terrace, Liverpool	
1880	C.	Haycraft, J. Berry, M.D., D.Sc., Professor of Physiology in the University College of South Wales and Monmouthshire, Cardiff	
1892	C.	Heath, Thomas, B.A., formerly Assistant Astronomer, Royal Observatory, Edinburgh, 11 Cluny Drive, Edinburgh	
1893		Hehir, Patrick, M.D., F.R.C.S.E., M.R.C.S., L.R.C.P.E., Surgeon-Captain, Indian Medical Service, Principal Medical Officer, H.H. the Nizam's Army, Hyderabad, Deccan, India	
1890	C.	Helme, T. Arthur, M.D., M.R.C.P., M.R.C.S., Tan y vron, Rhosneigr, Ty Croes, R.S.O., Anglesey 250	
1900		Henderson, John, D.Sc., A.Inst.E.E., Kinnoul, Warwick's Bench Road, Guildford, Surrey	
1908		* Henderson, William Dawson, M.A., B.Sc., Ph.D., Lecturer, Zoological Laboratories, University, Bristol	
1890	C.	Hepburn, David, M.D., Professor of Anatomy in the University College of South Wales and Monmouthshire, Cardiff	
1881	C. N.	Herdman, W. A., D.Sc., LL.D., F.R.S., Past Pres. L.S., Professor of Natural History in the University of Liverpool, Croxteth Lodge, Ullet Road, Liverpool	
1916		* Herring, Percy Theodore, M.D., F.R.C.P.Ed., Professor of Physiology, University of St Andrews, Hepburn Gardens, St Andrews 255	1917–
1894		Hill, Alfred, M.D., M.R.C.S., F.I.C., Valentine Mount, Freshwater Bay, Isle of Wight	
1902		Hinxman, Lionel W., B.A., formerly of the Geological Survey of Scotland, 8 Pier Terrace, West Bay, Bridport, Dorset	
1904		Hobday, Frederick T. G., F.R.C.V.S., 6 Berkely Gardens, Kensington, London, W.	

Date of Election.			Service on Council, etc.
1885		Hodgkinson, W. R., M.A., Ph.D., F.I.C., F.C.S., C.B.E., Professor of Chemistry and Physics at the Ordnance College, Woolwich, 89 Shooter's Hill Road, Blackheath, Kent	
1911		Holland, William Jacob, LL.D. St Andrews, etc., Director Carnegie Institute, Pittsburg, Pa., 5545 Forbes Street, Pittsburg, Pa., U.S.A. 260	
1881	C. N.	Horne, John, LL.D., F.R.S., F.G.S., formerly Director of the Geological Survey of Scotland, 20 Merchiston Gardens, Edinburgh	1902–05, 1906–07, 1914–15. V-P 1907–1913. P 1915–19.
1896		Horne, J. Fletcher, M.D., F.R.C.S.E., The Poplars, Barnsley	
1904	C.	* Horsburgh, Ellice Martin, M.A., D.Sc., Lecturer in Technical Mathematics, University of Edinburgh, 11 Granville Terrace, Edinburgh	
1897		Houston, Sir Alex. Cruikshanks, K.B.E., C.V.O., M.B., C.M., D.Sc., 19 Fairhazel Gardens, South Hampstead, London, N.W.	
1912	C. B.	* Houstoun, Robert Alexander, M.A., Ph.D., D.Sc., Lecturer in Physical Optics, University, Glasgow, 45 Kirklee Road, Glasgow 265	
1893		Howden, Robert, M.A., M.B., C.M., D.Sc., Professor of Anatomy in the University of Durham, 14 Burdon Terrace, Newcastle-upon-Tyne	
1883	C.	Hoyle, William Evans, M.A., D.Sc., M.R.C.S., Director of the Welsh National Museum: Crowland, Llandaff, Wales	
1910		Hume, William Fraser, D.Sc. (Lond.), Director, Geological Survey of Egypt, Helwân, Egypt	
1916		* Hunter, Charles Stewart, L.R.C.P.E., L.R.C.S.E., D.P.H., Medical Officer of Health, Carnoustie, Dalhousie Villa, Carnoustie	
1911		Hunter, Gilbert Macintyre, M.Inst.C.E., M.Inst.E.S., M.Inst.M.E., Resident Engineer Nitrate Railways, Iquique, Chile, and Maybole, Ayrshire 270	
1887	C.	Hunter, James, F.R.C.S.E., F.R.A.S., Rosetta, Liberton, Midlothian	
1887	C.	Hunter, William, M.D., M.R.C.P.L. and E., M.R.C.S., 103 Harley Street, London	
1908		Hyslop, Theophilus Bulkeley, M.D., M.R.C.P.E., 5 Portland Place, London, W.	
1912		* Inglis, Robert John Mathieson, A.M.Inst.C.E., 31 Buckingham Terrace, Glasgow, W.; Tantah, Peebles	
1904	C.	Innes, R. T. A., Director, Government Observatory, Johannesburg, Transvaal 275	
1917		* Irvine, James Colquhoun, Ph.D., D.Sc., F.R.S., Professor of Chemistry, University, St Andrews	
1914		Jack, John Noble	
1875		Jack, William, M.A., LL.D., Emeritus Professor of Mathematics in the University of Glasgow	1888–91.
1889		James, Alexander, M.D., F.R.C.P.E., 9 Randolph Crescent, Edinburgh	
1901		* Jardine, Robert, M.D., M.R.C.S., F.R.F.P.S. Glas., 20 Royal Crescent, Glasgow 280	
1912	C.	* Jeffrey, George Rutherford, M.D. (Glasg.), F.R.C.P. (Edin.), etc., Bootham Park Private Mental Hospital, York	
1906	C.	* Jehu, Thomas John, M.A., M.D., F.G.S., Professor of Geology in the University of Edinburgh: 23 Great King Street, Edinburgh	1917–
1900		* Jerdan, David Smiles, M.A., D.Sc., Ph.D., Temora, Colinton, Midlothian	
1916		* Johnston, Col. Sir Duncan A., K.C.M.G., C.B., Colonel Royal Engineers, 8 Lansdowne Crescent, Edinburgh	
1895		Johnston, Col. Henry Halcro, C.B., late A.M.S., D.Sc., M.D., F.L.S., Orphir House, Kirkwall, Orkney 285	
1903	C.	* Johnston, Thomas Nicol, M.B., C.M., Pogbie, Humbie, East Lothian	
1874		Jones, Francis, M.Sc., Lecturer on Chemistry, 17 Whalley Road, Whalley Range, Manchester	
1888		Jones, John Alfred, M.Inst.C.E., Fellow of the University of Madras, Sanitary Engineer to the Government of Madras, c/o Messrs Parry & Co., 70 Gracechurch Street, London	
1915		Kemnal, Sir James Hermann Rosenthal, Managing Director and Engineer-in-Chief of Babcock & Wilcox, Ltd., Kemnal Manor, Chislehurst, Kent	
1912		Kennedy, Robert Foster, M.D. (Queen's Univ., Belfast), M.B., B.Ch. (R.U.I.), Assistant Professor of Neurology, Cornell University, New York, 20 West 50th Street, New York, U.S.A. 290	
1909		Kenwood, Henry Richard, M.B., Chadwick Professor of Hygiene in the University of London, 126 Queen's Road, Finsbury Park, London, N.	

Date of Election.			Service on Council, etc.
1908		* Kerr, Andrew William, F.S.A. Scot., Royal Bank House, St Andrew Square, Edinburgh	
1891		Kerr, Joshua Law, M.D., Worthen, Shropshire	
1913		* Kerr, Walter Hume, M.A., B.Sc., Lecturer on Engineering Drawing and Structural Design in the University of Edinburgh	
1908		Kidd, Walter Aubrey, M.D., 2 Suffolk Square, Cheltenham 295	
1886	C. N.	Kidston, Robert, LL.D., F.R.S., F.G.S. (VICE-PRESIDENT), 12 Clarendon Place, Stirling	1891–94, 1903–06. Sec. 1909–16. V-P 1917–
1907		* King, Archibald, M.A., B.Sc., formerly Rector of the Academy, Castle Douglas; Junior Inspector of Schools, La Maisonnette, Clarkston, Glasgow	
1880		King, W. F., Lonend, Russell Place, Trinity, Leith	
1918		* Kingon, Rev. John Robert Lewis, M.A. (Edin. and Cape of Good Hope), F.L.S., Missionary of the U.F. Church of Scotland, St Andrew's Manse, Port Elizabeth, C.P., South Africa	
1878		Kintore, The Right Hon. the Earl of, P.C., G.C.M.G., M.A. Cantab., LL.D. Cambridge, Aberdeen, and Adelaide, Keith Hall, Inverurie, Aberdeenshire 300	
1901		* Knight, Rev. G. A. Frank, M.A., 5 Granby Terrace, Hillhead, Glasgow	
1907		* Knight, James, M.A., D.Sc., F.C.S., F.G.S., Head Master, John Street Higher Grade School, Bridgeton, Glasgow, The Shieling, Uddingston, by Glasgow	
1880	C. K.	Knott, C. G., D.Sc., LL.D., Lecturer on Applied Mathematics in the University of Edinburgh, formerly Professor of Physics, Imperial University, Japan (GEN. SECRETARY), 42 Upper Gray Street, Edinburgh	1894–97, 1898–1901, 1902–05. Sec. 1905–12. Gen. Sec. 1912–
1878	C.	Lang, P. R. Scott, M.A., B.Sc., Professor of Mathematics, University of St Andrews	
1910	C.	* Lauder, Alexander, D.Sc., Lecturer in Agricultural Chemistry, Edinburgh and East of Scotland College of Agriculture, 13 George Square, Edinburgh 305	1917–
1885	C.	Laurie, A. P., M.A., D.Sc., J.P., Principal of the Heriot-Watt College, Edinburgh	1908–11, 1913–16.
1894	C.	Laurie, Malcolm, B.A., D.Sc., F.L.S., 4 Wordsworth Road, Harpenden, Herts	
1910	C. B.	* Lawson, A. Anstruther, B.Sc., Ph.D., D.Sc., F.L.S., Professor of Botany, University of Sydney, New South Wales, Australia	
1905		* Lawson, David, M.A., M.D., L.R.C.P. and S.E., Druimdarroch, Banchory, Kincardineshire	
1910	C.	* Lee, Gabriel W., D.Sc., Palæontologist, Geological Survey of Scotland, 33 George Square, Edinburgh 310	
1903		* Leighton, Gerald Rowley, M.D., Local Government Board, 125 George Street, Edinburgh	
1910		Levie, Alexander, F.R.C.V.S., D.V.S.M., Veterinary Surgeon, Lecturer on Veterinary Science, Veterinary Infirmary, 12 Derwent Street, Derby	
1916	C.	* Levy, Hyman, M.A., B.Sc., Research Assistant, Aeronautical Section, National Physical Laboratory, Teddington, Middlesex	
1914	C. N.	Lewis, Francis John, D.Sc., F.L.S., Professor of Biology, University of Alberta, Edmonton South, Alberta, Canada	
1918		* Lidstone, George James, F.F.A., F.I.A., Manager and Actuary of the Scottish Widows' Fund Life Assurance Society, 8 Eglinton Crescent, Edinburgh 315	1919–
1905		* Lightbody, Forrest Hay, 53 Queen Street, Edinburgh	
1889		Lindsay, Rev. James, M.A., D.D., B.Sc., F.R.S.L., F.G.S., M.R.A.S., Corresponding Member of the Royal Academy of Sciences, Letters and Arts, of Padua, Associate of the Philosophical Society of Louvain, Annick Lodge, Irvine	
1912		* Lindsay, John George, M.A., B.Sc. (Edin.), Rector of Dunfermline High School	
1912		* Linlithgow, The Most Honourable the Marquis of, Hopetoun House, South Queensferry	
1903		Liston, William Glen, M.D., Captain, Indian Medical Service, c/o Grindlay, Groom & Co., Bombay, India 320	
1903		* Littlejohn, Henry Harvey, M.A., M.B., B.Sc., F.R.C.S.E., Professor of Forensic Medicine, Dean of the Faculty of Medicine in the University of Edinburgh, 11 Rutland Street, Edinburgh	

Date of Election.			Service on Council, etc.
1898		* Lothian, Alexander Veitch, M.A., B.Sc., Training College, Cowcaddens, Glasgow	
1884		Low, George M., Actuary, 11 Moray Place, Edinburgh	
1888		Lowe, D. F., M.A., LL.D., formerly Headmaster of Heriot's Hospital School, Lauriston, 19 George Square, Edinburgh	
1900		Lusk, Graham, Ph.D., M.A., Professor of Physiology, Cornell University Medical College, New York, N.Y., U.S.A. 325	
1894		Mabbott, Walter John, M.A., Rector of County High School, Duns, Berwickshire	
1887		M'Aldowie, Alexander M., M.D., 8 Holland Road, Cheltenham	
1917		* Macalister, Sir Donald, K.C.B., Principal of the University of Glasgow, The University, Glasgow	
1907		MacAlister, Donald Alexander, A.R.S.M., F.G.S., The Bath Club, 34 Dover Street, London, W.	
1883		M'Bride, P., M.D., F.R.C.P.E., 10 Park Avenue, Harrogate, and Hill House, Withypool, Dunster, Somerset 330	
1903		* M'Cormick, Sir W. S., M.A., LL.D., Secretary to the Carnegie Trust for the Universities of Scotland, 13 Douglas Crescent, Edinburgh	1910–13.
1918		* M'Culloch, Rev. James David, D.D., 43 Brougham Street, Greenock	
1905		* Macdonald, Hector Munro, M.A., F.R.S., Professor of Mathematics, University of Aberdeen, 52 College Bounds, Aberdeen	1908–11.
1897	C.	* Macdonald, James A., M.A., B.Sc., H.M. Inspector of Schools, Stewarton, Kilmacolm	
1904		* Macdonald, John A., M.A., B.Sc., King Edward VII School, Johannesburg, Transvaal 335	
1904		Macdonald, William, M.S.Agr., Sc.D., Ph.D., D.Sc., Editor, *Agricultural Journal* of South Africa, Rand Club, Johannesburg, Transvaal	
1886		Macdonald, William J., M.A., LL.D., 15 Comiston Drive, Edinburgh	
1901	C.	* MacDougall, R. Stewart, M.A., D.Sc., Professor of Biology, Royal Veterinary College, Edinburgh, 9 Dryden Place, Edinburgh	1914–17.
1910		Macewen, Hugh Allen, M.B., Ch.B., D.P.H. (Lond. and Camb.), Local Government Board, Whitehall, London, S.W.	
1888	C.	McFadyean, Sir John, Kt., M.B., B.Sc., LL.D., Principal, and Professor of Comparative Pathology in the Royal Veterinary College, Camden Town, London 340	
1885	C.	Macfarlane, J. M., D.Sc., Professor of Botany and Director of the Botanic Garden, University of Pennsylvania, Philadelphia, Pennsylvania, U.S.A.	
1897		* MacGillivray, Angus, C.M., M.D., D.Sc., F.S.A. (Scot.), 23 South Tay Street, Dundee	
1878		M'Gowan, George, F.I.C., Ph.D., 21 Montpelier Road, Ealing, London, W. 5	
1903		* M'Intosh, Donald C., M.A., D.Sc., Education Offices, Elgin	
1911		M'Intosh, John William, A.R.C.V.S., Dollis Hill Farm, Cricklewood, London, N.W. 2 345	
1869	C. N.	M'Intosh, William Carmichael, M.D., LL.D., F.R.S., F.L.S., Emeritus Professor of Natural History in the University of St Andrews, Pres. Ray Society, Nevay Park, Meigle	1885–88.
1895	C.	Macintyre, John, M.D., 179 Bath Street, Glasgow	
1914		* M'Kendrick, Archibald, F.R.C.S.E., D.P.H., L.D.S., 11 Rothesay Place, Edinburgh	
1873	C. B.	M'Kendrick, John G., M.D., F.R.C.P.E., LL.D., F.R.S., Emeritus Professor of Physiology in the University of Glasgow, Maxieburn, Stonehaven	1875–78, 1885–88, 1893–94, 1900–02. V-P 1894–1900.
1912	C.	M'Kendrick, Anderson Gray, M.B., Major, Indian Medical Service, Officiating Statistical Officer to the Government of India, The Pasteur Institute, Kasauli, India 350	
1900	C.	* M'Kendrick, John Souttar, M.D., F.R.F.P.S.G., 2 Buckingham Terrace, Hillhead, Glasgow	
1910	C.	* Mackenzie, Alister, M.A., M.D., D.P.H., Principal, College of Hygiene and Physical Training, Dunfermline	
1916	C.	* Mackenzie, John E., D.Sc., Lecturer in Chemistry, University of Edinburgh, Major-Adjutant, O.T.C., 2A Ramsay Garden, Edinburgh	
1894		Mackenzie, Robert, M.D., Napier, Nairn	
1904	C.	* Mackenzie, Sir W. Leslie, M.A., M.D., D.P.H., LL.D., Medical Member of the Local Government Board for Scotland, 4 Clarendon Crescent, Edinburgh 355	
1918		* Mackie, Wm., M.A., M.D., D.P.H., 13 North Street, Elgin	
1910		* MacKinnon, James, M.A., Ph.D., Professor of Ecclesiastical History, Edinburgh University, 12 Lygon Road, Edinburgh	

Date of Election.			Service on Council, etc.
1904		* Mackintosh, Donald James, M.V.O., M.B., C.M., LL.D., Supt., Western Infirmary, Glasgow	
1899		Maclean, Ewan John, M.D., M.R.C.P. Lond., 12 Park Place, Cardiff	
1888	C.	Maclean, Magnus, M.A., D.Sc., LL.D., M.Inst.C.E., M.I.E.E., Professor of Electrical Engineering in the Royal Technical College, 51 Kerrsland Terrace, Hillhead, Glasgow 360	1916-19.
1913		* M'Lellan, Dugald, M.Inst.C.E., District Engineer, Caledonian Railway, 20 Kingsburgh Road, Murrayfield, Edinburgh	
1916	C.	* M'Lintock, W. F. P., D.Sc. (Edin.), Royal Scottish Museum, Edinburgh	
1907	C.	* Macnair, Peter, Curator of the Natural History Collections in the Glasgow Museums, Kelvingrove Museum, Glasgow	
1917		* Macpherson, Rev. Hector Copland, M.A., F.R.A.S., Minister of the U.F. Church of Scotland, Loudoun United Free Manse, Newmilns, Ayrshire	
1898	C.	Mahâlanobis, S. C., B.Sc., Professor of Physiology, Presidency College, Calcutta, India 365	
1913		Majumdar, Tarak Nath, D.P.H. (Cal.), L.M.S., F.C.S., Health Officer, Calcutta, IV, 37 Lower Chitpore Road, Calcutta, India	
1917		* Malcolm, Louis William Gunther, M.A. (Melbourne), Capt. R.G.A., Christ's College, Cambridge	
1908		Mallik, Devendranath, Sc.D., B.A., Professor of Mathematics, Astronomical Observatory, Presidential College, Calcutta, India	
1912		Maloney, William Joseph, M.D. (Edin.), Professor of Neurology at Fordham University, New York City, N.Y., U.S.A.	
1913		Marchant, Rev. James, F.R.A.S., F.L.S., Director, National Council for Promotion of Race-Regeneration, 20 Bedford Square, London, W.C. 370	
1909	C. B.	* Marshall, C. R., M.D., M.A., Professor of Materia Medica, Marischal College, Aberdeen	1915-18.
1882	C.	Marshall, D. H., M.A., Professor, Union and Alwington Avenue, Kingston, Ontario, Canada	
1901	C.	Marshall, F. H. A., Sc.D., Lecturer on Agricultural Physiology in the University of Cambridge, Christ's College, Cambridge	
1912		* Martin, Sir Thomas Carlaw, LL.D., J.P., Director, Royal Scottish Museum, 18 Blackford Road, Edinburgh	
1913		Masson, George Henry, M.D., D.Sc., M.R.C.P.E., Port of Spain, Trinidad, British West Indies 375	
1885	C.	Masson, Orme, D.Sc., F.R.S., Professor of Chemistry in the University of Melbourne	
1898	C. B.	* Masterman, Arthur Thomas, M.A., D.Sc., Inspector of Fisheries, Board of Agriculture, Whitehall, London	1902-04.
1911		Mathews, Gregory Macalister, F.L.S., F.Z.S., Foulis Court, Fair Oaks, Hants	
1906		* Mathieson, Robert, F.C.S., St Serf's, Innerleithen	
1902		Matthews, Ernest Romney, A.M.Inst.C.E., F.G.S., Chadwick Professor of Municipal Engineering in the University of London, University College, Gower Street, London, W.C. 380	
1917		* Maylard, A. Ernest, M.B., B.Sc. (Lond.), F.R.F.P.S. (Glasgow), 1 Windsor Terrace, W., Great Western Road, Glasgow	
1901	C.	* Menzies, Alan W. C., M.A., B.Sc., Ph.D., F.C.S., Professor of Chemistry, Princeton University, Princeton, New Jersey, U.S.A.	
1917		* Merson, George Fowlie, Manufacturing Technical Chemist, 9 Hampton Terrace, Edinburgh	
1888		Methven, Cathcart W., M.Inst.C.E., F.R.I.B.A., Durham, Natal, S. Africa	
1902	C.	Metzler, William H., A.B., Ph.D., Corresponding Fellow of the Royal Society of Canada, Professor of Mathematics, Syracuse University, Syracuse, N.Y., U.S.A. 385	
1885	C. B.	Mill, Hugh Robert, D.Sc., LL.D., Hill Crest, Dorman's Park, E. Grinstead, London	
1908		* Miller, Alexander Cameron, M.D., F.S.A. Scot., Craig Linnhe, Fort-William, Inverness-shire	
1910		* Miller, John, M.A., D.Sc., Professor of Mathematics, Royal Technical College, 2 Northbank Terrace, North Kelvinside, Glasgow	
1909		Mills, Bernard Langley, M.D., F.R.C.S.E., M.R.C.S., D.P.H., Lt.-Col. R.A.M.C., formerly Army Specialist in Hygiene, c/o National Provincial Bank, Fargate, Sheffield	
1905		* Milne, Archibald, M.A., D.Sc., Lecturer on Mathematics and Science, Edinburgh Provincial Training College, 108 Comiston Drive, Edinburgh 390	
1905		* Milne, C. H., M.A., Head Master, Daniel Stewart's College, 4 Campbell Road, Murrayfield, Edinburgh	
1904	C.	* Milne, James Robert, D.Sc., Natural Philosophy Dept., University, Edinburgh	

Date of Election.			Service on Council, etc.
1886		Milne, William, M.A., B.Sc., 70 Beechgrove Terrace, Aberdeen	
1899		* Milroy, T. H., M.D., B.Sc., Professor of Physiology in Queen's College, Belfast, Meloyne, Malone Park, Belfast	
1889	C.	Mitchell, A. Crichton, D.Sc., Hon. Doc. Sc. (Genève), formerly Director of Public Instruction in Travancore, India (CURATOR OF LIBRARY AND MUSEUM), The Observatory, Eskdalemuir, Langholm, Dumfriesshire 395	1915–16. Cur. 1916–
1897		Mitchell, George Arthur, M.A., 9 Lowther Terrace, Kelvinside, Glasgow	
1900		* Mitchell, James, M.A., B.Sc., Monydrain, Lochgilphead	
1911		Modi, Edalji Manekji, D.Sc., LL.D., Litt.D., F.C.S., etc., Proprietor and Director of Arthur Road Chemical Works, Meher Buildings, Tardeo, Bombay, India	
1906	C.	Moffat, Rev. Alexander, M.A., B.Sc., Professor of Physical Science, Christian College, Madras, India	
1890	C.	Mond, R. L., M.A. Cantab., F.C.S., Combe Bank, near Sevenoaks, Kent 400	
1887	C.	Moos, N. A. F., L.C.E., B.Sc., Professor of Physics, Elphinstone College, and Director of the Government Observatory, Colaba, Bombay, India	
1896		* Morgan, Alexander, M.A., D.Sc., Principal, Edinburgh Provincial Training College, 1 Midmar Gardens, Edinburgh	
1919		* Morris, Robert Owen, M.A., M.D., C.M. (Edin.), D.P.H. (Liverpool), Tuberculosis Institute, Newtown, N. Wales	
1892	C.	Morrison, J. T., M.A., B.Sc., Professor of Physics and Chemistry, Victoria College, Stellenbosch, Cape Colony	
1914		Mort, Spencer, M.D., Ch.B., F.R.C.S.E., Lieut.-Col. R.A.M.C., Medical Officer in Charge, Edmonton Military Hospital, Silver Street, Upper Edmonton, London, N. 405	
1901		Moses, O. St John, I.M.S., M.D., D.Sc., F.R.C.S., Captain, Professor of Medical Jurisprudence, c/o Messrs King, Hamilton & Co., 4 and 5 Koila Ghat Street, Calcutta, India	
1892	C. K.	Mossman, R. C., Fernbank, South Morton Street, Joppa, Edinburgh	
1916		* Muir, Robert, M.A., M.D., Sc.D., F.R.S., Professor of Pathology, University of Glasgow, 16 Victoria Crescent, Dowanhill, Glasgow	
1874	C. K. V. J.	Muir, Sir Thomas, C.M.G., M.A., LL.D., F.R.S., Superintendent-General of Education for Cape Colony, Education Office, Cape Town, and Elmcote, Sandown Road, Rondebosch, South Africa	1885–88. V-P 1888–91.
1888	C.	Muirhead, George, Commissioner to His Grace the Duke of Richmond and Gordon, K.G., Speybank, Fochabers 410	
1907		Muirhead, James M. P., J.P., F.R.S.L., F.S.S., Park House, Maitland Park Road, London, N.W. 3	
1887		Mukhopâdhyay, Asûtosh, M.A., LL.D., F.R.A.S., M.R.I.A., Professor of Mathematics at the Indian Association for the Cultivation of Science, 77 Russa Road North, Bhowanipore, Calcutta, India	
1891	C.	Munro, Robert, M.A., M.D., LL.D., Hon. Memb. R.I.A., Hon. Memb. Royal Society of Antiquaries of Ireland, Elmbank, Largs, Ayrshire	1894–97, 1900–03. V-P 1903–08.
1896		* Murray, Alfred A., M.A., LL.B., 20 Warriston Crescent, Edinburgh	
1907		Musgrove, James, M.D., F.R.C.S. Edin. and Eng., LL.D., Emeritus-Professor of Anatomy, University of St Andrews, The Swallowgate, St Andrews 415	
1902		Mylne, Rev. R. S., M.A., B.C.L. Oxford, F.S.A. Lond., Great Amwell, Herts	
1888		Napier, A. D. Leith, M.D., C.M., M.R.C.P., 46 Austral Terrace, Malvern, S. Australia	
1897		Nash, Alfred George, B.Sc., F.R.G.S., C.E., Belretiro, Mandeville, Jamaica, W.I.	
1898		Newman, Sir George, M.D., D.P.H., Cambridge, Lecturer on Preventive Medicine, St Bartholomew's Hospital, University of London: Grim's Wood, Harrow Weald, Middlesex	
1884		Nicholson, J. Shield, M.A., D.Sc., Professor of Political Economy in the University of Edinburgh, 3 Belford Park, Edinburgh 420	1885–87, 1892–95, 1897–1900.
1880	C.	Nicol, W. W. J., M.A., D.Sc., 15 Blacket Place, Edinburgh	
1878		Norris, Richard, M.D., M.R.C.S. Eng., 3 Walsall Road, Birchfield, Birmingham	
1888		Ogilvie, Sir F. Grant, C.B., M.A., B.Sc., LL.D., Secretary of the Board of Education for the Science Museum and the Geological Survey, and Director of the Science Museum, 15 Evelyn Gardens, London, S.W.	1901–03.
1888		Oliphant, James, M.A., 11 Heathfield Park, Willesden Green, London	

Date of Election.			Service on Council, etc.
1886		Oliver, James, M.D., F.L.S., Physician to the London Hospital for Women, 123 Harley Street, London, W. 425	
1895	C.	Oliver, Sir Thomas, M.D., LL.D., F.R.C.P., Professor of Physiology in the University of Durham, 7 Ellison Place, Newcastle-upon-Tyne	
1915		* Orr, Lewis P., F.F.A., Manager of Scottish Life Assurance Co., 14 Learmonth Gardens, Edinburgh	
1914		* Oswald, Alfred, Lecturer in German, Glasgow Provincial Training College, 11 Nelson Terrace, Hillhead, Glasgow	
1908		Page, William Davidge, F.C.S., F.G.S., M.Inst.M.E., 10 Clifton Dale, York	
1905		Pallin, William Alfred, F.R.C.V.S., Veterinary-Major, Royal Horse Guards, London 430	
1914		Pare, John William, M.D., C.M., L.D.S., Lecturer in Dental Anatomy, National Dental Hospital, 9A Cavendish Square, London, W.	
1901		* Paterson, David, F.C.S., Lea Bank, Rosslyn, Midlothian	
1918		* Paterson, Rev. William Paterson, D.D., LL.D., Professor of Divinity, University, Edinburgh, 3 Royal Terrace, Edinburgh	
1886	C.	Paton, D. Noël, M.D., B.Sc., LL.D., F.R.C.P.E., F.R.S. (Vice-President), Professor of Physiology in the University of Glasgow, University, Glasgow	1894–97, 1904–06, 1909–12. V-P 1918–
1919	C.	* Patterson, Thomas Stewart, D.Sc. (London and Glasgow), Ph.D. (Heidelberg), Professor of Organic Chemistry in the University of Glasgow, 10 Oakfield Terrace, Hillhead, Glasgow 435	
1892		Paulin, Sir David, Actuary, 6 Forres Street, Edinburgh	
1881	C. N.	Peach, Benjamin N., LL.D., F.R.S., F.G.S., formerly District Superintendent and Acting Palæontologist of the Geological Survey of Scotland, 72 Grange Loan, Edinburgh	1905–08, 1911–12. V-P 1912–17.
1907		* Pearce, John Thomson, B.A., B.Sc., School House, Tranent	
1914		Pearson, Joseph, D.Sc., F.L.S., Director of the Colombo Museum, and Marine Biologist to the Ceylon Government, Colombo Museum, Ceylon	
1904		* Peck, James Wallace, C.B., M.A., Senior Assistant-Secretary, Ministry of Food, London, Caledonian Club, St. James's Sq., London, S.W. 1 440	
1889		Peck, Sir William, F.R.A.S., Town's Astronomer, City Observatory, Calton Hill, Edinburgh	
1887	C. B.	Peddie, Wm., D.Sc. (Vice-President), Professor of Natural Philosophy in University College, Dundee, The Weisha, Ninewells, Dundee	1904–07, 1908–11. V.P. 1919–
1893		Perkin, Arthur George, F.R.S., Grosvenor Lodge, Grosvenor Road, Leeds	
1913	C.	Philip, Alexander, M.A., LL.B., Writer, The Mary Acre, Brechin	
1889		Philip, Sir R. W., M.A., M.D., F.R.C.P.E., 45 Charlotte Square, Edinburgh 445	
1907	C.	Phillips, Major Charles E. S., O.B.E., 54 Bedford Gardens, London, W. 8.	
1914		* Pilkington, Basil Alexander, "Kambla," Davidson's Mains	
1905		* Pinkerton, Peter, M.A., D.Sc., Rector, High School, Glasgow, 44 Hamilton Park Terrace, Hillhead, Glasgow	
1908	C.	* Pirie, James Hunter Harvey, B.Sc., M.D., F.R.C.P.E., Superintendent of the Routine Division of The South African Institute for Medical Research, P.O. Box 1038, Johannesburg, South Africa	
1911		* Pirie, James Simpson, Civil Engineer, 28 Scotland Street, Edinburgh 450	
1906		Pitchford, Herbert Watkins, F.R.C.V.S.	
1919		* Porritt, B. D., M.Sc. (Lond.), F.I.C., Chief Chemist, North British Rubber Co., Ltd., Edinburgh, 64 Strathearn Road, Edinburgh	
1888		Prain, Sir David, Lt.-Col., Indian Medical Service (Retired), C.M.G., C.I.E., M.A., M.B., LL.D., F.L.S., F.R.S., For. Memb. K. Svensk. Vetensk. Akad.; Hon. Memb. Soc. Lett. ed Arti d. Zelanti, Acireale; Pharm. Soc. Gt. Britain; Corr. Memb. K. Bayer Akad. Wiss., etc.; Director, Royal Botanic Gardens, Kew, Surrey	
1902		* Preller, Charles du Riche, M.A., Ph.D., A.M.Inst.C.E., M.I.E.E., F.G.S., 61 Melville Street, Edinburgh	
1892		Pressland, Arthur J., M.A. Camb., Edinburgh Academy 455	
1875	C.	Prevost, E. W., Ph.D., Weston, Ross, Herefordshire	
1915		Price, Frederick William, M.D., M.R.C.P. Edin., Physician to the Great Northern Hospital, London, 133 Harley Street, London, W.	

Date of Election.				Service on Council, etc.
1908		* Pringle, George Cossar, M.A., Rector of Peebles Burgh and County High School, Bloomfield, Peebles		
1903		* Pullar, Laurence, Dunbarney, Bridge of Earn, Perthshire		
1911		Purdy, John Smith, M.D., C.M. (Aberd.), D.P.H. (Camb.), F.R.G.S., Chief Health Officer for Tasmania, Islington, Hobart, Tasmania	460	
1898		* Purves, John Archibald, D.Sc., 52 Queen Street, Exeter		
1897	C.	* Rainy, Harry, M.A., M.B., C.M., F.R.C.P.Ed., 16 Great Stuart Street, Edinburgh		
1899	C.	* Ramage, Alexander G., Marchfield, Davidson's Mains, Midlothian		
1884		Ramsay, E. Peirson, M.R.I.A., F.L.S., C.M.Z.S., F.R.G.S., F.G.S., Fellow of the Imperial and Royal Zoological and Botanical Society of Vienna, formerly Curator of Australian Museum, Sydney, N.S.W. : "Truro," Queensborough Road, Croydon, N.S.W.		
1914		* Ramsay, Peter, M.A., B.Sc., Head Mathematical Master, George Watson's College, 63 Comiston Drive, Edinburgh	465	
1911		* Rankin, Adam A., British Astronomical Association, West of Scotland Branch, 24 Woodend Drive, Jordanhill, Glasgow		
1891		Rankine, John, K.C., M.A., LL.D., Professor of the Law of Scotland in the University of Edinburgh, 23 Ainslie Place, Edinburgh		
1904		Ratcliffe, Joseph Riley, M.B., C.M., c/o The Librarian, The University, Birmingham		
1900		Raw, Nathan, C.M.G., M.D., M.P., 58 Harley Street, Cavendish Square, London, W. 1.		
1883	C.	Readman, J. B., D.Sc., F.C.S., Belmont, Hereford	470	
1902		Rees-Roberts, John Vernon, M.D., D.Sc., D.P.H., 11 Oak Hill Park, Hampstead, London, N.W. 3		
1902		Reid, George Archdall O'Brien, M.B., C.M., 9 Victoria Road South, Southsea, Hants		
1913		Reid, Harry Avery, F.R.C.V.S., D.V.H., Bacteriologist and Pathologist, Department of Agriculture, Wellington, New Zealand		
1908	C.	* Rennie, John, D.Sc., Lecturer on Parasitology and Experimental Zoology, University of Aberdeen, 60 Desswood Place, Aberdeen		
1914		Renshaw, Graham, M.D., M.R.C.S., L.R.C.P., L.S.A., Editor of the *Avicultural Magazine*, Sale Bridge House, Sale, Manchester	475	
1913		* Richardson, Harry, M.Inst.E.E., M.Inst.M.E., General Manager and Chief Engineer, Electricity Supply, Dundee and District, The Cottage, Craigie, Broughty Ferry		
1908		Richardson, Linsdall, F.G.S., 10 Oxford Parade, Cheltenham, Glos.		
1875		Richardson, Ralph, W.S., 29 Eglinton Crescent, Edinburgh		
1916	C.	* Ritchie, James, M.A., D.Sc., Royal Scottish Museum, 20 Upper Gray Street, Edinburgh		
1914	C.	* Ritchie, James Bonnyman, D.Sc., Science Master, Kelvinside Academy, Glasgow	480	
1906	C.	* Ritchie, William Thomas, M.D., F.R.C.P.E., Athelstaneford, Colinton, Midlothian		
1898	C.	Roberts, Alexander William, D.Sc., F.R.A.S., Lovedale, South Africa		
1919		* Roberts, Alfred Henry, O.B.E., M.Inst.C.E., Superintendent and Engineer, Leith Docks, 46 Netherby Road, Edinburgh		
1880		Roberts, D. Lloyd, M.D., F.R.C.P.L., 23 St John's Street, Manchester		
1900		* Robertson, Joseph M'Gregor, M.B., C.M., 26 Buckingham Terrace, Glasgow	485	
1902	C.	* Robertson, Robert A., M.A., B.Sc., Lecturer on Botany in the University of St Andrews		
1919		* Robertson, William Alexander, F.F.A., Century Insurance Co., Ltd., 18 Charlotte Square ; 12 Lonsdale Terrace, Edinburgh		
1896	C.	* Robertson, W. G. Aitchison, D.Sc., M.D., F.R.C.P.E., The Grange, Ashford, Middlesex		
1910	C.	* Robinson, Arthur, M.D., M.R.C.S. (Vice-President), Professor of Anatomy, University of Edinburgh, 35 Coates Gardens, Edinburgh		1910-12. Sec. 1912-18. V-P 1918-
1916		* Ronald, David, Civil Engineer, Chief Engineer, Scottish Board of Health, 125 George Street, Edinburgh	490	
1881		Rosebery, The Right Hon. the Earl of, K.G., K.T., LL.D., D.C.L., F.R.S., Dalmeny Park, Edinburgh		

Date of Election.			Service on Council, etc
1909	C.	* Ross, Alex. David, M.A., D.Sc., F.R.A.S., Professor of Mathematics and Physics, University of Western Australia, Perth, Western Australia	
1906		* Russell, Alexander Durie, B.Sc., Mathematical Master, Falkirk High School, 14 Heugh Street, Falkirk	
1902	C. K.	* Russell, James, 22 Glenorchy Terrace, Edinburgh	
1906		Saleeby, Caleb William, M.D., 10 Campden Mansions, Kensington, London, W. 8 495	
1916		* Salvesen, The Hon. Lord E. T., Judge of the Court of Session, Dean Park House, Edinburgh	
1914		* Salvesen, Theodore Emile, 37 Inverleith Place, Edinburgh	
1912	C.	* Sampson, Ralph Allen, M.A., D.Sc., F.R.S., Astronomer Royal for Scotland, Professor of Astronomy, University, Edinburgh, Royal Observatory, Edinburgh	1912–15. 1919– V-P 1915–18.
1903		* Samuel, Sir John S., K.B.E., 177 West Regent Street, Glasgow	
1903		* Sarolea, Charles, Ph.D., D.Litt., Professor of French, University of Edinburgh, 21 Royal Terrace, Edinburgh 500	
1900	C.	* Schafer, Sir Edward Albert Sharpey, M.D., LL.D., D.Sc., F.R.S., Professor of Physiology in the University of Edinburgh	1900–03, 1906–09. 1918–19 V-P 1913–17.
1919		* Scott, Alexander, M.A., D.Sc., Carnegie Scholar, 1912–14; 1851 Exhibition Scholar, 1914–16; lectured (temp.) on Petrology, Oxford, 1914–15, and at Glasgow University, 1917–18; Physical Chemist in charge of Radiometric Laboratory, Glasgow University, 1916–18; Chief Assistant to Principal, Pottery Laboratory, Stoke-on-Trent	
1885	C.	Scott, Alexander, M.A., D.Sc., F.R.S., 34 Upper Hamilton Terrace, London, N.W.	
1919		* Scott, Alexander Ritchie, B.Sc. (Edin.), D.Sc. (Lond.), Director of Returns, Local Authorities Dept., Ministry of Food, Deputy Director of Statistics, Ministry of Food, 79 Fawnbrake Avenue, Herne Hill, London, S.E. 24	
1917		* Scott, Henry Harold, M.D., M.R.C.P., L.R.C.P. (London), M.R.C.S. (Eng.), D.P.H., Bacteriologist and Pathologist to the Government of Hong Kong 505	
1908		* Simpson, George Freeland Barbour, M.D., F.R.C.P.E., F.R.C.S.E., 43 Manor Place, Edinburgh	
1900	C.	* Simpson, James Young, M.A., D.Sc., Professor of Natural Science in the New College, Edinburgh, 25 Chester Street, Edinburgh	
1911	C.	Simpson, Sutherland, M.D., D.Sc. (Edin.), Professor of Physiology, Medical College, Cornell University, Ithaca, N.Y., U.S.A., 118 Eddy Street, Ithaca, N.Y., U.S.A.	
1900		Sinhjee, Sir Bhagvat, G.C.I.E., M.D., LL.D. Edin., H.H. the Thakur Sahib of Gondal, Gondal, Kathiawar, Bombay, India	
1903		* Skinner, Robert Taylor, M.A., Head Master, Donaldson's Hospital, Edinburgh 510	
1901		* Smart, Edward, B.A., B.Sc., Tillyloss, Tullylumb Terrace, Perth	
1891	C. K.	Smith, Alexander, B.Sc., Ph.D., LL.D., Department of Chemistry, Columbia University, New York, N.Y., U.S.A.	
1882	C.	Smith, C. Michie, C.I.E., B.Sc., F.R.A.S., formerly Director of the Kodaikânal and Madras Observatories, Winsford, Kodaikânal, South India	
1917		* Smith, Sir George Adam, M.A., D.D., LL.D., Litt.D., Principal and Vice-Chancellor of Aberdeen University, Chanonry Lodge, Old Aberdeen	
1915		* Smith, James Lorrain, M.A., M.D., F.R.S., Professor of Pathology, University of Edinburgh, 9 Carlton Terrace, Edinburgh 515	1918–
1911		* Smith, Stephen, B.Sc., Engineer, 31 Grange Loan, Edinburgh	
1907	C.	Smith, William Ramsay, D.Sc., M.D., C.M., Permanent Head of the Health Department, South Australia, Belair, South Australia	
1880		Smith, Sir William (Robert), M.D., D.Sc., LL.D., Principal of The Royal Institute of Public Health, Em.-Professor of Forensic Medicine and Toxicology in King's College, University of London, 36 Russell Square, London, W.C. 1	
1919		* Smith, William Wright, M.A. (Edin.), Assistant Keeper, Royal Botanic Garden, Edinburgh, 6 Lennox Row, Trinity, Edinburgh	
1899		Snell, Ernest Hugh, M.D., B.Sc., D.P.H. Camb., Medical Officer of Health, Coventry 520	

Date of Election.			Service on Council, etc.
1880		Sollas, W. J., M.A., D.Sc., LL.D., F.R.S., Fellow of University College, Oxford, and Professor of Geology and Palæontology in the University of Oxford	
1910		* Somerville, Robert, B.Sc., Science Master, High School, Dunfermline, 31 Cameron Street, Dunfermline	
1889		Somerville, Wm., M.A., D.Sc., D.Oec., Sibthorpian Professor of Rural Economy and Fellow of St John's College in the University of Oxford, 121 Banbury Road, Oxford	
1911	C.	* Sommerville, Duncan M'Laren Young, M.A., D.Sc., Professor of Pure and Applied Mathematics, Victoria College, Wellington, New Zealand	
1882		Sorley, James, 73 Onslow Square, London, S.W. 7 525	
1896		* Spence, Frank, M.A., B.Sc., 25 Craiglea Drive, Edinburgh	
1874	C.	Sprague, T. B., M.A., LL.D., Actuary, West Holme, Woldingham, Surrey	1885–87.
1906		Squance, Major Thomas Coke, M.D., M.S., F.R.M.S., F.S.A.Scot., Physician and Pathologist in the Sunderland Infirmary, President Sunderland Antiquarian Society, Sunderland Naturalists' Association, 13 Esplanade West, Sunderland	
1891		Stanfield, Richard, Professor of Mechanics and Engineering in the Heriot-Watt College, Edinburgh	
1915		* Steggall, John Edward Aloysius, M.A., Professor of Mathematics at University College, Dundee, in St Andrews University, Woodend, Perth Road, Dundee 530	
1912	C.	Stephenson, John, M.B., D.Sc. (Lond.), Indian Medical Service, Professor of Biology, Government College, Lahore, India	
1910		* Stephenson, Thomas, F.C.S., Editor of the *Prescriber*, Examiner to the Pharmaceutical Society, 6 South Charlotte Street, Edinburgh	
1916		* Steuart, D. R., F.I.C., Chemist to the Broxburn Oil Company, Osborne Cottage, Broxburn	
1886	C.	Stevenson, Charles A., B.Sc., M.Inst.C.E., 28 Douglas Crescent, Edinburgh	
1884		Stevenson, David Alan, B.Sc., M.Inst.C.E., 84 George Street, Edinburgh 535	
1919		* Stevenson, David Alan, B.Sc., A.M.Inst.C.E., Captain R.M.E., seconded to Admiralty, 28 Douglas Crescent, Edinburgh	
1888	C.	Stewart, Charles Hunter, D.Sc., M.B., C.M., Professor of Public Health in the University of Edinburgh, Usher Institute of Public Health, Warrender Park Road, Edinburgh	
1902		* Stockdale, Herbert Fitton, LL.D., Director of the Royal Technical College, Glasgow, Clairinch, Upper Helensburgh, Dumbartonshire	
1889	C.	Stockman, Ralph, M.D., F.R.C.P.E., Professor of Materia Medica and Therapeutics in the University of Glasgow	1903-05
1906		Story, Fraser, Professor of Forestry, University College, Bangor, North Wales 540	
1907		* Strong, John, C.B.E., M.A., LL.D., Professor of Education in the University of Leeds	
1903		Sutherland, David W., M.D., M.R.C.P., Captain, Indian Medical Service, Professor of Pathology and Materia Medica, Medical College, Lahore, India	
1905		Swithinbank, Harold William, Denham Court, Denham, Bucks	
1912		* Syme, William Smith, M.D. (Edin.), 11 Lynedoch Crescent, Glasgow	
1885	C.	Symington, Johnson, M.D., LL.D., F.R.C.S., F.R.S., formerly Professor of Anatomy in the Queen's University of Belfast 545	1892–93.
1917	C.	* Tait, John, D.Sc., M.D., Professor of Physiology, M'Gill University, Montreal, Canada	
1904		* Tait, John W., B.Sc., Rector of Leith Academy, 18 Netherby Road, Leith	
1898	C.	Tait, William Archer, D.Sc., M.Inst.C.E., 72A George Street, Edinburgh (Society's Representative on George Heriot's Trust)	1914–17, 1918–
1895		Talmage, James Edward, D.Sc., Ph.D., F.R.M.S., F.G.S., Professor of Geology, University of Utah, Salt Lake City, Utah, U.S.A.	
1890	C.	Tanakadate, Aikitu, Professor of Natural Philosophy in the Imperial University of Japan, Tokyo, Japan 550	
1870		Tatlock, Robert R., F.C.S., City Analyst's Office, 156 Bath Street, Glasgow	
1899		* Taylor, James, M.A., Mathematical Master in the Edinburgh Academy	
1917	C.	* Taylor, William White, M.A., D.Sc., Lecturer on Chemical Physiology, University, Edinburgh, Park Villa, Liberton, Edinburgh	
1892		Thackwell, J. B., M.B., C.M., 423A Battersea Park Road, London, S.W.	
1885	C.	Thompson, D'Arcy W., C.B., D.Litt., F.R.S., Professor of Natural History, University, St Andrews, 44 South Street, St Andrews 555	* 1892–95, 1896–99, 1907–10, 1912–15. V-P 1916–19.

Date of Election,			Service on Council etc.
1917	C.	* Thompson, John M'Lean, M.A., D.Sc., F.L.S., Lecturer in Plant Morphology, Department of Botany, University, Glasgow, 2 Second Avenue, King's Park, Cathcart, Glasgow	
1905		* Thoms, Alexander, 7 Playfair Terrace, St Andrews	
1887		Thomson, Andrew, M.A., D.Sc., F.I.C., 17 Riselaw Road, Edinburgh	
1896		* Thomson, George Ritchie, M.B., C.M., General Hospital, Johannesburg, Transvaal	
1903		Thomson, George S., F.C.S., Ferma Albion, Marculesci, Roumania 560	
1906		* Thomson, Gilbert, M.Inst.C.E., 164 Bath Street, Glasgow	
1887	C.	Thomson, J. Arthur, M.A., LL.D., Regius Professor of Natural History in the University of Aberdeen	1906–08.
1906	C.	Thomson, James Stuart, M.Sc., Ph.D., Zoological Department, University, Manchester	
1880		Thomson, John Millar, LL.D., F.R.S., Professor of Chemistry in King's College, London, Rose Lynn, Havant Road, Emsworth, Hants	
1899		* Thomson, R. Tatlock, F.C.S., 156 Bath Street, Glasgow 565	
1912	C.	Thomson, Robert Black, M.B. Edin., Professor of Anatomy, The University, Cape Town	
1870		Thomson, Spencer C., Actuary, 10 Eglinton Crescent, Edinburgh	
1882		Thomson, Wm., M.A., B.Sc., LL.D., Registrar, University of South Africa, Somerset House, Vermeulen Street, Pretoria	
1876	C.	Thomson, William, Royal Institution, Manchester	
1917		* Thorneycroft, Wallace, Coal and Iron Master, Plean House, Plean, Stirling 570	
1917		* Tovey, Donald Francis, B.A., Professor of Music, University, Edinburgh, 2 St Margaret's Road, Edinburgh	
1914		Tredgold, Alfred Frank, M.D. (Durham), L.R.C.P., M.R.C.S., Hon. Consulting Physician to National Association for the Feeble-minded, 6 Dapdune Crescent, Guildford, Surrey	
1915		* Trotter, George Clark, M.D., Ch.B. Edin., D.P.H. (Aberdeen), Medical Officer of Health, Paisley, Remuera, Paisley	
1905		* Turner, Arthur Logan, M.D., F.R.C.S.E., 27 Walker Street, Edinburgh	
1906	C.	* Turner, Dawson F. D., B.A., M.D., F.R.C.P.E., M.R.C.P., Lecturer on Medical Physics, Surgeons' Hall, Physician in charge of Radium Treatment, Royal Infirmary, Edinburgh, 37 George Square, Edinburgh 575	
1895		Turton, Albert H., M.I.M.M., 233 George Road, Erdington, Birmingham	
1898	C.	* Tweedie, Charles, M.A., B.Sc., Lecturer on Mathematics in the University of Edinburgh, Duns, Berwickshire	
1918	C.	* Tyrrell, G. W., A.R.C.Sc., F.G.S., Chief Assistant and Lecturer in Petrology, Geological Department, University, Glasgow	
1910		Vincent, Swale, M.D. Lond., D.Sc. Edin., etc., Professor of Physiology, University of Manitoba, Winnipeg, Canada	
1891	C. B.	Walker, James, D.Sc., Ph.D., LL.D., F.R.S. Professor of Chemistry in the University of Edinburgh, 5 Wester Coates Road, Edinburgh 580	1903–05, 1910–13. V-P 1916–19.
1873	C.	Walker, Robert, M.A., LL.D., University, Aberdeen	
1902		* Wallace, Alexander G., M.A., 56 Fonthill Road, Aberdeen	
1886	C.	Wallace, R., F.L.S., Professor of Agriculture and Rural Economy in the University of Edinburgh	
1898		Wallace, Wm., M.A., Belvedere, Alberta, Canada	
1891		Walmsley, R. Mullineux, D.Sc., Principal of the Northampton Institute, Clerkenwell, London 585	
1901	C.	* Waterston, David, M.A., M.D., F.R.C.S.E., Professor of Anatomy, University, St Andrews	1916–19.
1911		* Watson, James A. S., B.Sc., etc., Assistant in Agriculture, University of Edinburgh, 30 Mayfield Terrace, Edinburgh	
1900		* Watson, Thomas P., M.A., B.Sc., Principal, Keighley Institute, Keighley	
1907		* Watt, Andrew, M.A., Secretary to the Scottish Meteorological Society, 6 Woodburn Terrace, Edinburgh	1912–14.
1911		Watt, James, W.S., F.F.A., 24 Rothesay Terrace, Edinburgh 590	
1911		* Watt, Rev. Lauchlan Maclean, B.D., Minister of St Stephen's Parish, 7 Royal Circus, Edinburgh	
1896		Webster, John Clarence, B.A., M.D., F.R.C.P.E., Professor of Obstetrics and Gynæcology, Rush Medical College, Shediac, N.B., Canada	
1907	B. C.	* Wedderburn, Ernest Maclagan, M.A., LL.B., W.S., D.Sc., 6 Succoth Gardens, Edinburgh	1913–16.
1903	C.	* Wedderburn, J. H. Maclagan, M.A., D.Sc., P.O. Box 53, Princeton, N.J., U.S.A.	

Date of Election.			Service on Council, etc.
1904		Wedderspoon, William Gibson, M.A., LL.D., Indian Educational Service, Senior Inspector of Schools, Burma, The Education Office, Rangoon, Burma 595	
1896		Wenley, Robert Mark, M.A., D.Sc., D.Phil., Litt.D., LL.D., D.C.L., Professor of Philosophy in the University of Michigan, Ann Arbor, U.S.A.	
1909	C.	* Westergaard, Reginald Ludovic Andreas Emil, Ph.D., formerly Professor of Technical Mycology, Heriot-Watt College, Elmscroft, Lundin Links, Fife	
1916		* White, J. Martin, Esq., of Balruddery, Balruddery, near Dundee	
1896	C.	White, Philip J., M.B., Professor of Zoology in University College, Bangor, North Wales	
1911		* Whittaker, Charles Richard, F.R.C.S. (Edin.), F.S.A. (Scot.), Lynwood, Hatton Place, Edinburgh 600	
1912	C.	* Whittaker, Edmund Taylor, Sc.D., F.R.S., Professor of Mathematics in the University of Edinburgh (SECRETARY), 35 George Square, Edinburgh	1912–15. Sec. 1916–
1918		* Whyte, Rev. Charles, M.A., LL.D., F.R.A.S., U.F. Church Manse, Kingswells, Aberdeen	
1918		* Wight, John Thomas, General Manager, Hydraulic Gears, Ltd., Beavor Lane, Hammersmith, London, W. 6; Dartbeigh, Ascot, Berks.	
1879		Will, John Charles Ogilvie, of Newton of Pitfodels, M.D., 17 Bon-Accord Square, Aberdeen	
1908		* Williamson, Henry Charles, M.A., D.Sc., Naturalist to the Fishery Board for Scotland, Marine Laboratory, Aberdeen 605	
1910	C.	* Williamson, William, F.L.S., 79 Morningside Drive, Edinburgh	
1900		Wilson, Alfred C., F.C.S., Voewood Croft, Stockton-on-Tees	
1911		* Wilson, Andrew, M.Inst.C.E., 66 Netherby Road, Trinity, Edinburgh	
1902		* Wilson, Charles T. R., M.A., F.R.S., 14 Cranmer Road, Cambridge, Sidney Sussex College, Cambridge	
1895		Wilson-Barker, David, R.N.R., F.R.G.S., late Captain-Superintendent Thames Nautical Training College, H.M.S. "Worcester," off Greenhithe, Kent, Flimwell Grange, near Hawkhurst, Kent 610	
1882		Wilson, George, M.A., M.B., LL.D.	
1908		* Wood, Thomas, M.D., Eastwood, 182 Ferry Road, Bonnington, Leith	
1886	C.	Woodhead, Sir German Sims, K.B.E., M.D., F.R.C.P.E., Professor of Pathology in the University of Cambridge	1887–90.
1884		Woods, G. A., M.R.C.S., 1 Hammelton Road, Bromley, Kent	
1911		* Wrigley, Ruric Whitehead, B.A. (Cantab.), Assistant Astronomer, Royal Observatory, Edinburgh 615	
1890		Wright, Johnstone Christie, Conservative Club, Edinburgh	
1896		* Wright, Sir Robert Patrick, LL.D., Chairman of the Board of Agriculture for Scotland, Kingarth, Colinton, Midlothian	
1882		Young, Frank W., F.C.S., H.M. Inspector of Science and Art Schools, 32 Buckingham Terrace, Botanic Gardens, Glasgow	
1892		Young, George, Ph.D., "Bradda," Church Crescent, Church End, Finchley, London, N.	
1896	C.	* Young, James Buchanan, M.B., D.Sc., Dalveen, Braeside, Liberton 620	
1904		Young, R. B., M.A., D.Sc., F.G.S., Professor of Geology and Mineralogy in the South African School of Mines and Technology, Johannesburg, Transvaal 621	

LIST OF HONORARY FELLOWS OF THE SOCIETY

At January 15, 1920.

HIS MOST GRACIOUS MAJESTY THE KING.

FOREIGNERS (LIMITED TO THIRTY-SIX BY LAW I).

Elected

1916 Charles Barrois, Professor of Geology and Mineralogy, Université, Lille, France: 41 rue Pascal, Lille.

1905 Waldemar Christofer Brögger, Professor of Mineralogy and Geology, K. Frederiks Universitet, Christiania, Norway.

1916 Douglas Houghton Campbell, Professor of Botany, Leland Stanford Junior University, California, U.S.A.

1910 Hugo de Vries, Professor of Plant Anatomy and Physiology, Lunteren, Holland.

1916 Marcel Eugène Emile Gley, Professor of Physiology, Collège de France, Paris, Membre de l'Académie de Médecine, Paris: 14, rue Monsieur le Prince, Paris.

1910 Karl F. von Goebel, Professor of Botany, Universität, München, Germany.

1916 Camillo Golgi, Professor of Pathology, Università, Pavia, Italy.

1916 William Crawford Gorgas, Major-General, Surgeon General, U.S. Army, War Department, Washington

1916 Gio. Battista Grassi, Professor of Comparative Anatomy, Regia Università, Roma, Italy: Via Agostino Depretis N. 91, Rome.

1905 Paul Heinrich von Groth, Professor of Mineralogy, Universität, München, Germany.

1913 George Ellery Hale, Director of Mount Wilson Solar Observatory (Carnegie Institution of Washington), Pasadena, California, U.S.A.

1883 Julius Hann, Emeritus Professor of Cosmical Physics, Universität, Wien, Austria.

1910 Jacobus Cornelius Kapteyn, Professor of Astronomy, Universiteit, Groningen, Holland.

1897 Gabriel Lippmann, Professor of Physics, Université, Paris, France.

1895 Carl Menger, Professor of Political Economy, Universität, Wien, Austria: Wien ix, Fuchstallerg 2, Austria.

1910 Albert Abraham Michelson, Professor of Physics, University, Chicago, U.S.A.

1897 Fridtjof Nansen, Professor of Oceanography, K. Frederiks Universitet, Christiania, Norway.

1908 Henry Fairfield Osborn, Professor of Zoology, Columbia University and American Museum of Natural History, New York, N.Y., U.S.A.

1908 Ivan Petrovitch Pawlov, Emeritus Professor of Physiology, Kais. Inst. Exper. Med., Petrograd: Wedenskaja Strasse 4, Petrograd, Russia.

1889 Georg Hermann Quincke, Emeritus Professor of Physics, Bergstrasse 41, Heidelberg, Germany.

1913 Santiago Ramón y Cajal, Professor of Histology and Pathological Anatomy, Universidad, Madrid, Spain.

1908 Augusto Righi, Professor of Experimental Physics, Regia Università, Bologna, Italy.

1913 Vito Volterra, Professor of Mathematical Physics, Regia Università, Rome, Italy.

1916 Eugenius Warming, Emeritus Professor of Botany at the University of Copenhagen and Director of the Botanical Garden: Bjerregaardsvej 5, Copenhagen, Valby.

Total, 24.

BRITISH SUBJECTS (LIMITED TO TWENTY BY LAW I).

1916 Sir Francis Darwin, Kt., D.Sc., M.B., F.R.S., Hon. Fellow, Christ's College, Cambridge, 10 Madingley Road, Cambridge.

1900 Sir David Ferrier, Kt., M.A., M.D., LL.D., F.R.S., Emer.-Professor of Neuro-Pathology, King's College, London, 34 Cavendish Square, London, W.

1900 Andrew Russell Forsyth, M.A., Sc.D., LL.D., Math.D., F.R.S., Chief Professor of Mathematics in the Imperial College of Science and Technology, London, formerly Sadlerian Professor of Pure Mathematics in the University of Cambridge, Imperial College of Science and Technology, London, S.W.

1910 Sir James George Frazer, D.C.L., LL.D., Litt.D., F.B.A., Fellow of Trinity College, Cambridge, Professor of Social Anthropology in the University of Liverpool, Trinity College, Cambridge.

1916 James Whitbread Lee Glaisher, M.A., Sc.D., F.R.S., Fellow of Trinity College, Cambridge.

1908 Sir Alexander B. W. Kennedy, Kt., LL.D., F.R.S., Past Pres. Inst. C.E., A7, Albany, Piccadilly, London, W.

Elected

1913 Horace Lamb, M.A., Sc.D., D.Sc., LL.D., F.R.S., Professor of Mathematics in the University of Manchester.

1916 John Newport Langley, Sc.D., LL.D., F.R.S., Fellow of Trinity College, Professor of Physiology in the University of Cambridge, Hedgerley Lodge, Madingley Road, Cambridge.

1908 Sir Edwin Ray Lankester, K.C.B., LL.D., F.R.S., 29 Thurloe Place, S. Kensington, London, S.W.

1916 Charles Lapworth, F.R.S., LL.D., M.Sc., F.G.S., Emeritus Professor of Geology and Physiography in the University of Birmingham, 38 Calthorpe Road, Edgbaston, Birmingham.

1910 Sir Joseph Larmor, Kt., M.A., D.Sc., LL.D., D.C.L., F.R.S., M.P. University of Cambridge since 1911, Lucasian Professor of Mathematics in the University of Cambridge, St John's College, Cambridge.

1900 Archibald Liversidge, M.A., LL.D., F.R.S., Emer.-Professor of Chemistry in the University of Sydney, Fieldhead, Coombe Warren, Kingston, Surrey.

1916 Alexander Macalister, M.A., M.D., LL.D., F.R.S., F.S.A., Fellow of St John's College, Cambridge, Professor of Anatomy in the University of Cambridge.

1916 Sir Arthur Schuster, Ph.D., D.Sc., LL.D., D. ès Sc. Geneva, Secretary of the Royal Society, London, Emer.-Professor of Physics in the University of Manchester.

1908 Charles Scott Sherrington, M.A., M.D., LL.D., F.R.S., Waynflete Professor of Physiology in the University of Oxford, Physiological Laboratory, Oxford.

1913 Sir William Turner Thiselton-Dyer, K.C.M.G., C.I.E., M.A., LL.D., F.R.S., formerly Director of the Royal Botanic Gardens, Kew: The Ferns, Witcombe, Gloucester.

1905 Sir Joseph John Thomson, D.Sc., LL.D., F.R.S., Cavendish Professor of Experimental Physics, University of Cambridge, Trinity College, Cambridge.

1900 Sir Thomas Edward Thorpe, Kt., C.B., D.Sc., LL.D., F.R.S., formerly Principal of the Government Laboratories, Imperial College of Science and Technology, South Kensington, London, S.W., Whinfield, Salcombe, South Devon.

Total, 18.

CHANGES IN FELLOWSHIP DURING SESSION 1918–19.

ORDINARY FELLOWS OF THE SOCIETY ELECTED.

ARTHUR R. CUSHNY, M.A., M.D., LL.D., F.R.S.

WILLIAM JOHN DUNDAS, W.S., LL.D.

ROBERT OWEN MORRIS, M.A., M.D., C.M., D.P.H.

THOMAS STEWART PATTERSON, D.Sc., Ph.D.

B. D. PORRITT, M.Sc. (Lond.).

ALFRED H. ROBERTS, O.B.E., M.Inst.C.E.

WILLIAM ALEXANDER ROBERTSON, F.F.A.

ALEXANDER SCOTT, M.A., D.Sc.

ALEXANDER RITCHIE SCOTT, B.Sc. (Edin.), D.Sc. (Lond.).

WILLIAM WRIGHT SMITH, M.A. (Edin.).

DAVID ALAN STEVENSON, B.Sc., A.M.Inst.C.E.

ORDINARY FELLOWS DECEASED.

ARCHIBALD C. ADAMS, A.M.Inst.Mech.E., A.M.Inst.E.E.

J. MACKAY BERNARD, of Dunsinnan, B.Sc.

THOMAS FAIRLEY, F.I.C.

W. S. GREENFIELD, M.D., F.R.C.P.E., LL.D.

W. LAMOND HOWIE, F.C.S.

GEORGE W. JONES, M.A., B.Sc., LL.B.

ANDREW KING, M.A., F.I.C.

SIR J. H. A. MACDONALD, P.C., G.C.B., K.C., LL.D., F.R.S., M.Inst.E.E.

R. C. MACLAGAN, M.D., F.R.C.P.E.

R. SYDNEY MARSDEN, M.D., C.M., D.Sc., D.P.H., F.I.C., M.R.I.A.

SIR MITCHELL MITCHELL-THOMSON, BART.

FREDERICK PHILLIPS, M.Sc.

SIR BOVERTON REDWOOD, BART., D.Sc., F.I.C., F.C.S., A.Inst.C.E.

EDWIN O. SACHS.

SIR JAMES SAWYER, KT., M.D.

E. WYNSTON WATERS.

LLIAM WRIGHT WILSON, F.R.C.S.E., M.R.C.S.

HONORARY FELLOWS DECEASED.

EMIL FISCHER.

EDWARD CHARLES PICKERING.

LORD RAYLEIGH, O.M., P.C., D.C.L., LL.D., D.Sc., F.R.S.

MAGNUS GUSTAF RETZIUS.

Additions to Library by Gift or Purchase.

The Application of Meteorology to Gunnery. Experimental Establishment, Shoeburyness. Fol. August 1918. (*Presented by Capt. E. M. Wedderburn.*)

Arsenikkommissionen. Betänkande avgivet av Sakkunniga . . . Upptäcka, Förebygga och Motverka Faran av Kronisk Arsenik förgiftning. Bib. i–xix. 4to. Lund, 1919. (*Presented.*)

Astrographic Catalogue, 1900·0. Hyderabad Section, Dec. −16° to −21°. From photographs taken and measured at the Nizamiah Observatory, Hyderabad, under the direction of R. J. Pocock. Vol. II. Measures of Rectangular Co-ordinates and Diameters of 61378 Star-Images on plates with centres in Dec. −18°. 4to. Edinburgh, 1918. (*Presented by the Nizamiah Observatory.*)

Belot (Émile). L'Origine des Formes de la Terre et des Planètes. 8vo. Paris, 1918. (*Presented.*)

A Bibliography of Indian Geology and Physical Geography. With an Annotated Index of Minerals of Economic Value. Compiled by T. H. D. La Touche. La. 8vo. Calcutta, 1917. (*Presented.*)

Black (F. A.). Planetary Rotation Periods and Group Ratios. 8vo. Edinburgh and London, 1919. (*Presented by the Author.*)

The Carmichael Lectures, 1918. Lectures on the Ancient History of India . . . . . By D. R. Bhandarkar. 8vo. Calcutta, 1919. (*Presented by the University of Calcutta.*)

Carnegie (Col. David). The Promotion of Co-operation between Employers and Employed. 8vo. London, 1919. (*Presented by the Author.*)

Carnegie Endowment for International Peace. (Division of Economics and History): The Industrial Development and Commercial Policies of the Three Scandinavian Countries. By Pool Drachmann. La. 8vo. London, Edinburgh, New York, Toronto, Melbourne, and Bombay, 1915.

——— Epidemics Resulting from Wars. By Dr Friedrich Prinzing.

——— The Colonial Tariff Policy of France. By Arthur Girault.

——— Economic Protectionism. By Josef Grunzel. La. 8vo. 1916. (*Presented.*)

Catalogue des Coléoptères de la Région Malgache décrits ou mentionnés par L. Fairmaire (1849–1906), par le Docteur René Marie et Pierre Lesne. 8vo. Paris, 1917. (*Presented by the Muséum National d'Histoire Naturelle.*)

Chaudhuri (T. C.) Modern Chemistry and Chemical Industry of Starch and Cellulose. 8vo. Calcutta, 1918. (*Presented.*)

Department of Scientific and Industrial Research. Advisory Council. Bulletin No. 3. A Study on the Performance of "Night-Glasses." By L. C. Martin. 8vo. London, 1919.

——— Report of the Food Investigation Board for the Year 1918. 8vo. London, 1919.

——— Food Investigation Board. Special Report No. 2. The Literature of Refrigeration. 8vo. London, 1919. (*Presented.*)

Disclosures from Germany. I. The Lichnowsky Memorandum. II. Memoranda and Letters of Dr Muehlon. III. The Dawn in Germany? 8vo. New York, 1918. (*Presented by the American Association for International Conciliation.*)

Escherich (Prof. Dr Karl). Die Bekämpfung Schädlicher Insekten. 8vo. Frankfurt a M., 1919. (*Presented.*)

Festskrift utgiven av Lunds Universitet vid dess Tvåhundrafemtioårs-jubileum, 1918. 4to, 8vo. Lund, 1918. (*Presented.*)

Galloway (T. Lindsay). A Method of determining the Magnetic Meridian as a Basis for Mining Surveys. 8vo. London, 1919. (*Presented by the Author.*)

Gedenkboek van het Bataafsch Genootschap der Proefondervindelijke Wijsbegeerte te Rotterdam, 1769–1919. 4to. Rotterdam, 1919. (*Presented.*)

Halkyard (Edward). The Fossil Foraminifera of the Blue Marl of the Côte des Basques, Biarritz. (From vol. lxii, pt. ii, of *Memoirs and Proceedings of the Manchester Literary and Philosophical Society.*) 8vo. Manchester, 1919. (*Presented.*)

Herdman (W. A.). Spolia Runiana. III. Distribution of certain Diatoms and Copepoda throughout the year. (Extracted from *Linnean Society's Journal: Botany*, vol. xliv; and *Zoology*, vol. xxxiv, 1918.) (*Presented by the Author.*)

Kaye (G. R.). The Astronomical Observatories of Jai Singh. 4to. Calcutta, 1918. (*Presented by the Archæological Society of India.*)

Lepper (George Henry). From Nebula to Nebula, or the Dynamics of the Heavens. 4th edition. 8vo. Pittsburgh, Pa., 1919. (*Presented by the Author.*)

Lister (Gulielma). The Mycetozoa: A short history of their study in Britain. *Essex Field Club Special Memoirs*, vol. vi. 8vo. Stafford, Essex, 1918. (*Presented.*)

The Ministry of Munitions Journal. Nos. 1–25, Dec. 1916–Dec. 1918. 4to. London, 1916–1918. (*Presented by the Editorial Committee of the Ministry of Munitions Journal.*)

Munitions Inventions Department. Report on the Investigations carried out by the Chemical Waste-Products Committee. 4to. 1919. (*Presented.*)

Munro (Robert). From Darwinism to Kaiserism. 8vo. Glasgow, 1919. (*Presented by the Author.*)

Notes Ptéridologiques. Fasc. v and vii. Par Le Prince Bonaparte. 8vo. Paris, 1917, 1918. (*Presented by the Author.*)

Petrovitch (Michel). Les Spectres Numériques. 8vo. Paris, 1919. (*Presented by the Author.*)

Philip (Alex.). Calendar Reform considered with reference to the Practical Requirements of Science, of Civil Life, and of the Church. 8vo. Brechin, 1919. (*Presented by the Author.*)

Preller (C. S. Du Riche). The Ancient Sea and Lake Basins of Central Italy. (Reprinted from *The Scottish Geographical Magazine*, vol. xxxv, May 1919. (*Presented by the Author.*)

——— Dalmatia. (Reprint from *The Scottish Geographical Magazine*, vol. xxxiv, Dec. 1918.)

Preller (C. S. Du Riche). Physiographic Analogies between Scotland and Italy. (Reprint from *The Scottish Geographical Magazine*, vol. xxxiv, Nov. 1918.) (*Presented by the Author.*)

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Ventosa (Vicente). Reflexiones acerca de la Resolucion de las Ecuaciones Algébricas Numéricas por el Método de Graffe. 2 pts. 8vo. Madrid, 1917, 1919. (*Presented by the Author.*)

Very (Frank W.). What Becomes of the Light of the Stars? (Reprinted from the *Popular Science Monthly*, March 1913.)

——— The Luminiferous Ether. I. Its Relation to the Electron and to a Universal Interstellar Medium. II. Its Relation to the Atom. 8vo. Boston, 1919. (*Presented.*)

Vialay (Alfred). Essai sur la Genèse et l'Evolution des Roches. Compléments. 8vo. Paris, 1918. (*Presented by the Author.*)

Wells (John Edwin). First Supplement to a Manual of the Writings in Middle English, 1050–1400. Published under the Auspices of the Connecticut Academy of Arts and Sciences. 8vo. New Haven, 1919. (*Presented.*)

Williamson (H. Chas.). On the Transport of Herring Spawn to the Southern Hemisphere. (From *The Annals of Applied Biology*, vol. v, No. 2, October 1918.) (*Presented by the Author.*)

LAWS OF THE SOCIETY.

Adopted July 3, 1916; amended December 18, 1916.

I.

THE ROYAL SOCIETY OF EDINBURGH, which was instituted by Royal Charter in 1783 for the promotion of Science and Literature, shall consist of Ordinary Fellows (hereinafter to be termed Fellows) and Honorary Fellows. The number of Honorary Fellows shall not exceed fifty-six, of whom not more than twenty may be British subjects, and not more than thirty-six subjects of Foreign States.

Fellows only shall be eligible to hold office or to vote at any Meeting of the Society.

ELECTION OF FELLOWS.

II.

Each Candidate for admission as a Fellow shall be proposed by at least four Fellows, two of whom must certify from personal knowledge. The Official Certificate shall specify the name, rank, profession, place of residence, and the qualifications of the Candidate. The Certificate shall be delivered to the General Secretary before the 30th of November, and, subject to the approval of the Council, shall be exhibited in the Society's House during the month of January following. All Certificates so exhibited shall be considered by the Council at its first meeting in February, and a list of the Candidates approved by the Council for election shall be issued to the Fellows not later than the 21st of February.

III.

The election of Fellows shall be by Ballot, and shall take place at the first Ordinary Meeting in March. Only Candidates approved by the Council shall be eligible for election. A Candidate shall be held not elected, unless he is supported by a majority of two-thirds of the Fellows present and voting.

IV.

On the day of election of Fellows two scrutineers, nominated by the President, shall examine the votes and hand their report to the President, who shall declare the result.

V.

Each Fellow, after his election, is expected to attend an Ordinary Meeting, and sign the Roll of Fellows, he having first made the payments required by Law VI. He shall be introduced to the President, who shall address him in these words :

> *In the name and by the authority of THE ROYAL SOCIETY OF EDINBURGH, I admit you a Fellow thereof.*

PAYMENTS BY FELLOWS.

VI.

Each Fellow shall, before he is admitted to the privileges of Fellowship, pay an admission fee of two guineas, and a subscription of two guineas for the year of election. He shall continue to pay a subscription of two guineas at the beginning of each session so long as he remains a Fellow. A Fellow may compound for these contributions by a single payment of forty guineas, or on such other terms as the Council may from time to time fix.*

VII.

A Fellow who, after application made by the Treasurer, fails to pay any contribution due by him shall be reported to the Council, and, if the Council see fit, shall be declared no longer a Fellow. Notwithstanding such declaration all arrears of contributions shall remain exigible.

ELECTION OF HONORARY FELLOWS.

VIII.

Honorary Fellows shall be persons eminently distinguished in Science or Literature. They shall not be liable to contribute to the Society's Funds. Personages of the Blood Royal may be elected Honorary Fellows without regard to the limitation of numbers specified in Law I.

* Law VI does not apply to Fellows elected before 1917, whose terms of Fellowship are determined by the previously existing Laws.

IX.

Honorary Fellows shall be proposed by the Council. The nominations shall be announced from the Chair at the first Ordinary Meeting in June. The names shall be printed in the circular for the second Ordinary Meeting in June. The election shall be by Ballot, and shall take place at the first Ordinary Meeting in July after the manner prescribed in Laws III and IV for the election of Fellows.

EXPULSION OF FELLOWS.

X.

If, in the opinion of the Council, the conduct of any Fellow is injurious to the character or interests of the Society, the Council may, by registered letter, request him to resign. If he fail to do so within one month of such request, the Council shall call a Special Meeting of the Society to consider the matter. If a majority consisting of not less than two-thirds of the Fellows present and voting decide for expulsion, he shall be expelled by declaration from the Chair, his name shall be erased from the Roll, and he shall forfeit all right or claim in or to the property of the Society.

XI.

It shall be competent for the Council to remove any person from the Roll of Honorary Fellows if, in their opinion, his remaining on the Roll would be injurious to the character or interests of the Society. Reasonable notice of such proposal shall be given to each member of the Council, and, if possible, to the Honorary Fellow himself. Thereafter the decision on the question shall not be taken until the matter has been discussed at two Meetings of Council, separated by an interval of not less than fourteen days. A majority of two-thirds of the members present and voting shall be required for such removal.

MEETINGS OF THE SOCIETY.

XII.

A Statutory Meeting for the election of Council and Office-Bearers, for the presentation of the Annual Reports, and for such other business as may be arranged by the Council, shall be held on the fourth Monday of October. Each Session of the Society shall begin at the date of the Statutory Meeting.

XIII.

Meetings for reading and discussing communications and for general business, herein termed Ordinary Meetings, shall be held on the first and third Mondays of each month, from November to March and from May to July, inclusive,

with the exception that in January the Meetings shall be held on the second and fourth Mondays.

The Council shall have power to alter the date of any Ordinary Meeting, if it appears to them conducive to the interests of the Society.

XIV.

A Special Meeting of the Society may be called at any time by direction of the Council, or on a requisition to the Council signed by not fewer than six Fellows. The date and hour of such Meeting shall be determined by the Council, who shall give not less than seven days' notice of such Meeting. The notice shall state the purpose for which the Special Meeting is summoned; no other business shall be transacted.

PUBLICATION OF PAPERS.

XV.

The Society shall publish Transactions and Proceedings. The consideration of the acceptance, reading, and publication of papers is vested in the Council, whose decision shall be final. Acceptance for reading shall not necessarily imply acceptance for publication.

DISTRIBUTION OF PUBLICATIONS.

XVI.

Fellows who are not in arrear with their Annual Subscriptions and all Honorary Fellows shall be entitled gratis to copies of the Parts of the Transactions and the Proceedings published subsequently to their admission.

Copies of the Parts of the Proceedings shall be distributed by post or otherwise, as soon as may be convenient after publication; copies of the Transactions or Parts thereof shall be obtainable upon application, either personally or by an authorised agent, to the Librarian, provided the application is made within five years after the date of publication.

CONSTITUTION OF COUNCIL.

XVII.

The Council shall consist of a President, six Vice-Presidents, a Treasurer, a General Secretary, two Secretaries to the Ordinary Meetings (the one representing the Biological group and the other the Physical group of Sciences),* a Curator of the Library and Museum, and twelve ordinary members of Council.

* The Biological group includes Anatomy, Anthropology, Botany, Geology, Pathology, Physiology, Zoology; the Physical group includes Astronomy, Chemistry, Mathematics, Metallurgy, Meteorology, Physics.

ELECTION OF COUNCIL.

XVIII.

The election of the Council and Office-Bearers for the ensuing Session shall be held at the Statutory Meeting on the fourth Monday of October. The list of the names recommended by the Council shall be issued to the Fellows not less than one week before the Meeting. The election shall be by Ballot, and shall be determined by a majority of the Fellows present and voting. Scrutineers shall be nominated as in Law IV.

XIX.

The President may hold office for a period not exceeding five consecutive years; the Vice-Presidents, not exceeding three; the Secretaries to the Ordinary Meetings, not exceeding five; the General Secretary, the Treasurer, and the Curator of the Library and Museum, not exceeding ten; and ordinary members of Council, not exceeding three consecutive years.

XX.

In the event of a vacancy arising in the Council or in any of the offices enumerated in Law XVII, the Council shall proceed, as soon as convenient, to elect a Fellow to fill such vacancy for the period up to the next Statutory Meeting.

POWERS OF THE COUNCIL.

XXI.

The Council shall have the following powers:—(1) To manage all business concerning the affairs of the Society. (2) To decide what papers shall be accepted for communication to the Society, and what papers shall be printed in whole or in part in the Transactions and Proceedings. (3) To appoint Committees. (4) To appoint employees and determine their remuneration. (5) To award the various prizes vested in the Society, in accordance with the terms of the respective deeds of gift, provided that no member of the existing Council shall be eligible for any such award. (6) To make from time to time Standing Orders for the regulation of the affairs of the Society. (7) To control the investment or expenditure of the Funds of the Society.

At Meetings of the Council the President or Chairman shall have a casting as well as a deliberative vote.

DUTIES OF PRESIDENT AND VICE-PRESIDENTS.

XXII.

The President shall take the Chair at Meetings of Council and of the Fellows. It shall be his duty to see that the business is conducted in accordance with the Charter and Laws of the Society. When unable to be present at any Meetings or attend to current business, he shall give notice to the General Secretary, in order that his place may be supplied. In the absence of the President his duties shall be discharged by one of the Vice-Presidents.

DUTIES OF THE TREASURER.

XXIII.

The Treasurer shall receive the monies due to the Society and shall make payments authorised by the Council. He shall lay before the Council a list of arrears in accordance with Rule VII. He shall keep accounts of all receipts and payments, and at the Statutory Meeting shall present the accounts for the preceding Session, balanced to the 30th of September, and audited by a professional accountant appointed annually by the Society.

DUTIES OF THE GENERAL SECRETARY.

XXIV.

The General Secretary shall be responsible to the Council for the conduct of the Society's correspondence, publications, and all other business except that which relates to finance. He shall keep Minutes of the Statutory and Special Meetings of the Society and Minutes of the Meetings of Council. He shall superintend, with the aid of the Assistant Secretary, the publication of the Transactions and Proceedings. He shall supervise the employees in the discharge of their duties.

DUTIES OF SECRETARIES TO ORDINARY MEETINGS.

XXV.

The Secretaries to Ordinary Meetings shall keep Minutes of the Ordinary Meetings. They shall assist the General Secretary, when necessary, in superintending the publication of the Transactions and Proceedings. In his absence, one of them shall perform his duties.

DUTIES OF CURATOR OF LIBRARY AND MUSEUM.

XXVI.

The Curator of the Library and Museum shall have charge of the Books, Manuscripts, Maps, and other articles belonging to the Society. He shall keep the Card Catalogue up to date. He shall purchase Books sanctioned by the Council.

ASSISTANT-SECRETARY AND LIBRARIAN.

XXVII.

The Council shall appoint an Assistant-Secretary and Librarian, who shall hold office during the pleasure of the Council. He shall give all his time, during prescribed hours, to the work of the Society, and shall be paid according to the determination of the Council. When necessary he shall act under the Treasurer in receiving subscriptions, giving out receipts, and paying employees.

ALTERATION OF LAWS.

XXVIII.

Any proposed alteration in the Laws shall be considered by the Council, due notice having been given to each member of Council. Such alteration, if approved by the Council, shall be proposed from the Chair at the next Ordinary Meeting of the Society, and, in accordance with the Charter, shall be considered and voted upon at a Meeting held at least one month after that at which the motion for alteration shall have been proposed.

INDEX.

Index of Papers published in the "Transactions" during Session 1918–19.

(Arranged under the Authors' Names.)

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MODEL INDEX.

Schäfer, E. A.—On the Existence within the Liver Cells of Channels which can be directly injected from the Blood-vessels. Proc. Roy. Soc. Edin., vol. 1902, pp.

Cells, Liver,—Intra-cellular Canaliculi in.
E. A. Schäfer. Proc. Roy. Soc. Edin., vol. , 1902, pp. .

Liver,—Injection within Cells of.
E. A. Schäfer. Proc. Roy. Soc. Edin., vol. , 1902, pp. .

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